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RESEARCH ARTICLE

EFFECTS OF ADDITIVES IN FINITE JOURNAL BEARING USING FINITE DIFFERENCE METHOD

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Abstract

In this chapter, a generalized form of Reynolds equation for liquid film lubrication with slip at the bearing surface is derived, by considering the variation of fluid properties across the film thickness and is applied to study the effects of velocity slip and viscosity variation in finite journal bearing using FDM Technique. The effect of two layer increases the pressure and increases load.

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1. INTRODUCTION

In general, most of the lubricated systems can be considered to consist of moving (stationary) surfaces (plane/ curve or loaded/ unloaded) with a thin film of an external material (lubricant) between them. The presence of such a thin film between these surfaces not only helps to support considerable load but also minimizes the friction. The characteristics such as pressure in the film, frictional force at the surfaces, flow rate of lubricant etc., of the system depends upon the nature of the surfaces, nature of the lubricant film boundary conditions etc.

The equation governing the pressure generated in the lubricant film can be obtained by coupling the equation of motion with the equation of continuity and was first derived by Reynolds [18] and is known as "Reynolds Equation". In deriving this equation thermal effects, compressibility, viscosity variation, slip at the surfaces, inertia and surface roughness effects were ignored. Later this equation is modified by Cope [6] by including viscosity and density variation along the fluid film. The viscosity variation along the film thickness has been considered by Zienkiewicz and Cameron [5, 27, and 28] who also pointed out that temperature gradient and viscosity variation across the film may not be ignored. In the year 1962, Dowson [8] unified the various attempts in generalizing the Reynolds equation by considering the variation of fluid properties across as well as along the fluid film-thickness by neglecting slip effects at the bearing surfaces. Since then many workers have studied the effects of viscosity variation in lubricated systems by considering Reynolds equation with energy equation. A different approach which avoids the use of an energy equation in the study of viscosity variation has also been proposed by the workers [14, 15 and 23] by assuming a relationship between viscosity and film thickness.

It may be noted that the effects of velocity-slip at the surface, is important on the flow behavior of gases and liquids particularly when the film-thickness is very small [9, 10, and 11], the surface is very smooth [7] and the porous boundary [1, 20, and 22].

To study the effects of slip, Burgdorfer [4] modified the Reynolds equation for gas lubricated hydrodynamic bearings with “slip flow” under isothermal condition and pointed out that if $0 < \frac{h}{\lambda} < 1$ gas flow may be continuous and analysis can be carried out with modified slip boundary condition. Hsing and Malanoski [10] studied the effect of molecular mean free path in spiral-grooved thrust bearing and Tseng [25] used the Reynolds equation with slip to study the rarefaction effects of gas - lubricated bearings in the magnetic recording disk file.

The boundary conditions for the slip flow at the surface, in the case of gas bearing, can be written as, Kennard [4] and Burgdorfer [3]

$$u_{slip} = \alpha \left(\frac{2-f}{f} \right) \lambda \left(\frac{\partial u}{\partial z} \right)_{wall} + \dots \quad (1)$$

where f is the reflection coefficient, λ is the mean free path and α is a numerical constant. Because α and f are closed to unity, it may be considered that $\alpha \left(\frac{2-f}{f} \right)$ is unity Burgdorfer [6]. As molecular mean free path λ depends upon the fluid viscosity, pressure and temperature, it can be approximated by the following relation, Tseng [25]

$$\lambda = \frac{16}{5\sqrt{2\pi}} \frac{\eta}{p} \sqrt{RT} \quad (2)$$

where R is the gas constant and T is the temperature and η is the viscosity of the gas and p is its pressure.

The effect of slip is also important on the behavior of liquids especially when the bearing surface is very smooth and is operating at higher temperatures where the viscosity of the base oil decreases near the surface [2, 13]. This effect has been studied for liquids [2, 4, and 13]. The slip velocity at the wall can be written as

$$u_{slip} = \frac{1}{\beta} \left(\eta \frac{\partial u}{\partial z} \right)_{wall} \quad (3)$$

where β is the coefficient of sliding friction at the wall and η is the liquid viscosity.

2. MATHEMATICAL FORMULATION OF THE PROBLEM

The Physical configuration of the journal bearing is shown in fig (1). Here c be the clearance of the bearing, $c = r-R$ and $\varepsilon = \frac{e}{c}$ be the eccentricity ratio

h is the total film thickness equation for journal bearing and is given by

$$h = c(1 + \varepsilon \cos \theta)$$

$$\frac{\partial h}{\partial \theta} = -\varepsilon \sin \theta \quad (4)$$

The equation of governing to the fluid flow in the bearing derived [Appendix-D] is given by

$$\frac{\partial}{\partial x} \left[F \frac{h^3}{12\mu} \frac{\partial p}{\partial x} \right] + \frac{\partial}{\partial y} \left[\frac{h^3}{12\mu} F \frac{\partial p}{\partial y} \right] = U \frac{\partial h}{\partial x} \tag{5}$$

Where

$$F(h, a, \beta) = \frac{\left(1 - \frac{a}{h}\right)^3 (k-1) + 1}{k} + \frac{6\mu}{h\beta} \tag{6}$$

$x=R\theta, dx=Rd\theta$

The non-dimensional parameters are

$$\bar{y} = \frac{y}{L} \Rightarrow y = \bar{y}L, dy = Ld\bar{y}$$

$$\bar{a} = \frac{a}{c}, \bar{h} = \frac{h}{c}, \lambda = \frac{L}{2R} \tag{7}$$

By substituting equation (7) in (3), then the modified Reynolds equation in a non dimensional form can be written as

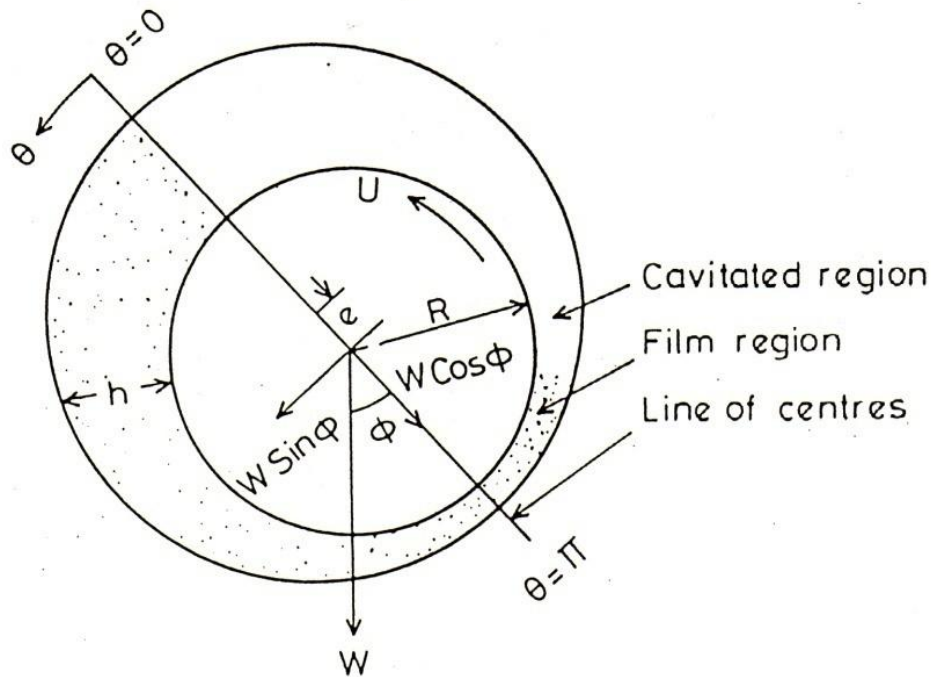


Fig (1): Journal bearing configuration

$$\frac{\partial}{R\partial\theta}\left[\frac{\bar{F}}{12\mu}\frac{\bar{h}^3}{R}\frac{\partial p}{\partial\theta}\right]+\frac{\partial}{L\partial y}\left[\frac{\bar{h}^3}{12\mu}\frac{\bar{F}}{L}\frac{\partial p}{\partial y}\right]=\frac{U}{R}\frac{\partial\bar{h}}{\partial\theta} \quad (8)$$

$$\frac{\partial}{\partial\theta}\left[\frac{\bar{F}}{12\mu}\frac{\bar{h}^3}{\partial\theta}\frac{\partial p}{\partial\theta}\right]+\frac{1}{4\lambda^2}\frac{\partial}{\partial y}\left[\frac{\bar{h}^3}{12\mu}\frac{\bar{F}}{\partial y}\frac{\partial p}{\partial y}\right]=RU\frac{\partial\bar{h}}{\partial\theta} \quad (9)$$

where $\lambda^2 = \frac{L^2}{4R^2}$

$$\bar{F}(\bar{h}, \bar{a}, \beta) = \frac{\left(1 - \frac{\bar{a}}{\bar{h}}\right)^3 (k-1) + 1}{k} + \frac{6\mu}{h\beta} \quad (10)$$

$$\bar{h} = c(1 + \varepsilon \cos \theta)$$

By solving the above equation (9), we get the non- dimensional pressure as

$$\bar{p} = \frac{pc^2}{\mu UR} \quad (11)$$

Now the equation (8) reduced to

$$\frac{\partial}{\partial\theta}\left[\frac{\bar{F}\bar{h}^3}{\partial\theta}\frac{\partial p}{\partial\theta}\right]+\frac{1}{4\lambda^2}\frac{\partial}{\partial y}\left[\frac{\bar{F}\bar{h}^3}{\partial y}\frac{\partial p}{\partial y}\right]=-12\varepsilon\sin\theta \quad (12)$$

The boundary conditions for fluid film pressure are

$$\begin{aligned} \bar{p} &= 0 \text{ at } \theta = 0 \\ \bar{p} &= 0 \text{ at } \theta = \pi \end{aligned} \quad (13)$$

The modified Reynolds equation is solved numerically using Finite difference method. The film domain under consideration is divided by grid points as shown in fig (2). In finite increment format, the terms of equation (12) can be written as

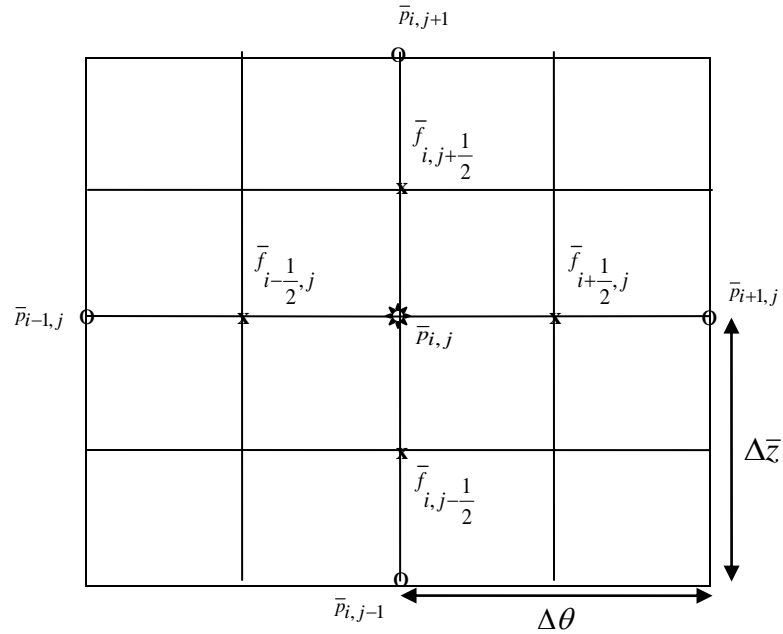


Fig 2: Grid point notation for film domain

$$\begin{aligned}
 & \frac{1}{\Delta\theta} \left[\bar{F}_{i+\frac{1}{2},j} \left(\frac{\bar{P}_{i,j} - \bar{P}_{i-1,j}}{\Delta\theta} \right) \right] - \left[\bar{F}_{i-\frac{1}{2},j} \left(\frac{\bar{P}_{i+1,j} - \bar{P}_{i,j}}{\Delta\theta} \right) \right] + \\
 & \frac{1}{4\lambda^2} \frac{1}{\Delta y} \left[\bar{F}_{i,j+\frac{1}{2}} \left(\frac{\bar{P}_{i,j} - \bar{P}_{i,j-1}}{\Delta y} \right) - \left[\bar{F}_{i,j-\frac{1}{2}} \left(\frac{\bar{P}_{i,j+1} - \bar{P}_{i,j}}{\Delta y} \right) \right] \right] = -12\varepsilon \sin \theta
 \end{aligned}
 \tag{14}$$

By solving the above we get

$$\begin{aligned} & \frac{1}{(\Delta\theta)^2} \left[\bar{F}_{i+1/2,j} \bar{p}_{i,j} - \bar{F}_{i+1/2,j} \bar{p}_{i+1,j} - \bar{F}_{i-1/2,j} \bar{p}_{i+1,j} + \bar{F}_{i-1/2,j} \bar{p}_{i,j} \right] + \\ & \frac{1}{4\lambda^2} \frac{1}{\Delta y^2} \left[\bar{F}_{i,j+1/2} \bar{p}_{i,j} - \bar{F}_{i,j+1/2} \bar{p}_{i,j-1} - \bar{F}_{i,j-1/2} \bar{p}_{i+1,j} + \bar{F}_{i,j-1/2} \bar{p}_{i,j} \right] = -12\varepsilon \sin \theta \\ & 4\lambda^2 r^2 \left[\bar{F}_{i+1/2,j} \bar{p}_{i,j} - \bar{F}_{i+1/2,j} \bar{p}_{i-1,j} - \bar{F}_{i-1/2,j} \bar{p}_{i+1,j} + \bar{F}_{i-1/2,j} \bar{p}_{i,j} \right] + \\ & \left[\bar{F}_{i,j+1/2} \bar{p}_{i,j} - \bar{F}_{i,j+1/2} \bar{p}_{i,j-1} - \bar{F}_{i,j-1/2} \bar{p}_{i,j+1} + \bar{F}_{i,j-1/2} \bar{p}_{i,j} \right] = -12\varepsilon 4\lambda^2 \sin \theta \Delta y^2 \end{aligned} \tag{15}$$

By substituting this equation in (12), we get

$$\begin{aligned} & \bar{p}_{i,j} \left[4\lambda^2 r^2 \left(\bar{F}_{i+1/2,j} + \bar{F}_{i-1/2,j} \right) + \left(\bar{F}_{i,j+1/2} + \bar{F}_{i,j-1/2} \right) \right] = -12\varepsilon 4\lambda^2 \Delta y^2 \sin \theta + 4\lambda^2 r^2 \bar{F}_{i+1/2,j} \bar{p}_{i-1,j} \\ & + 4\lambda^2 r^2 \bar{F}_{i-1/2,j} \bar{p}_{i+1,j} + \bar{F}_{i,j+1/2} \bar{p}_{i,j-1} + \bar{F}_{i,j-1/2} \bar{p}_{i,j+1} \\ & c_0 \bar{p}_{i,j} = 4\lambda^2 r^2 \bar{F}_{i+1/2,j} \bar{p}_{i-1,j} + 4\lambda^2 r^2 \bar{F}_{i-1/2,j} \bar{p}_{i+1,j} + \bar{F}_{i,j+1/2} \bar{p}_{i,j-1} + \bar{F}_{i,j-1/2} \bar{p}_{i,j+1} + -12\varepsilon 4\lambda^2 \Delta y^2 \sin \theta \\ & \bar{p}_{i,j} = c_1 \bar{p}_{i-1,j} + c_2 \bar{p}_{i+1,j} + c_3 \bar{p}_{i,j-1} + c_4 \bar{p}_{i,j+1} + c_5 \end{aligned} \tag{16}$$

The coefficient $c_0, c_1, c_2, c_3, c_4, c_5$, defined as

$$\begin{aligned} c_0 &= \left[4\lambda^2 r^2 \left(\bar{F}_{i+1/2,j} + \bar{F}_{i-1/2,j} \right) + \left(\bar{F}_{i,j+1/2} + \bar{F}_{i,j-1/2} \right) \right], \\ c_1 &= 4\lambda^2 r^2 \bar{F}_{i+1/2,j} / c_0 \\ c_2 &= 4\lambda^2 r^2 \bar{F}_{i-1/2,j} / c_0 \\ c_3 &= \bar{F}_{i,j+1/2} / c_0 \\ c_4 &= \bar{F}_{i,j-1/2} / c_0 \\ c_5 &= -48\lambda^2 \Delta y^2 \varepsilon \sin \theta / c_0 \\ \bar{r} &= \frac{\Delta z}{\Delta \theta} \end{aligned} \tag{17}$$

The pressure p is calculated numerically with grid spacing of $\Delta\bar{\theta} = 0.05$ and $\Delta\bar{z} = 9^0$

The load carrying capacity of the bearing is W , generated by the film pressure and is obtained as

$$W = \int_{\theta=0}^{\theta=\pi} \int_{z=0}^{z=1/2} p \cos \theta d\theta.dz \tag{18}$$

By using (12)in (17), we get the non dimensional load as

$$\bar{W} = \frac{WC^2}{\mu UR} = \int_{\theta=0}^{\theta=\pi} \int_{z=0}^{z=1/2} p \cos \theta d\bar{\theta} d\bar{z} \quad (19)$$

$$\approx \bar{w} = \sum_{i=0}^M \sum_{j=0}^N \bar{p}_{ij} \Delta\bar{\theta} \Delta\bar{z} \quad (20)$$

Where M+1 and N+1 are the grid point numbers in the x and z direction respectively.

3. RESULT AND DISCUSSION

The pressure in equation (16), the mesh of the film domain has 20 equal intervals along the bearing length and circumference. The coefficient matrix of the system of algebraic equation is of pentadiagonal form. These equations have been solved by using sci-lab tools.

Pressure

Fig (3) shows that the non-dimensional pressure \bar{p} for different peripheral layer thickness \bar{a} . From this we can say the pressure increases with the decrease of peripheral layer thickness \bar{a} and the corresponding pressure values are also shown. Fig (4) shows that the non-dimensional pressure \bar{p} for different slip parameter $\bar{\beta}$. From this we can say the pressure increases with the increase of slip parameter β and the corresponding pressure values are also shown. Fig (5) shows that the non-dimensional pressure \bar{p} for different k. From this we can say the pressure increases with the increase of k and the corresponding pressure values are also shown.

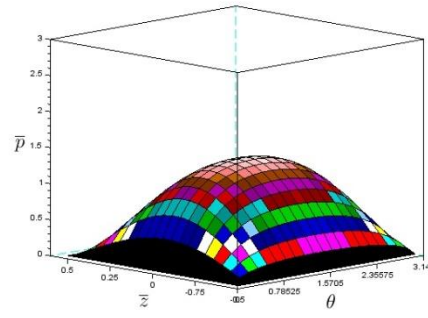
Load carrying capacity

Fig (6) shows that the variation of non-dimensional load carrying capacity \bar{W} with ϵ for different values of peripheral layer thickness \bar{a} . From this we can say that the load capacity increases with the increase of eccentricity ϵ and it decreases with the increase of peripheral layer thickness and the corresponding values of load for different \bar{a} are shown in table (1).

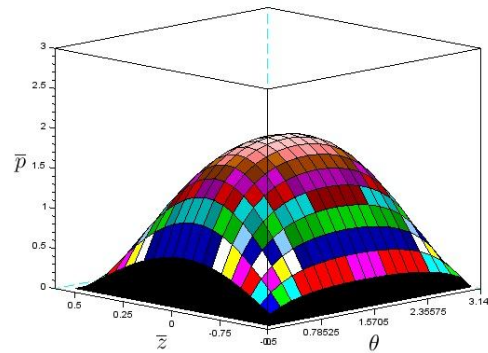
Fig (7) shows that the variation of non-dimensional load carrying capacity \bar{W} with ϵ for different values of slip parameter $\bar{\beta}$. From this we can say that the load capacity increases with the increase of eccentricity ϵ and also increases with the increase of slip parameter β , the corresponding values of load for different $\bar{\beta}$ are shown in table (2).

Fig (8) shows that the variation of non-dimensional load carrying capacity \bar{W} with ϵ for different values of k. From this we can say that the load capacity increases with the increase of eccentricity ϵ and also increases with the increase of k, the corresponding values of load for different k are shown in table (3).

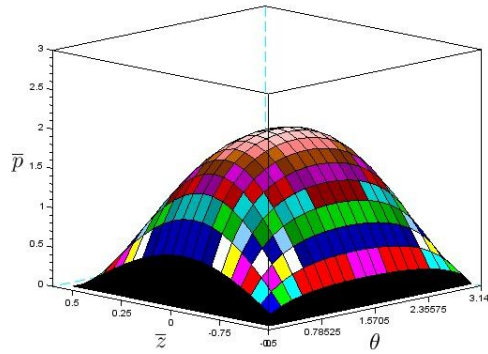
4. GRAPHS



$(\bar{a}=0.1)1.342797$

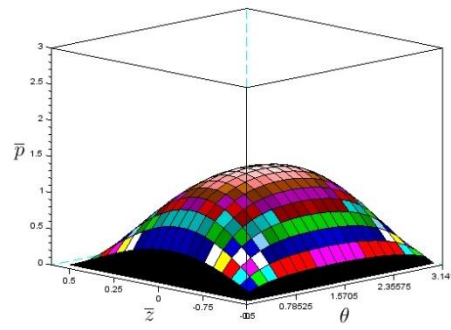


$(\bar{a}=0.01)1.8938013$

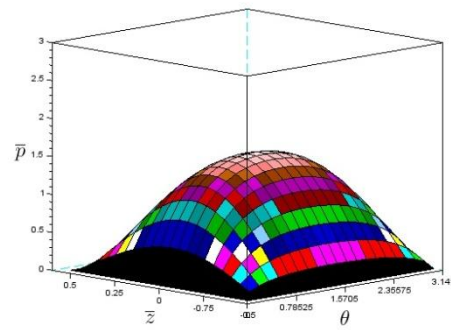


$(\bar{a}=0.001)1.975432$

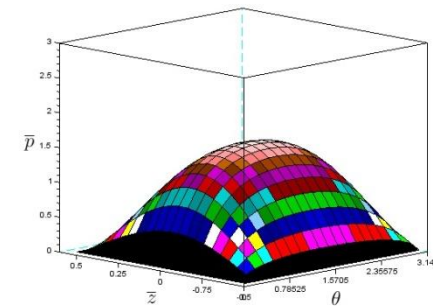
Fig 3: Non-Dimensionless pressure \bar{p} for different values of \bar{a} with $k=0.5, r=1.5, \varepsilon=0.4, \lambda=0.75, \bar{\beta}=10$



$(\bar{\beta}=10)1.342797$

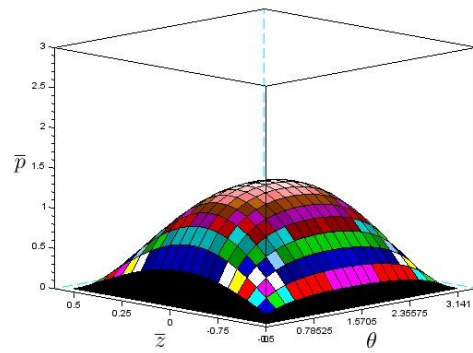


$(\bar{\beta}=100)1.5408451$

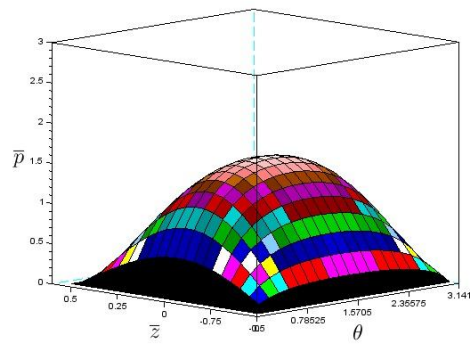


$(\bar{\beta}=1000)1.566559$

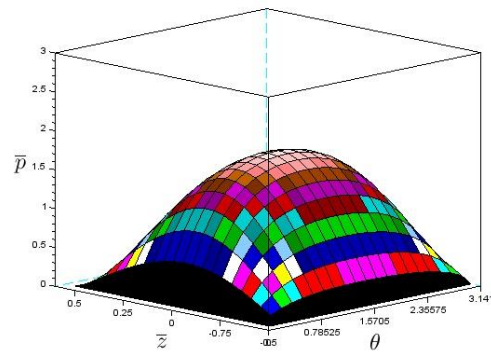
Fig 4: Non-Dimensionless pressure \bar{p} for different values of $\bar{\beta}$ with $k=0.2$, $r=1.5$, $\varepsilon=0.4$, $\lambda=0.75$, $\bar{a}=0.1$



(k=0.5)1.342797



(k=1)1.5695005



(k=2)1.7145365

Fig 5: Non-Dimensionless pressure \bar{p} for different values of k with $\bar{a}=0.1$, $r=1.5$, $\varepsilon=0.4$, $\lambda=0.75$ $\bar{\beta}=10$

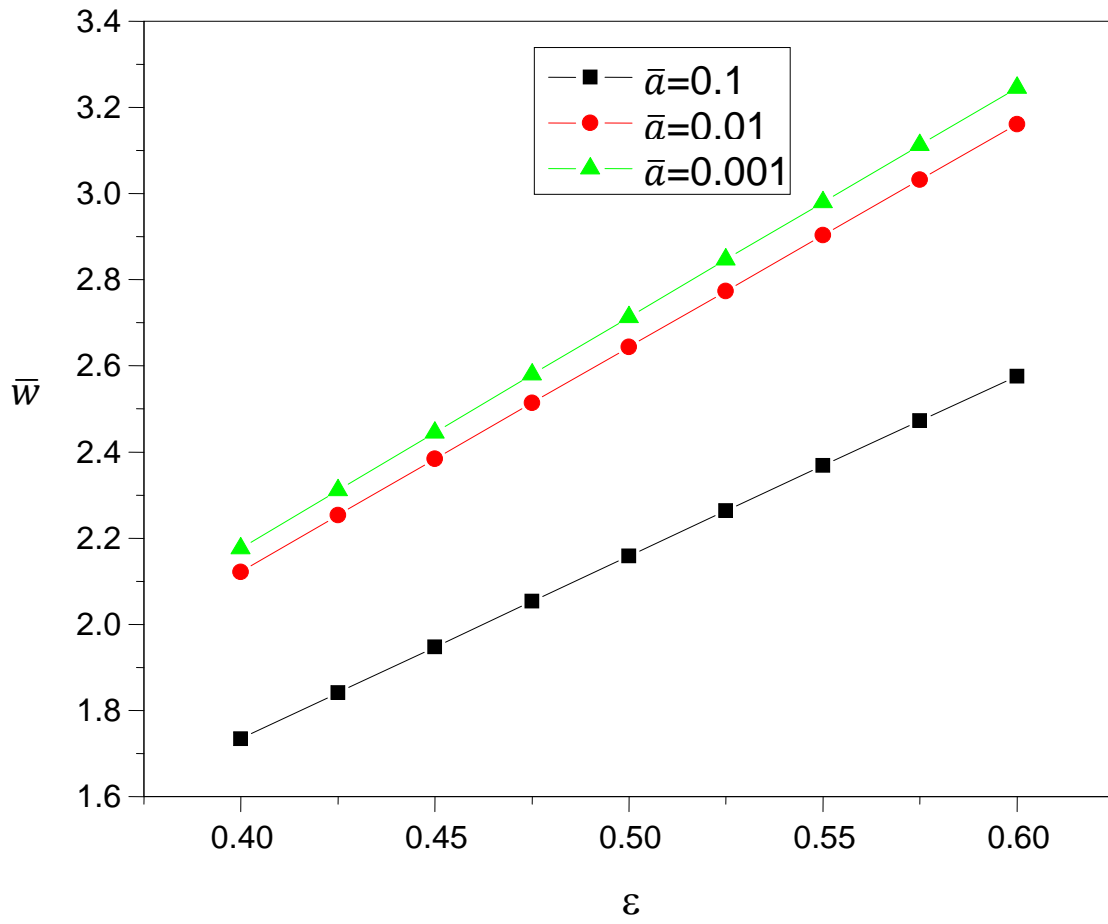


Fig 6: Dimensionless load \bar{W} Vs ϵ for different \bar{a} at $k=0.5$

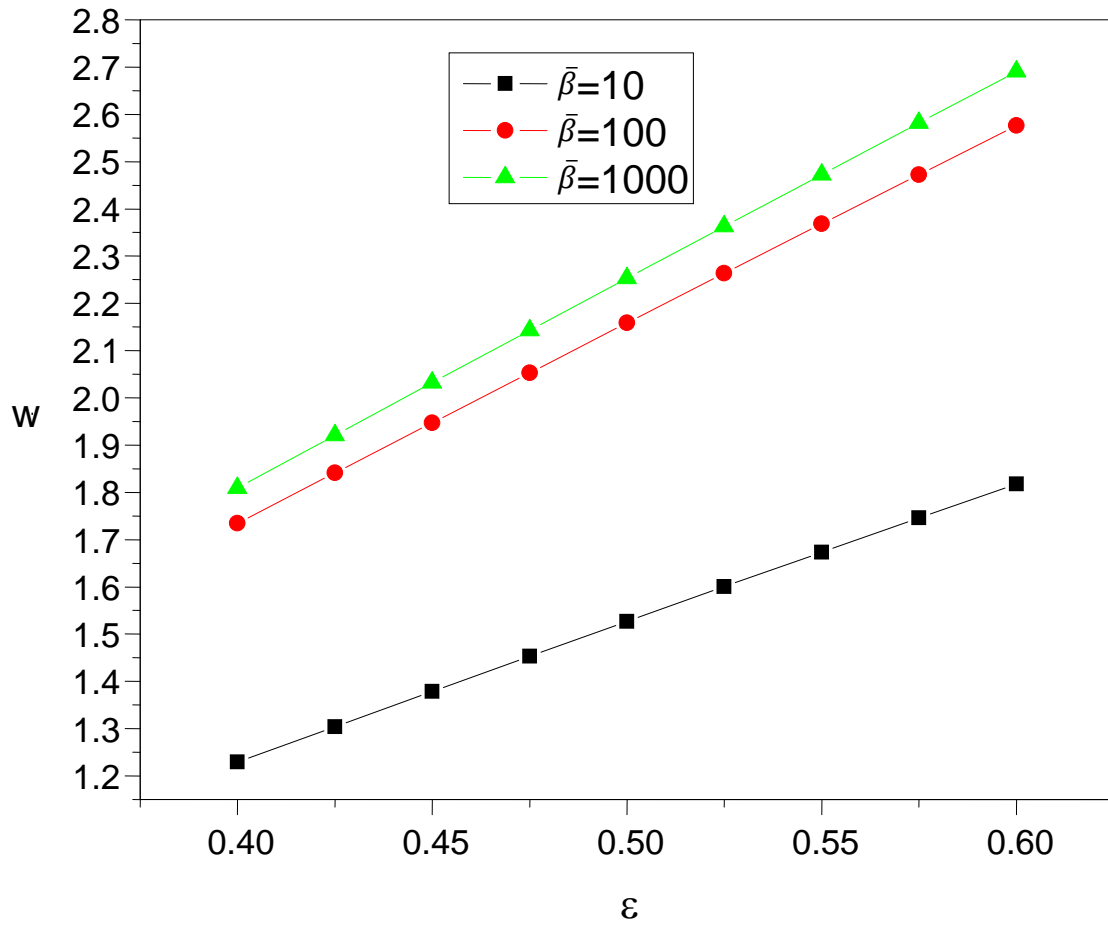


Fig 7: Dimensionless load \bar{W} Vs ε for different $\bar{\alpha}$ at $k=1$

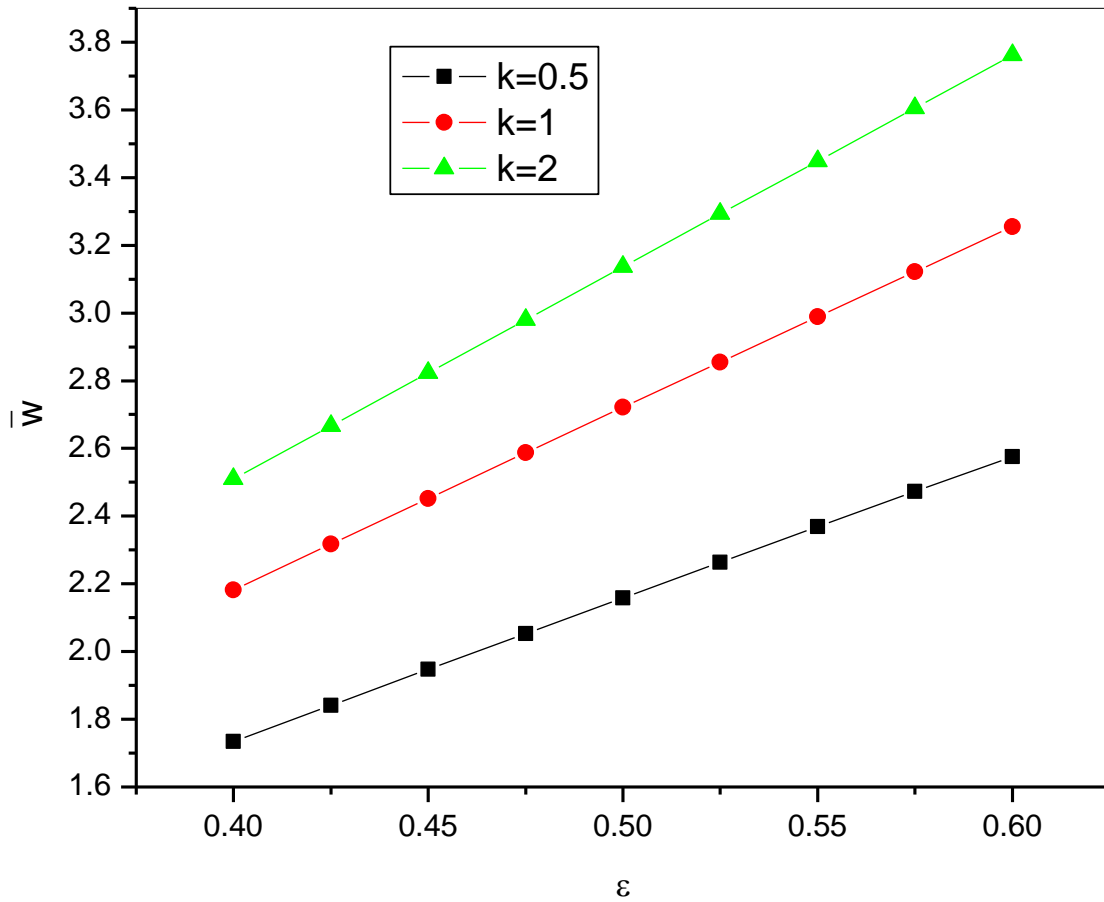


Fig 8: Dimensionless load \bar{W} Vs ϵ for different $\bar{\alpha}$ at $k=1.5$

5. TABLES

ϵ	$\bar{\beta}=10$	$\bar{\beta}=100$	$\bar{\beta}=1000$
0.4	1.2292188	1.734273	1.8091845
0.425	1.3041889	1.8408931	1.9206303
0.45	1.3788031	1.9471677	2.0317586
0.475	1.4530354	2.053071	2.1425461
0.5	1.5268589	2.1585764	2.2529689
0.525	1.6002452	2.2636559	2.3630023
0.55	1.6731648	2.3682807	2.4726206
0.575	1.7455865	2.4724205	2.5817977
0.6	1.8174773	2.5760435	2.6905061

ϵ	$\bar{a}=0.1$	$\bar{a}=0.01$	$\bar{a}=0.001$
0.4	1.734273	2.1217389	2.1762447
0.425	1.8408931	2.2528299	2.3109091
0.45	1.9471677	2.3836248	2.4453124
0.475	2.053071	2.5140997	2.5794337
0.5	2.1585764	2.6442288	2.713251
0.525	2.2636559	2.7739848	2.8467404
0.55	2.3682807	2.9033382	2.9798765
0.575	2.4724205	3.0322567	3.1126317
0.6	2.5760435	3.1607055	3.2449756

ϵ	K=0.5	K=1	K=1.5
0.4	1.734273	2.1825482	2.5099525
0.425	1.8408931	2.3176283	2.6665624
0.45	1.9471677	2.4524518	2.8231338
0.475	2.053071	2.5869983	2.9796645
0.5	2.1585764	2.7212463	3.1361519
0.525	2.2636559	2.8551724	3.2925926
0.55	2.3682807	2.9887519	3.4489821
0.575	2.4724205	3.1219575	3.6053147
0.6	2.5760435	3.2547598	3.7615825

6. SUMMARY

In this paper a journal bearing considered and studied the effect of additives in lubrication of journal bearing. The generalized Reynolds equation for two layer fluid is applied to finite journal bearing. The finite modified Reynolds type equation is obtained solved numerically by using FDM technique with a grid space of $\theta=9^0$ and $\Delta z=0.05$. The effect of two layer increases the pressure and increases load when $k>1$ and decreases for $K<1$.

NOMENCLATURE

a	peripheral layer thickness
p	pressures
h	Film thickness
μ	Viscosity of the fluid
x,y,z	Cartesian coordinates
θ	Circumference angle
c	Clearance

e	Eccentricity
R	Radius of the shaft
K	Ratio of viscosity near the surface to the purely hydrodynamic
u, v, w	Velocity component of the film in x,y,z direction
U	Velocity
ε	Eccentricity ratio

REFERENCES

1. Beavers, G.S., Joseph, D.D., "Boundary condition at a naturally permeable wall", J. Fluid Mech., Vol.30.(1967), P.197.
2. Bird, R. B., "Theory of diffusion", In advances in Chemical Engineering, T. B. Drew and J. W. Hoopes, Jr. (Eds.), Academic Press, N. Y., Vol. 1 (1956), p. 195.
3. Bramhall, A. D., Hutton, J. F., "Wall effect in the flow of lubricating greases in plunger viscometers", Brit. J. App. Phys., Vol. 11 (1960), p. 363.
4. Burgdorfer, A., "The influence of molecular mean free path on the performance of hydrodynamic gas lubricated bearing", J. Bas. Eng. Vol. 81D (1959), p. 94.
5. Cameron, A. "The viscous wedge", Trans. ASME, Vol. 1 (1958), p. 248.
6. Cope, W.F., "The hydrodynamic theory of film lubrication", Proc.Soc.Lond., 197 (1949), p.201.
7. Davenport, T. C., "The Rheology of lubricants", Wiley N. Y., Vol. 19 (1973), p. 100.
8. Dowson, D., "A generalized Bramhall, A. D., Hutton, J. F., "Wall effect in the flow of lubricating greases in plunger viscometers", Brit. J. App. Phys., Vol. 11 (1960), p. 363.
9. Gross, W.A., "Gas film lubrication", Wiley, N.Y.(1962).
10. Hsing, F. C., Malanoski, S. B., "Mean free path effect in spiral – grooved thrust bearing", J. Lub. Tech., Trans. ASME, Vol. 91F (1969), p. 69.
11. Kennard, E. H., "Kinetic theory of gases", McGraw Hill Book Comp., Inc. N. Y., (1938), p. 292.
12. Khonasari,M.M., and Brewe, D., "On the performance of finite journal bearing lubricated with micropolar Fluid", Trans.Tribol, Vol. 32 (2), pp. 155-160, 1989.
13. Lamb, H., "Hydrodynamic", Dover, N. Y. (1945), p. 576.
14. Quale, E.B., Wiltshire, F.R., "The performance of hydrodynamic lubricating films with viscosity variation perpendicular to the direction of motion", ASME Publication. Paper No. 72, Lub.Eng.
15. Murti, P.R.K., "Hydrodynamic lubrication of long porous bearings", Wear, Vol. 18, pp. 449- 460, 1971.
16. Naduvinamani, N.B.,and Kashinath, B., "Surface roughness effects on the static and dynamic behaviour of squeeze film lubrication of short journal bearings with micropolar fluids." J. Engg. Tribol, Vol. 222, pp. 1-11, 2008.
17. Prakash, J. and Sinha, P., "Cyclic Squeeze films in micropolar fluid lubricated journal bearings". Trans. ASME. J. Lubr. Technol, Vol. 98, pp. 412-417, 1975.
18. Reynolds, O., "On the theory of lubrication and its application to Mr. Beauchamp Tower Experiment", Phil. Roy. Soc. Lon., 177 Part 1 (1886), p.157.
19. Reynolds equation for fluid film lubrication", Int. J. Mech. Sci., Vol. 4 (1962), p. 159.

20. Richardson, S., "A model for the boundary condition of a porous material, Part", *J. Fluid Mech.*, Vol.49(1971),p.327.
21. Sinha, P., "Dynamically loaded micropolar fluid lubricated journal bearings with special reference to the squeeze films under fluctuating loads", *Wear*, Vol. 45, pp. 279-292, 1977.
22. Taylor, G. I., "A model for the boundary condition of a porous Material Part1", *J.Fluid Mech.*, Vol.49(1971), p.319
23. Tipei, N., "Theory of lubrication", Stanford University Press, London(1962).
24. Tipei, N., Nica, A., "On the field of temperature in lubricated films", *Trans. ASME,J. Basic Eng.*, Vol. 89 (1967), P.483.
25. Tseng, R. C., "Rarefaction effects of gas lubricated bearings in a magnetic recording disk file", *J. Lub. Tech.*, *Trans. ASME*, Vol. 97F (1975), p.624.
26. Uma, S., "The analysis of double layered porous slider bearings", *Wear*, Vol. 42, pp.205-215, 1977.
27. Zienkiewicz. O.C., "A note on theory of hydrodynamic lubrication of parallel surface thrust bearing", *Ninth Int.Confr.App. Mech.*, (1951), P.251.
28. Zienkiewicz. O.C., " Temperature distribution within lubricating films between parallel surfaces and its effect on the pressure developed", *Proc.Confr. Lub. Wear IMF*, (1957), P. 135.