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RESEARCH ARTICLE

A Heuristic Algorithm for Single Machine Scheduling Problem with OR-Precedence Constraints

Siamak.Kheybari[1], Reza Salehpour*[2], mahmoud moradi[3], Mohammad Mohammadpour omran[4]

1. Department of Industrial Engineering, Shomal University, Amol, Iran.
2. Department of Physics, Sharif University of Technology, Tehran, Iran.
3. Department of Management, University of Guilan, Rasht, Iran.
4. Department of Industrial Engineering, Iran University of Science and Technology, Tehran, Iran.

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Corresponding e-mail:
rezasalehpour@gmail.com

This paper introduces the single machine scheduling problem in which the jobs are dependent to each other via the OR-precedence constraints. In the literature basic problem is proved to be classified in to the NP-Hard class of complexity while the objective function investigates minimizing total weighted completion times (TWCT). The importance of this type of precedence relations emerges while the successive jobs can be performed by finishing at least only one of the predecessors. An Integer Linear Programming (ILP) model is presented. Because of the complexity of the considered problem a heuristic algorithm is developed. It should be noted that the Heuristic's running time is distinguished, since it is almost zero for all the problem sizes.

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Introductionand literature review

Scheduling is a decision-making process that is used on a regular basis in many manufacturing and service industries and also plays an important role in most manufacturing and production systems as well as in most information processing environments. Baker and Trietsch [1] addressed that the research on Single machine scheduling problem (SMSP) is very important because it can provide good ideas for complex systems. In order to completely understand the behavior of a complex system, it is vital to understand its parts, and quite often the single machine problem appears as a part of a larger scheduling problem. SMSP has received much attention in recent years, and many papers have been published in this area. A scheduling problem described by a triplet $\alpha \mid \beta \mid \gamma$. The field α describes the machine environment and contains just one entry. The possible machine environments specified in the α field are: Single machine (1); and parallel machine and etc. The β field provides details of processing characteristics and constraints and may contain no entry at all, a single entry, or multiple entries. The γ field describes the objective to be minimized and often contains a single entry. Pinedo [2] indicates the mentioned fields in details.

This paper considers the case of dealing with OR-precedence (OR-prec) relations among the tasks in order to minimize TWCT that is notated as $1 \mid OR-prec \mid \sum w_j c_j$. The importance of this kind of precedence relations is mentioned by Gillies and Liu [3] that gave an industrial example that explains an engine head has to be fixed by four bolts. However, one of the bolts may secure the engine head well enough to allow further work on it. If the set *X* consists of the four jobs to secure the bolts and *j* represents the further work on the engine head, then the waiting condition (*X*, *j*) obviously models the desired temporal dependencies among the jobs. Another motivation of this issue is studied by Goldwasser and Motwani [4]. They studied this kind of precedence relations on the industrial case of partially disassembling a given product to reach asingle part or component. In order to remove a certain part, one previously may have to remove other parts which can be modeled by traditional (AND) precedence constraints. However, one may choose to remove that same part of the product from another geometric direction, in which case some other parts must be removed previously. This freedom of choice can be modeled by AND/OR precedence constraints. The third industrial real example is given by Dinic [5], who considers the setup of new technologies and products. In his model, a new technology requires certain products; on the other hand, a new product can be obtained as an output of one of several new technologies. The latter requirement leads to an OR-constraint, while the first requirement is a classical AND-constraint (AND-prec).

So to the best of our knowledge the problem of SMSP with OR-prec with the purpose of minimizing TWCT is not considered so far and no optimal and near optimal solutions has not been introduced. The only consideration has been investigated by Johaness [6] that proved the problem of $1|OR-prec|\sum w_j c_j$ is strongly NP-hard on a single machine, but he also studied this problem with the objective functions of makespan and total completion time that his study resulted in introducing a polynomial-time algorithm for these both objectives. Scanning of the literature is also provides a further proof on the claim of being novel of the considered problem, which is presented at the following statements.SMSP has been considered under several functional constraints such as: release time, deteriorating jobs, constraints networks, learning effect, etc [7-14]. Furthermore, AND/OR-networks are an important generalization of ordinary precedence constraints in various scheduling contexts. AND/OR-networks consist of traditional AND-prec constraints, where a job can only be started after all its predecessors are completed. [3-6, 15,16] considered the SMSP with AND-Networks and OR- prec constraint, where a job is ready for processing as soon as any one of its predecessors is completed. The more general AND/OR-networks are found useful in a variety of applications such as resource constrained project scheduling [17, 18]. Objectives of the previous studies could be categorized as: Makespan (C_{max}) [19-23]; Maximum Lateness (L_{max}) [16, 24-26]; Total weighted completion time (*TWCT*) [6, 10, 27-31]; Total weighted tardiness (*TWT*) [32]; Total completion time (*TCT*) [33, 34], etc. It is noticeable that the most of these problems are known to be NP-Hard, especially the considered problem with OR-prec relations is known as NP-hard in the strong sense [6].

Rest of this paper is organized as follows: section 2 introduces the mathematical formulation of the problem and proposed lower bounds. In section 3 we introduce a heuristic that is based on WSPT (Weighted Shortest Processing Time) rule. Finally the computational results are presented at section 4. The paper results and future researches are concluded at section 5.

2. Formulation

The problem is defined as, there are *N* jobs dependent to each other by OR-prec network that should be scheduled and sequenced on a single machine in order to minimize total weighted completion times. The used parameters and notations are explained at below.

2.1. Parameters and variables

N	Number of all Jobs
I	Set of all tasks
i,j	$\in I$
W_j	Weight of jobs
p_j	Processing time of jobs
M	Large positive constant
D	OR-precedence matrix, D is a N*N matrix that $D_{i,j}$= 1 if job *i* be a predecessor for job *j* in the OR-
S_j	Starting time of jobs
$y_{i,j}$	Is a binary variable equals to 1 if job *j* sequenced after job *i*

2.2. Mathematical model

Objective function

$$Min = \sum_{j=1}^{N} w_j \left(s_j + p_j\right) \tag{1}$$

Subject to:

$$s_i + p_i \le s_j + M \times (1 - y_{i,j}) \qquad \forall\, i \in I\,,\, j > i \tag{2}$$

$$s_j + p_j \le s_i + M \times y_{i,j} \qquad \forall\, i \in I\,,\, j > i \tag{3}$$

$$D_{i,j} \times y_{i,j} + \sum_k D_{j,k}\left(1 - y_{j,k}\right) \ge 1 \qquad \forall\, i, j \in I\,, i > j, k > j \tag{4}$$

$$s_j \ge 0 \qquad \forall\; j \in I \tag{5}$$

Equation (1) depicts the total weighted completion time that should be minimized. Since job preemption is not allowed, $s_j + p_j$ is the completion time of job j. It is sufficient to define $y_{i,j}$ only for i, $i < j$ because $y_{i,j} = 1 - y_{j,i}$. For any pair of jobs i and j, either j follows i or i follows j, so either $s_i + p_i \le s_j$ or $s_j + p_j \le s_i$. These are called "disjunctive constraints", meaning that one or the other must hold for a solution to be feasible. Using precedence variables, the pair of disjunctive constraints between jobs i and j can be written in linear form as (2) and (3). Equation (4) depicts the OR-prec constraints in the problem and it should be written for some of j that they have got predecessors and assures that at least one predecessor of a job must be completed before it. Finally, Number (5) is non-negativity constraint. In the proposed model there are N linear variables (s_j) and *N(N-1)/2* integer variables ($y_{i,j}$). Thus the proposed model will change to traditional $1\|\sum w_j c_j$, which is optimally solvable by WSPT rule by elimination of constraint (4).
Since applying mathematical models for solving Large-scale problems take a long time, a heuristic based WSPT rule has been developed to solve the problem in a reasonable computational time.

2.3. Proposed Lower Bounds

Since there is not any bench mark solution for the considered problem, it is beneficent to create lower bound (LB) to check the quality of resulted proposed algorithms' solutions. In this section two lower bounds are introduced. The first one is WSPT rule and the second one will be resulted from the mathematical model.

2.3.1. WSPT as lower bound

As told in section 2.2 , $1|OR-prec|\sum w_j c_j \subset 1\|\sum w_j c_j$. Zahedi-seresht and Abbassi [35] proved that for the problem of:

$$\begin{aligned} &\min \quad CX \\ &\text{Subject to: } AX \ge b \\ &X \ge 0 \end{aligned} \tag{6}$$

If any constraint added to (6), feasible space of it will change from (7) to (8):

$$S = \{x \mid a^j x \le b_j, j = 1,...,m, x \ge 0\} \tag{7}$$

$$S' = \{x \mid a^j x \le b_j, j = 1,...,m, a^{m+1} x \ge b_{m+1}, x \ge 0\} \tag{8}$$

Where S and S' are the feasible spaces of problem (6) and the problem with added constraint, respectively. Next, they showed that $S' \subseteq S$ and finally they proved that $Z_0^* \ge Z^*$, where Z_0^* is the objective function of the problem with feasible space (8) and Z^* is the objective function of problem (6).

On the basis of this proof the objective function of basic TWCT ($1 \| \sum w_j c_j$) is lower than or equal to the objective function of TWCT with OR-prec ($1 | OR-prec | \sum w_j c_j$).

2.3.2. Lower bound based on jobs earliest start times

This LB is based on the earliest start time of jobs which is shown as following;

Objective function (1) is included by two parameters (w_j, p_j) and one variable (s_j), the only effective variable on the objective function. So it is possible to determine a LB by assuming that all jobs independently start at the earliest time, however this will not lead to a feasible solution. This LB is proved simply by follows.

Lemma. By assuming that all jobs start at earliest start time, its' objective function is lower than equal to objective of considered problem.

Proof. Let S_E^j be the earliest start time of job j, which is evaluated completely independent from other jobs except the predecessors.

$Obj^1 = \sum_{j=1}^{N} w_j (s_j + p_j) = \sum_{j=1}^{N} w_j . s_j + \sum_{j=1}^{N} w_j . p_j$. Since $\sum_{j=1}^{N} w_j . p_j$ is constant value, so just by minimizing the $\sum_{j=1}^{N} w_j . s_j$, Obj^1 is minimized. It is obvious that $S_E^j \le S_j \succ \sum_{j=1}^{N} w_j . s_E^j \le \sum_{j=1}^{N} w_j . s_j$,

Thus $\sum_{j=1}^{N} w_j . s_E^j + \sum_{j=1}^{N} w_j . p_j \le \sum_{j=1}^{N} w_j . s_j + \sum_{j=1}^{N} w_j . p_j \succ \sum_{j=1}^{N} w_j (s_E^j + p_j) \le \sum_{j=1}^{N} w_j (s_j + p_j)$. Finally $\sum_{j=1}^{N} w_j (s_E^j + p_j) \le Obj^1$. Proof is complete □.

3. **Heuristic**

The search for optimal solutions for Large-sized problems is time consuming, but an effective heuristic can supply a time-saving approximate solution with a small margin of error. In this section the proposed heuristic is described which is based on the WSPT rule [2], the priority function of $\rho_j = w_j / p_j \quad \forall j \in I$, which can obtain the optimal solution for the problem of $1 \| \sum w_j c_j$ with NP-complete category of complexity. The NP-hard nature of the problem with additional AND/OR prec relations does not allow to find polynomial time algorithm able to solve optimally the problem. So a heuristic with ability to solve this kind of problems in a trivial computational time and near or optimal solutions can be beneficial. The full trend of the proposed heuristic is clarified in next section.

3.1. Procedure of the heuristic algorithm

The main consideration of this paper investigates OR-prec relations, so proposed heuristic is performed by this logic and proceeds as following statements.

Step 0: $let \ X = 0$, S=Φ, A= Φ,

Step 1: updating set S, prepare the set of tasks that have not predecessor or their predecessor is done based on the OR-prec logic, $I' = I' - A$, then $S = I'$,
Step 2: compute priority function $\rho_j \ \forall j \in S$,
Step2: selection
Select job j from set S with highest ρ and assign it in the first available sequence position. If there are several jobs with identical ρ , select one of them choicely. $X = X + 1$,
$A = A + j$,
Step3: check terminal condition
If $X = N$, go to step 4 else go to step1,
Step4: terminate the algorithm and show π and *TWCT*,
Where

A	Set of all assigned jobs
X	Number of iterations
I'	Set of all jobs that have not any precedence limitation, either assigned or not
S	Set of all jobs that are ready to assign in the sequence
Π	Resulted sequence of all jobs
TWCT	Total weighted completion time

6. Computational experiment

Instances are separately generated in three classes, Loose, mediocre and Tight. For all the test problems, the weight factors (w_j) and processing times (p_j) are generated from the discrete uniform distribution with range [1, 20] and [1, 10], respectively. And the tests are carried out 30 times for each case. Table 2 shows the results in details. The results of heuristic are reported in Table 2 which shows optimal solution, heuristic solution, running time, and percentage deviation from the optimal solution (PDFOS). Each of PDFOS is defined as $100 \times (heu - opt)/opt$, where heu is the heuristic objective value, opt is the optimal value
For example, in fourteenth experiment, the number of jobs is 12, the global optimum value and the running time of global solution are 2663 and 204, respectively. At the columns that show the heuristic results, heu , time and PDFOS are reported 2703, 0.1 and 1.5% , ordinary.
Proposed two lower bounds' resulted values show that the performance of WSPT rule is more near to optimal solutions. This can be derived more simply by skimming Table. 1 and also Fig1. , which shows TWCTs for both lower bounds (WSPT and ES) in comparison with optimum solutions values. Fig2 exhibits that WSPT is more likely to be selected as the main lower bound for the considered problem, since ES results are a bit far from *Opt* solutions.

Fig. 1. Values of Heuristic and Opt solutions

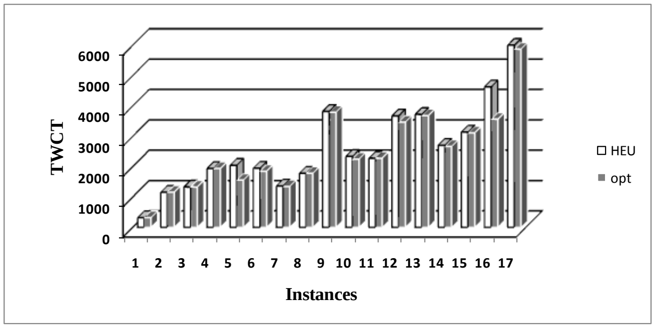
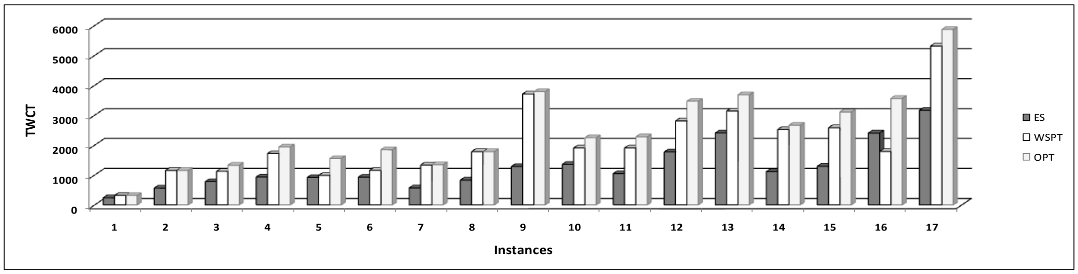


Table.1. Model and heuristic results

#	J_N	LB		Global		Heuristic			
		ES	**WSPT**	*Opt*	**time**	Heu	**time**	**PDFOS**	**average**
1	5	238	315	316	0	316	0.1	0.0%	316
2	6	562	1154	1154	1	1154	0.1	0.0%	1154
3	7	777	1121	1327	1	1327	0.1	0.0%	1327
4	7	933	1717	1936	0	1936	0.1	0.0%	1936
5	8	916	985	1551	2	2043	0.1	31.7%	1551
6	8	925	1158	1845	1	1943	0.1	5.3%	1845
7	9	567	1330	1344	2	1362	0.1	1.3%	1353
8	9	828	1781	1781	13	1781	0.1	0.0%	1781
9	10	1281	3708	3788	156	3820	0.1	0.8%	3788
10	10	1356	1904	2246	1	2334	0.1	3.9%	2246
11	10	1046	1904	2276	17	2276	0.1	0.0%	2325.5
12	11	1772	2809	3467	19	3671	0.1	5.9%	3560.8
13	11	2406	3139	3681	1	3719	0.1	1.0%	3704.6
14	12	1111	2519	2663	204	2703	0.1	1.5%	2663.7
15	13	1292	2582	3106	893	3139	0.1	1.1%	3107.8
16	13	2403	1787	3561	18	4623	0.1	29.8%	3805
17	14	3160	5313	5869	1427	6051	0.1	3.1%	5878.9
18	15	2801	4738	-		6083	0.1	-	6347.8
19	15	3738	5889	-		7465	0.1	-	7500.8
20	17	1737	5268	-		7166	0.1	-	6466.2
21	18	2296	6668	-		7417	0.1	-	7163.5
22	20	3428	6210	-		11377	0.1	-	10213.3

23	27	7228	17385	-	20996	0.1	-	22078.2
24	37	5309	24460	-	33561	0.1	-	29047.9
25	39	5393	22981	-	34858	0.1	-	36240.1
26	42	5458	30398	-	37040	0.1	-	40876
27	51	9463	41506	-	61527	0.1	-	61304.2
28	63	10071	65123	-	87498	0.1	-	88106.7
29	74	11584	67045	-	97042	0.1	-	113421
30	100	19446	134748	-	188257	0.1	-	212694.8

Fig.2. Lower bounds



Conclusion

This paper addresses the single-machine scheduling problem of minimizing total weighted completion time with OR-prec constraints. This subject has not been considered in the literature, except the only study which addresses its complexity. For this aim, an ILP model based on the basic assumptions of the problem is. The proposed model could not be able to solve the problems over the 14 jobs in a reasonable running time, so this limitation caused to design a heuristic algorithm. This paper also introduced two kinds of lower bounds that only one of them recognized to be consider, since the ES based lower bound did not result in considerable good solutions. So solution quality of algorithm is compared by proposed better lower bound. For the future research it is suggested that, the simultaneous consideration of OR-prec constraints and objective function can be extended to other conditions of single machine and other fields of scheduling problems. In the aspect of solution methodology, other exact methods like Branch and Bound or near optimal methods like meta- heuristics or developed form of the proposed heuristic also can be suggested as future research.

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