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RESEARCH ARTICLE

A NEW OPTIMAL SOLUTION METHOD FOR TRAPEZOIDAL FUZZY TRANSPORTATION PROBLEM

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Abstract

In this paper, fuzzy transportation problem is investigated using zero termination method. The Transportation costs, supply and demand values are considered to lie in an interval of values. Robust Ranking method is applied to arrange the fuzzy numbers in a specific interval. Also, a new equation is formed using α-cut method to find the optimal solution. The fuzzy transportation problem can be transformed into crisp transportation problem using Robust Ranking method in the linear programming problem and it is solved by using zero termination method .Fuzzy modified distribution method is proposed to find the optimal solution in terms of fuzzy numbers. The solution procedure is illustrated with a numerical example.

Introduction

Transportation problem deals Transportation problem with the distribution of a product from various sources to different destinations of demand, in such a manner that the total transportation cost is minimized. In order to solve a transportation problem, the decision parameters such as availability, requirement and the unit transportation cost of the model were tried at crisp values. But in real life, supply, demand and unit life transportation cost are uncertain due to several factors. These imprecise data may be represented by fuzzy numbers. To deal with this uncertain situations in transportation problems many researchers [3, 5, 6, 9] have proposed fuzzy and interval programming techniques for solving the transportation problem. The concept of fuzzy mathematical programming on a general level was first proposed by Tanaka etal 1974 in the frame work of fuzzy decision (Bellman and Zadeh, 1965).

Abhinav Bansal (2011) has paid specific attention to trapezoidal fuzzy numbers of type (a, b, c , d) without any non negative restriction and the related mathematical expressions for the evaluation of power functions and other algebraic relations are derived using simple analytical mathematics.

Das et al (1999), proposed a method, using fuzzy technique to solve interval transportation problems by considering the right bound and the midpoint interval and (A.Sengubta et al**,** 2003) proposed a new orientated method to solve interval transportation problems by considering the midpoint and width of the interval in the objective function. (Stephen Dinagara et al, 2009) proposed method of finding the initial fuzzy feasible solution to a fuzzy transportation problem. But most of the existing techniques provide only crisp solution for fuzzy transportation problem. In general, most of the authors obtained the crisp optimal solution to a given fuzzy transportation problem. In this paper, we propose a new algorithm to find the initial fuzzy feasible solution to a fuzzy transportation problem without converting it to be a classical transportation problem.

For a decision making process in a fuzzy environment, ranking the fuzzy numbers play a vital role. The fuzzy numbers got to be ranked before the decision maker takes the decision. (R.R.Yager, 1991) ranking method is one of the robust ranking technique which satisfies the properties of compensation, linearity and additive. A. (Manimaran et al, 2013) have transformed a fuzzy assignment problem into a crisp assignment problem in the LPP form using LINGO 9.0.

Also, The arithmetic behavior of trapezoidal numbers has not already been dealt enough using α-cut approach till date.

In section 2, we recall the basic concepts and results of Trapezoidal fuzzy numbers and the fuzzy transportation problem with Trapezoidal fuzzy number and their related results. In Section 3, we propose a new algorithm of fuzzy interval transportation problem. In Section 4, we propose a new algorithm to find the initial fuzzy feasible solution for the given fuzzy transportation problem and obtained the fuzzy optimal solution, applying the zero termination method. Section 5 deals with application of Robust Rankin Method in our paper. In Section 6, we brief the method of solving a fuzzy transportation problem using zero termination method on Trapezoidal fuzzy number. Numerical example is illustrated. Section 7 explains how the α –cut method is used to create a new equation to satisfy the optimal solution.

2. Preliminaries

In this section we present some necessary definitions.

2.1 Definition

A real fuzzy number $\tilde{a}$ is a fuzzy subset of the real number R with membership function $\mu_{\tilde{a}}$ satisfying the following conditions.

1. $\mu_{\tilde{a}}$ is continuous from R to the closed interval [0,1]
2. $\mu_{\tilde{a}}$ is strictly increasing and continuous on $[a_1, a_2]$
3. $\mu_{\tilde{a}}$=1in [a_2,a_3]
4. $\mu_{\tilde{a}}$ is strictly decreasing and continuous on $[a_3, a_4]$ where a_1, a_2 , a_3 & a_4 are real numbers, and the fuzzy number denoted by $\tilde{a}$= $[a_1, a_2, a_3, a_4]$is called a trapezoidal fuzzy number.

2.2 Definition.

The fuzzy number is a trapezoidal number, and its membership function $\mu_{\tilde{a}}$ is represented by the figure given below.

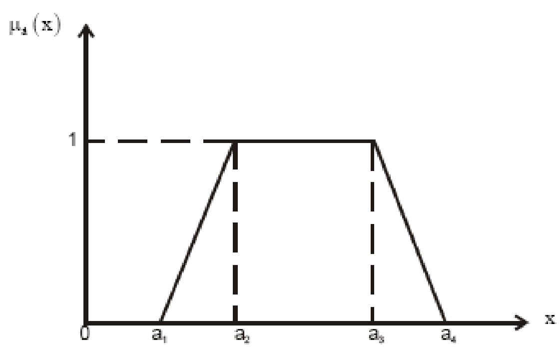

Fig.1. Membership function of a fuzzy number ã

2.3 Definition.

We define a ranking function R: F(R) → R, which maps each fuzzy number into the real line ,F(μ) represents the set of all trapezoidal fuzzy numbers. If R be any ranking function, Then R ($\tilde{a}$) =$(a_1 + a_2 + a_3 + a_4)/4$.

2.4 Arithmetic operations

Let $\tilde{a}$=[a_1, a_2 , a_3 ,a_4]and $\tilde{b}$=[b_1,b_2 , b_3 ,b_4] be two trapezoidal fuzzy numbers then the arithmetic operations on $\tilde{a}$ and $\tilde{b}$ are defined as follows :

Addition: $\tilde{a} + \tilde{b}= (a_1+b_1, a_2+b_2, a_3+b_3, a_4+b_4)$

$\tilde{a} + \tilde{b}= (a_1+b_4, a_2+b_3, a_3+b_2, a_4+b_1)$

Subtraction: $\tilde{a}-\tilde{b}= (a_1-b_4 , a_2-b_3 , a_3-b_2 , a_4-b_1)$

Multiplication:
Case 1:

$\tilde{a} * \tilde{b} =$
$[min(a_1b_1, a_1b_4, a_4b_1, a_4b_4), min(a_2b_2, a_2b_3, a_3b_2, a_3b_3), max(a_2b_2, a_2b_3, a_3b_2, a_3b_3), max(a_1b_1, a_1b_4, a_4b_1, a_4b_4)]$

Case2:

$$\tilde{a} * \tilde{b} = \left(\frac{a_1}{4}(b_1 + b_2 + b_3 + b_4), \frac{a_2}{4}(b_1 + b_2 + b_3 + b_4), \frac{a_3}{4}(b_1 + b_2 + b_3 + b_4), \frac{a_4}{4}(b_1 + b_2 + b_3 + b_4)\right) \text{ if } R(\tilde{a}) > 0$$

$$\tilde{a} * \tilde{b} = \left(\frac{a_4}{4}(b_1 + b_2 + b_3 + b_4), \frac{a_3}{4}(b_1 + b_2 + b_3 + b_4), \frac{a_2}{4}(b_1 + b_2 + b_3 + b_4), \frac{a_1}{4}(b_1 + b_2 + b_3 + b_4)\right) \text{ if } R(\tilde{a}) < 0$$

Case 3:

$\tilde{a} * \tilde{b} = (a_1, a_2, a_4, a_4) * (b_1, b_2, b_3, b_4) = (a_1b_1, a_2b_2, a_3b_3, a_4b_4)$

3. Fuzzy Transportation problem

Let us consider a transportation system based on fuzzy with m fuzzy origins and n fuzzy destinations. Let us further assume that the transportation cost of one unit of product from ith fuzzy origin to jth fuzzy destination be denoted by $C_{ij} =[c_{ij}^{(1)}, c_{ij}^{(2)}, c_{ij}^{(3)}, c_{ij}^{(4)}]$, the availability of commodity at fuzzy origin i be $s_i =[s_i^{(1)}, s_i^{(2)}, s_i^{(3)}, s_i^{(4)}]$, commodity needed at the fuzzy destination j be $d_j =[d_j^{(1)}, d_j^{(2)}, d_j^{(3)}, d_j^{(4)}]$. The quantity transported from ith fuzzy origin to jth fuzzy destination be $X_{ij} =[x_{ij}^{(1)}, x_{ij}^{(2)}, x_{ij}^{(3)}, x_{ij}^{(4)}]$.

Now, the fuzzy transportation problem based on supply s_i, demand d_i and the transported quantity X_{ij} can be related in a table as follows.

	1	2…	N	Fuzzy capacity
1	C_{11} X_{11}	$C_{12…}$ X_{12}…	C_{1n} X_{1n}	s_1
2 . . .	C_{21} X_{21}	$C_{22…}$ X_{22}…	C_{1n} X_{2n}	s_2
M	C_{m1} X_{m1}	$C_{m2…}$ Xm_2…	C_{mn} X_{mn}	S_m
Fuzzy demand	d_1	d_2	d_n	$\sum d_j = \sum s_i$

Where $C_{ij} =[c_{ij}^{(1)}, c_{ij}^{(2)}, c_{ij}^{(3)}, c_{ij}^{(4)}]$, $X_{ij} =[x_{ij}^{(1)}, x_{ij}^{(2)}, x_{ij}^{(3)}, x_{ij}^{(4)}]$, $s_i =[s_i^{(1)}, s_i^{(2)}, s_i^{(3)}, s_i^{(4)}]$ and $d_i =[d_i^{(1)}, d_i^{(2)}, d_i^{(3)}, d_i^{(4)}]$

The linear programing model representing the fuzzy transportation is given by

$$Minimize\ Z = \sum_{i=1}^{m} \sum_{j=1}^{n} [c_{ij}^{(1)}, c_{ij}^{(2)} c_{ij}^{(3)}, c_{ij}^{(4)}]\ [X_{ij}^{(1)}, X_{ij}^{(2)}, X_{ij}^{(3)}, X_{ij}^{(4)}]$$

Subject to the constraints

$$\sum_{i=1}^{n} [x_{ij}^{(1)}, x_{ij}^{(2)}, x_{ij}^{(3)}, x_{ij}^{(4)}] = [s_i^{(1)}, s_i^{(2)}, s_i^{(3)}, s_i^{(4)}]$$

for i=1,2,……..m(Row sum)

$$\sum_{i=1}^{m}[X_{ij}^{(1)},X_{ij}^{(2)},X_{ij}^{(3)},X_{ij}^{(4)}]=[d_j^{(1)},d_j^{(2)},d_j^{(3)},d_j^{(1)}]$$

for j=1,2…n(Column sum)

$$[x_{ij}^{(1)},x_{ij}^{(2)},x_{ij}^{(3)},x_{ij}^{(4)}]\geq 0$$

The given fuzzy transportation problem is said to be balanced if

$$\sum_{i=1}^{m}[s_i^{(1)},s_i^{(2)},s_i^{(3)},s_i^{(4)}]=\sum_{j=1}^{n}[d_j^{(1)},d_j^{(2)},d_j^{(3)},d_j^{(4)}]$$

i.e., if the total fuzzy capacity is equal to the total fuzzy demand

3.1. Fuzzy Solution:

A set of allocation x_{ij} which satisfies the row and column restriction is called a fuzzy solution.

4. The Computational Procedure for Fuzzy Modified Distribution Method

4.1. Zero Termination Method

The procedure of Zero Termination method is as follows,

Step 1: Construct the transportation table

Step 2: Select the smallest unit transportation cost value for each row and subtract it from all costs in that row. In a similar way this process is repeated column wise.

Step 3: In the reduced cost matrix obtained from step 2, there will be at least one zero in each row and column. Then we find the termination value of all the zeros in the reduced cost matrix, using the following rule;

The zero termination cost is T= Sum of the costs of all the cells adjacent to zero/ Number of non- zero cells added

Step 4: In the revised cost matrix with zero termination costs has a unique maximum T, allot the supply to that cell. If it has one (or) more Equal max values, then select the cell with the least original cost and allot the maximum possible.

Step 5: After the allotment, the columns and rows corresponding to exhausted demands and supplies are trimmed. The resultant matrix must possess at least one zero in each row and column, else repeat step (2)

Step 6: Repeat step (3) to step (5) until the optimal solution is obtained.

4.2. Fuzzy modified distribution method.

This proposed method is used for finding the optimal basic feasible solution in fuzzy environment and the following step by step procedure is utilized to find out the same.

1. Find a set of numbers$[u_i^{(1)},u_i^{(2)},u_i^{(3)},u_i^{(4)}]$ and $[v_j^{(1)},v_j^{(2)},v_j^{(3)},v_j^{(4)}]$ for each row and column satisfying $[u_i^{(1)},u_i^{(2)},u_i^{(3)},u_i^{(4)}]$+ $[v_j^{(1)},v_j^{(2)},v_j^{(3)},v_j^{(4)}]$=$[c_{ij}^{(1)},c_{ij}^{(2)},c_{ij}^{(3)},c_{ij}^{(4)}]$ for each occupied cell. To start with we assign a fuzzy zero to any row or column having maximum number of allocations. If this maximum number of allocation is more than one, choose any one arbitrarily.
2. For each empty (un occupied) cell, we find fuzzy sum $[u_i^{(1)},u_i^{(2)},u_i^{(3)},u_i^{(4)}]$ and $[v_j^{(1)},v_j^{(2)},v_j^{(3)},v_j^{(4)}]$
3. Find out for each empty cell the net evaluation value $[Z_{ij}^{(1)},Z_{ij}^{(2)},Z_{ij}^{(3)},Z_{ij}^{(4)}]$=$[c_{ij}^{(1)},c_{ij}^{(2)},c_{ij}^{(3)},c_{ij}^{(4)}]$- $[u_i^{(1)},u_i^{(2)},u_i^{(3)},u_i^{(4)}]$+ $[v_j^{(1)},v_j^{(2)},v_j^{(3)},v_j^{(4)}]$ this step gives the optimality conclusion

i.) If all $[Z_{ij}^{(1)},Z_{ij}^{(2)},Z_{ij}^{(3)},Z_{ij}^{(4)}]$> [-2,-1,0,1,2] the solution is optimal and a unique solution exists.

ii) .If $[Z_{ij}^{(1)}, Z_{ij}^{(2)}, Z_{ij}^{(3)}, Z_{ij}^{(4)}] \geq$ [-2,-1,0,1,2] then the solution is fuzzy optimal. But an alternate solution exists.,

iii) .If at least one $[Z_{ij}^{(1)}, Z_{ij}^{(2)}, Z_{ij}^{(3)}, Z_{ij}^{(4)}] <$[-2,-1,0,1,2] the solution is not fuzzy optimal. In this case we go to next step, to improve the total fuzzy transportation minimum cost.

4. Select the empty cell having the most negative value of $[Z_{ij}^{(1)}, Z_{ij}^{(2)}, Z_{ij}^{(3)}, Z_{ij}^{(4)}]$ from this cell we draw a closed path drawing horizontal and vertical lines with corner cells being occupied. Assign sign + and – alternately and find the fuzzy minimum allocation from the cells having negative sign. This allocation should be added to the allocation having negative sign.

5. The above step yield a better solution by making one (or more) occupied cell as empty and one empty cell as occupied. For this new set of fuzzy basic feasible allocation repeat the steps from [i] till an fuzzy optimal basic feasible solution is obtained.

5. Robust Ranking Method.

Robust ranking technique [14] which satisfies compensation ,linearity and addictive properties and provides results which are consistent with human intuition. Given a convex fuzzy number a, the Robust Ranking Index is defined by

$$R(a) = \int_0^1 0.5(a_\alpha^L, a_\alpha^U),\ d\alpha$$

where (a_α^L, a_α^U) is the α – level cut of the fuzzy number a.

In this paper we use this method for ranking the objective values. The Robust‘s ranking index R(a) gives the representative value of the fuzzy number a. It satisfies the linearity and additive property:

If G = $l_E + m_Y$ and U= $k_A - t_N$,where l ,m, k and t are constants, then we

Have R(G) = l R(E)+m R(Y) and R(U)= K R(A)-t R(N). On the basis of this property the fuzzy assignment problem can be transformed into a crisp assignment problem in Linear programming problem form. The ranking Technique of the Robust is

If R(S) ≤R(I),then S ≤ I, (ie) min {S,I}=S.

For the assignment problem (1) ,with fuzzy objective function

$$\text{Min } Z = \sum_{i=1}^{n}\sum_{j=1}^{n} R(a_{ij}) x_{ij}$$

We apply Robust’s ranking method [14] (using the linearity and additive property) to get the minimum objective value z^* from the formulation

$$R(z^*) = \min z = \sum_{i=1}^{n}\sum_{j=1}^{n} R(a_{ij}) x_{ij}$$

Now we calculate R(-2,0,2,8) by applying the Robust’s ranking method. The membership function of the trapezoidal number (-2,0,2,8)

$$\mu(x) = \begin{cases} \dfrac{x-(-2)}{0-(-2)}, & -2 < x < 0 \\ \dfrac{8-x}{8-2}, & 2 < x < 8 \end{cases}$$

The α – cut of a fuzzy number (-2,0,2,8) is $(a_\alpha^L, a_\alpha^U) = (2\alpha - 2, 8 - 6\alpha)$ for which

$$R(a_{11}) = R(-2,0,2,8) = \int_0^1 0.5(a_\alpha^L, a_\alpha^U) d\alpha = \int_0^1 0.5(2\alpha - 2 + 8 - 6\alpha) d\alpha = 2$$

Proceeding similarly, the Robust’s ranking indices for the fuzzy costs a_{11} are calculated as:

$$R(a_{12}) = 2\ , R(a_{13}) = 2,\ R(a_{14}) = 1,\quad R(a_{15}) = 3$$

$R(a_{21}) = 10$, $R(a_{22}) = 8,$ $R(a_{23}) = 5,$ $R(a_{24}) = 4,$ $R(a_{25}) = 7$
$R(a_{31}) = 7$, $R(a_{32}) = 6,$ $R(a_{33}) = 6,$ $R(a_{34}) = 8,$ $R(a_{35}) = 5$
$R(a_{41}) = 4$, $R(a_{42}) = 3,$ $R(a_{43}) = 4,$ $R(a_{44}) = 4,$ $R(a_{45}) = 15$

We replace these values for their corresponding a_{ij} in which results in a conventional transportation problem in the L.P.P from. We solve it by using zero termination method.

6. Numerical example

To solve the following fuzzy transportation problem starting with the fuzzy initial fuzzy basic feasible solution obtained by Zero Termination Method

.	D1	D2	D3	D4	Fuzzy capacity
O1	[-2,0,2,8]	[-2,0,2,8]	[-2,0,2,8]	[-1,0,1,4]	[0,2,4,6]
O2	[4,8,12,16]	[4,7,9,12]	[2,4,6,8]	[1,3,5,7]	[2,4,9,13]
O3	[2,4,9,13]	[0,6,8,10]	[0,6,8,10]	[4,7,9,12]	[2,4,6,8]
Fuzzydemand	[1,3,5,7]	[0,2,4,6]	[1,3,5,7]	[1,3,5,7]	[4,10,19,27]

Since $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j = [4,10,19,27]$, *the problem is balanced fuzzy transportation problem.*

There exists a fuzzy initial basic feasible solution.

	D1	D2	D3	D4	Fuzzy Capacity
O1	[-2,0,2,8]	[-2,0,2,8] [-7,-1,5,11]	[-2,0,2,8]	[-1,0,1,4] [-11,-3,5,13]	[0,2,4,6]
O2	[4,8,12,16]	[4,7,9,12]	[2,4,6,8] [1,3,5,7]	[1,3,5,7] [-12,-2,8,18]	[2,4,9,13]
O3	[2,4,9,13] [1,3,5,7]	[0,6,8,10] [-5,-1,3,7]	[0,6,8,10]	[4,7,9,12]	[2,4,6,8]
Fuzzy demand	[1,3,5,7]	[0,2,4,6]	[1,3,5,7]	[1,3,5,7]	

Since the number of occupied cell having m+n-1= 6 and are also independent , there exist a non-degenerate fuzzy basic feasible solution.

Therefore, the initial fuzzy transportation minimum cost is,

$[Z^{(1)},Z^{(2)},Z^{(3)},Z^{(4)}]=$

[-2,0,2,8][-7,-1,5,11]+[-1,0,1,4][**-11,3,5,13]+[2,4,6,8][1,3,5,7]+**
[1,3,5,7][-12,-,8,18]+[2,4,9,13][1,3,5,7]+[0,6,8,10][-5,-1,3,7]
= [-56 ,-2, 10, 88]+[-44,-3, 5, 52]+[2, 12, 30, 56]+[-84, -10, 40, 126]+
[2, 12, 45, 91]+[0, -8, 24, 70]

$[Z^{(1)}, Z^{(2)}, Z^{(3)}, Z^{(4)}]$ =[-180, 1, 154 483] = 114.5

To find the optimal solution:

Applying the fuzzy modified distribution method, we determine a set of numbers

$[u_i^{(1)}, u_i^{(2)}, u_i^{(3)}, u_i^{(4)}]$ and $[v_j^{(1)}, v_j^{(2)}, v_j^{(3)}, v_j^{(4)}]$ each row and column such that

$$[c_{ij}^{(1)}, c_{ij}^{(2)}, c_{ij}^{(3)}, c_{ij}^{(4)}]= [u_i^{(1)}, u_i^{(2)}, u_i^{(3)}, u_i^{(4)}] +[v_j^{(1)}, v_j^{(2)}, v_j^{(3)}, v_j^{(4)}]$$

for each occupied cell. Since 3rd row has maximum numbers of allocations, we give fuzzy number

$$[u_3^{(1)}, u_3^{(2)}, u_3^{(3)}, u_3^{(4)}] = [-2,-1,0,1,2].$$

The remaining numbers can be obtained as given below.

$[c_{31}^{(1)}, c_{31}^{(2)}, c_{31}^{(3)}, c_{31}^{(4)}]= [u_3^{(1)}, u_3^{(2)}, u_3^{(3)}, u_3^{(4)}] +[v_1^{(1)}, v_1^{(2)}, v_1^{(3)}, v_1^{(4)}]$

$[v_1^{(1)}, v_1^{(2)}, v_1^{(3)}, v_1^{(4)}]= [4, 5, 8, 11]$

$[c_{32}^{(1)}, c_{32}^{(2)}, c_{32}^{(3)}, c_{32}^{(4)}]= [u_3^{(1)}, u_3^{(2)}, u_3^{(3)}, u_3^{(4)}] +[v_2^{(1)}, v_2^{(2)}, v_2^{(3)}, v_2^{(4)}]$

$[v_2^{(1)}, v_2^{(2)}, v_2^{(3)}, v_2^{(4)}]= [-2, 5, 9, 12]$

$[c_{33}^{(1)}, c_{33}^{(2)}, c_{33}^{(3)}, c_{33}^{(4)}]= [u_3^{(1)}, u_3^{(2)}, u_3^{(3)}, u_3^{(4)}] +[v_3^{(1)}, v_3^{(2)}, v_3^{(3)}, v_3^{(4)}]$

$[v_3^{(1)}, v_3^{(2)}, v_3^{(3)}, v_3^{(4)}]= [-2, 5, 9, 12]$

$[c_{23}^{(1)}, c_{23}^{(2)}, c_{23}^{(3)}, c_{23}^{(4)}]= [u_2^{(1)}, u_2^{(2)}, u_2^{(3)}, u_2^{(4)}] +[v_3^{(1)}, v_3^{(2)}, v_3^{(3)}, v_3^{(4)}]$

$[u_2^{(1)}, u_2^{(2)}, u_2^{(3)}, u_2^{(4)}] =[-10,-5,1,10]$

$[c_{24}^{(1)}, c_{24}^{(2)}, c_{24}^{(3)}, c_{24}^{(4)}]= [u_2^{(1)}, u_2^{(2)}, u_2^{(3)}, u_2^{(4)}] +[v_4^{(1)}, v_4^{(2)}, v_4^{(3)}, v_4^{(4)}]$

$[v_4^{(1)}, v_4^{(2)}, v_4^{(3)}, v_4^{(4)}]= [-9, 2, 10, 17]$

$[c_{11}^{(1)}, c_{11}^{(2)}, c_{11}^{(3)}, c_{11}^{(4)}]= [u_1^{(1)}, u_1^{(2)}, u_1^{(3)}, u_1^{(4)}] +[v_1^{(1)}, v_1^{(2)}, v_1^{(3)}, v_1^{(4)}]$

$[u_1^{(1)}, u_1^{(2)}, u_1^{(3)}, u_1^{(4)}] = [-13,-8,-3,4]$

We find, for each empty cell of the sum

$$[u_i^{(1)}, u_i^{(2)}, u_i^{(3)}, u_i^{(4)}] \text{ and } [v_j^{(1)}, v_j^{(2)}, v_j^{(3)}, v_j^{(4)}].$$

Next we find net evaluation $[Z_{ij}^{(1)}, Z_{ij}^{(2)}, Z_{ij}^{(3)}, Z_{ij}^{(4)}]$ is given by:

	D1	D2	D3	D4	**Fuzzy Capacity**
O1	[-2,0,2,8] [-18,-4,10,24]	[-2,0,2,8] ***[-18,-6,5,23]**	[-2,0,2,8] ***[-18,-6,5,23]**	[-1,0,1,4] ***[-22,-7,7,26]**	[0,2,4,6]
O2	[4,8,12,16] ***[-17,-1,12,22]**	[4,7,9,12] ***[-18,-3,9,24]**	[2,4,6,8] [-12,-2,8,18]	[1,3,5,7] [-23,-5,13,31]	[2,4,9,13]
O3	[2,4,9,13] [-23,-7,9,25]	[0,6,8,10] [-12,-2,8,18]	[0,6,8,10] [-11,-3,5,13]	[4,7,9,12] ***[-18,-3,9,24]**	[2,4,6,8]
Fuzzy Demand	[1,3,5,7]	[0,2,4,6]	[1,3,5,7]	[1,3,5,7]	

Where $U_i = [u_i^{(1)}, u_i^{(2)}, u_i^{(3)}, u_i^{(4)}]$, $V_j = [v_j^{(1)}, v_j^{(2)}, v_j^{(3)}, v_j^{(4)}]$ and

$$*[Z_{ij}^{(1)}, Z_{ij}^{(2)}, Z_{ij}^{(3)}, Z_{ij}^{(4)}] = [c_{ij}^{(1)}, c_{ij}^{(2)}, c_{ij}^{(3)}, c_{ij}^{(4)}] - [u_i^{(1)}, u_i^{(2)}, u_i^{(3)}, u_i^{(4)}] + [v_j^{(1)}, v_j^{(2)}, v_j^{(3)}, v_j^{(4)}]$$

Since all $[Z_{ij}^{(1)}, Z_{ij}^{(2)}, Z_{ij}^{(3)}, Z_{ij}^{(4)}] > 0$ the solution is fuzzy optimal and unique.

Therefore the fuzzy optimal solution in terms of trapezoidal fuzzy numbers:

$$[Z^{(1)}, Z^{(2)}, Z^{(3)}, Z^{(4)}] = [X_{11}^{(1)}, X_{11}^{(2)}, X_{11}^{(3)}, X_{11}^{(4)}] = [-18,-4, 10, 24]$$

$$[X_{23}^{(1)}, X_{23}^{(2)}, X_{23}^{(3)}, X_{23}^{(4)}] = [-12,-2, 8, 18]$$

$$[X_{24}^{(1)}, X_{24}^{(2)}, X_{24}^{(3)}, X_{24}^{(4)}] = [-23,-5, 13, 31]$$

$$[X_{31}^{(1)}, X_{31}^{(2)}, X_{31}^{(3)}, X_{31}^{(4)}] = [-23,-7, 9, 25]$$
$$[X_{32}^{(1)}, X_{32}^{(2)}, X_{32}^{(3)}, X_{32}^{(4)}] = [-12,-2,8,18]$$
$$[X_{33}^{(1)}, X_{33}^{(2)}, X_{33}^{(3)}, X_{33}^{(4)}] = [-11,-3,5,13]$$

Hence, the total fuzzy transportation minimum cost is

Case 1:

$[Z^{(1)}, Z^{(2)}, Z^{(3)}, Z^{(4)}]$= **[-930,-148,318,1188]=107**

Case 2

$[Z^{(1)}, Z^{(2)}, Z^{(3)}, Z^{(4)}]$ = [**10,182,136,198**] = **106.5**

Case3

$[Z^{(1)}, Z^{(2)}, Z^{(3)}, Z^{(4)}]$ = **[-57,-81,318,1188]=342**

Case 4

$[Z^{(1)}, Z^{(2)}, Z^{(3)}, Z^{(4)}]$ = **[152,82,88,270]=148**

Case 5

$[Z^{(1)}, Z^{(2)}, Z^{(3)}, Z^{(4)}]$ = **[82,102,114,126]=106**

Case 6

$[Z^{(1)}, Z^{(2)}, Z^{(3)}, Z^{(4)}]$ = **[531,131,106,600]=342**

7. It represents the unoccupied cells:

	D1	**D2**	**D3**	**D4**	**Fuzzy Capacity**
O1	[-2,0,2,8] [-18,-4,10,24]	[-2,0,2,8] ***[-18,-6,5,23]**	[-2,0,2,8] ***[-18,-6,5,23]**	[-1,0,1,4] ***[-22,-7,7,26]**	[0,2,4,6]
O2	[4,8,12,16] ***[-17,-1,12,22]**	[4,7,9,12] ***[-18,-3,9,24]**	[2,4,6,8] [-12,-2,8,18]	[1,3,5,7] [-23,-5,13,31]	[2,4,9,13]
O3	[2,4,9,13] [-23,-7,9,25]	[0,6,8,10] [-12,-2,8,18]	[0,6,8,10] [-11,-3,5,13]	[4,7,9,12] ***[-18,-3,9,24]**	[2,4,6,8]
Fuzzy Demand	[1,3,5,7]	[0,2,4,6]	[1,3,5,7]	[1,3,5,7]	

Where $U_i = [u_i^{(1)}, u_i^{(2)}, u_i^{(3)}, u_i^{(4)}]$, $V_j = [v_j^{(1)}, v_j^{(2)}, v_j^{(3)}, v_j^{(4)}]$ and

*$[Z_{ij}^{(1)}, Z_{ij}^{(2)}, Z_{ij}^{(3)}, Z_{ij}^{(4)}] = [c_{ij}^{(1)}, c_{ij}^{(2)}, c_{ij}^{(3)}, c_{ij}^{(4)}] - [u_i^{(1)}, u_i^{(2)}, u_i^{(3)}, u_i^{(4)}] + [v_j^{(1)}, v_j^{(2)}, v_j^{(3)}, v_j^{(4)}]$

Since all $[Z_{ij}^{(1)}, Z_{ij}^{(2)}, Z_{ij}^{(3)}, Z_{ij}^{(4)}] > 0$ the solution is fuzzy optimal and unique.

Therefore the fuzzy optimal solution in terms of trapezoidal fuzzy numbers:

7.1 Computation of membership function:

Computation of membership functions of the fuzzy optimal solution of the fuzzy transportation problem. It is to find fuzzy membership functions of c_{ij} and x_{ij} for each cell (i, j).The membership function of fuzzy transportation cost for the occupied cells are fuzzy allocation and their α - level sets of fuzzy transportation cost and fuzzy allocation as follows:

$$\mu_{C11}(x) = \begin{cases} \dfrac{x-(-2)}{0-(-2)} & -2 \le x \le 0 \\ \dfrac{8-x}{8-2} & 2 \le x \le 8 \end{cases} \qquad \mu_{X11}(x) = \begin{cases} \dfrac{x-(-18)}{-4-(-18)} & -18 \le x \le -4 \\ \dfrac{24-x}{24-10} & 10 \le x \le 24 \end{cases}$$

$c_{11} = [x_1^{(\alpha)}, x_2^{(\alpha)}] = [2\alpha - 2, 8 - 6\alpha]$, $x_{11} = [x_1^{(\alpha)}, x_2^{(\alpha)}] = [14\alpha - 18, 24 - 14\alpha]$

$c_{11} \bullet x_{11} = [28\alpha^2 - 64\alpha + 36, 84\alpha^2 - 256\alpha + 192]$ ---------- (A)

Similarly

$c_{23} \bullet x_{23} = [20\alpha^2 - 4\alpha - 24 \ , \ 20\alpha^2 - 116\alpha + 144]$ ---------- (B)

$c_{24} \bullet x_{24} = [36\alpha^2 - 28\alpha - 23 \ , \ 36\alpha^2 - 188\alpha + 217]$ ---------- (c)

$c_{31} \bullet x_{31} = [32\alpha^2 - 14\alpha - 46 \ , \ 64\alpha^2 - 308\alpha + 325]$ ---------- (D)

$c_{32} \bullet x_{32} = [60\alpha^2 - 72\alpha \ , \ 20\alpha^2 - 136\alpha + 180]$ ---------- (E)

$c_{33} \bullet x_{33} = [48\alpha^2 - 66\alpha \ , 16\alpha^2 - 106\alpha + 130]$ --------- (F)

The Fuzzy Transportation cost is z = A+B+C+D+E+F

$$z = [224\alpha^2 - 248\alpha - 57 \ , \ 240\alpha^2 - 1110\alpha + 1188]$$

Solving the equations:

$$224\alpha^2 - 248\alpha - 57 - x_1 = 0 \quad \text{-------------------------------- (G)}$$

$$240\alpha^2 - 1110\alpha + 1188 - x_2 = 0 \quad \text{-------------------------------- (H)}$$

We get $\alpha = \dfrac{[248 + \{(248)^2 - 896(-57 - x_1)\}^{1/2}]}{448}$ $\quad \alpha = \dfrac{[1110 - \{(1110)^2 - 960(1188 - x_2)\}^{1/2}]}{480}$

and therefore

$$\mu_{.\cos t.Z(x))} = \left\{ \begin{array}{ll} \dfrac{248 + \{(248)^2 - 896(-57 - X_1)\}^{1/2}}{448} & -57 \le x_1 \le -81 \\ \dfrac{1110 - \{(1110)^2 - 960(1188 - X_2)\}^{1/2}}{480} & 318 \le x_2 \le 1188 \end{array} \right\}.$$

From the membership function of the optimal solution, we can find the grade of the fuzzy transportation cost which lies between

α	$[Z_{ij}^{(1)}, Z_{ij}^{(2)}, Z_{ij}^{(3)}, Z_{ij}^{(4)}]$	$\sum_{i,j} z_{ij}$	α	$[Z_{ij}^{(1)}, Z_{ij}^{(2)}, Z_{ij}^{(3)}, Z_{ij}^{(4)}]$	$\sum_{i,j} z_{ij}$
0	[-57, -81,318, 1188]	342	0.5	[-125, -81,318, 693]	201.25
0.1	[-79.56, -81,318, 1079.4]	309.115	0.6	[-125.16, -81,318, 608.4]	180.06
0.2	[-97.64, -81,318, 975.6]	278.74	0.7	[-120.84, -81,318, 528.6]	161.19
0.3	[-111.24, -81,318, 876.6]	250.59	0.8	[-112.04, -81,318, 453.6]	144.64
0.4	[-120.36, -81,318, 782.4]]	224.76	0.9	[-98.76, -81,318, 383.4]	130.41

- $\sum_{i,j} z_{ij}$ - T*he fuzzy transportation cost by using the measure function*`

8. Result and Discussion

In this paper, the fuzzy transportation problem has been converted into a crisp transportation problem using Robust Ranking Method. We have obtained an optimal solution for the fuzzy transportation problem of minimal cost using the fuzzy trapezoidal membership function. Using various multipliers, many optimal solutions have been derived. Robust ranking method is applied to arrange the fuzzy numbers in a specific interval. The optimal solutions are obtained using a new algorithm by applying the zero termination method.

The proposed method provides more options and this can serve an important tool in decision making problem. Also, a new equation is formed using alpha-cut method. The new equation exists and it satisfies the optimal solution.

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