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Fine-Minimal Separation Axioms in Fine Topological Spaces

K. Rajak
St. Aloysius College (Autonomous), Jabalpur

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***Corresponding Author**

K. Rajak

Abstract

In this paper, the author has introduced some new separation axioms called fine-minimal T_0, fine-minimal c-regular and fine minimal completely regular spaces in fine-topological spaces. Also studied some of their basic properties in fine-topological spaces.

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1. Introduction

Powar P. L. and Rajak K. [10], have investigated a special case of generalized topological space called fine topological space. In this space, the authors have defined a new class of open sets namely fine-open sets which contains all $\alpha -$open sets, $\beta -$open sets, semi-open sets, pre-open sets, regular open sets etc.. By using these fine-open sets they have defined fine-irresolute mappings which includes pre-continuous functions, semi-continuous function, $\alpha -$continuouos function, $\beta -$continuous functions, $\alpha -$irresolute functions, $\beta -$irresolute functions, etc (cf. [6]-[10]).

In the present paper, the author has introduced some new separation axioms called fine-minimal T_0, fine-minimal c-regular and fine-minimal completely regular spaces in fine topological spaces and also studied some basic properties of them.

2. Preliminaries

Throughout this paper, (X, τ) and (Y, σ) means topological spaces. For a subset A of a space X the closure and interior of A with respect to τ are denoted by $\mathrm{cl}(A)$ and $\mathrm{int}\,(A)$. We use the following definitions:

Definition 2.1 A proper non-empty open subset U of a topological spaces X is said to be a minimal open set if any open set which is contained in U is ϕ or U (cf. [1]).

Definition 2.2 A proper non-empty open subset U of a topological spaces X is said to be maximal open set if any open set which is containing U is X or U (cf. [2]).

Definition 2.3 A proper non-empty closed subset F of a topological spaces X is said to be minimal closed set if any closed set which is contained in F is ϕ or F (cf. [3]).

Definition 2.4 A proper non-empty closed subset F of a topological spaces X is said to be maximal closed set if any closed set which is containing in F is X or F (cf. [3]).

Definition 2.5 A topological space (X, τ) is said to be T_{min}-space if every non-empty proper open subset of X is minimal open set (cf. [4]).

Definition 2.6 Let X and Y be the topological spaces. A map $f: X \rightarrow Y$ is called

1) Minimal continuous [4] if $f^{-1}(M)$ is an open set in X for every minimal open set M in Y.
2) Minimal irresolute if [4] $f^{-1}(M)$ is minimal open set in X for every minimal open set M in Y.
3) Minimal open [4] if f(M) is an open set in Y for every minimal open set M in X.

4) Strongly minimal open [4] if f(M) is minimal open in Y for every minimal open set M in X.
5) Minimal closed [4] if f(F) is a closed set in Y for every minimal closed set F in X.
6) Strongly minimal closed [4] if f(F) is minimal closed set in Y for every minimal closed set F in X.

Definition 2.7 A topological space X is said to be minimal T_0 (min-T_0) space if for any two distinct points x, y in X, there exists a minimal open set containing one of them but not the other (cf. [5]).

Definition 2.8 A topological space X is said to be:

1) Minimal c-regular (min c-regular) if for every point $x \in X$ and each minimal closed set F in X such that $x \notin F$, there exist open sets U, V in X such that $x \in U, F \subset V$ and $U \cap V = \phi$ (cf. [5]).
2) Minimal o-regular (min o-regular) if for every point $x \in X$ and each closed set F in X such that $x \notin F$, there exist minimal open sets M, N in X such that $x \in M$, $F \subset N$, and $M \cap N = \phi$ (cf. [5]).
3) Minimal completely regular (min –completely regular) if for every point $x \in X$ and each minimal closed set F in X such that $x \notin F$, there exists a continuous map $f: X \to [0, 1]$ such f(x) = 0 and f(F) = {1} (cf. [5]).

Definition 2.9 Let (X, τ) be a topological space we define

$\tau(A_\alpha) = \tau_\alpha$ (say) $= \{G_\alpha(\neq X) : G_\alpha \cap A_\alpha = \phi$, for $A_\alpha \in \tau$ and $A_\alpha \neq \phi, X$, for some $\alpha \in J$, where J is the index set.$\}$

Now, we define

$$\tau_f = \{\phi, X, \cup_{\{\alpha \in J\}} \{\tau_\alpha\}\}$$

The above collection τ_f of subsets of X is called the fine collection of subsets of X and (X, τ, τ_f) is said to be the fine space X generated by the topology τ on X (cf. [10]).

Definition 2.10 A subset U of a fine space X is said to be a fine-open set of X, if U belongs to the collection τ_f and the complement of every fine-open sets of X is called the fine-closed sets of X and we denote the collection by F_f (cf. [10]).

Definition 2.11 Let A be a subset of a fine space X, we say that a point $x \in X$ is a fine limit point of A if every fine-open set of X containing x must contains at least one point of A other than x (cf. [10]).

Definition 2.12 Let A be the subset of a fine space X, the fine interior of A is defined as the union of all fine-open sets contained in the set A i.e. the largest fine-open set contained in the set A and is denoted by f_{Int} (cf. [10]).

Definition 2.13 Let A be the subset of a fine space X, the fine closure of A is defined as the intersection of all fine-closed sets containing the set A i.e. the smallest fine-closed set containing the set A and is denoted by f_{cl} (cf. [10]).

Definition 2.14 A function f : (X, τ, τ_f) → (Y, τ ′, τ_f') is called fine-irresolute (or f-irresolute) if $f^{-1}(V)$ is fine-open in X for every fine-open set V of Y (cf. [10]).

3. Fine-minimal open sets

In section, author has defined the minimal open sets, minimal closed sets, minimal continuous functions etc. in fine-topological space.

Definition 3.1 A proper non-empty fine-open subset U of a fine-topological spaces X is said to be a fine-minimal open set if any fine-open set which is contained in U is ϕ or X.

Definition 3.2 A proper non-empty fine-open subset U of a fine-topological spaces X is said to be a fine-maximal open set if any fine-open set which is containing in U is X or U.

Definition 3.3 A proper non-empty fine-closed subset F of a fine-topological spaces X is said to be a fine-minimal closed set if any fine-closed set which is contained in F is ϕ or F.

Definition 3.4 A proper non-empty fine-closed subset F of a topological spaces X is said to be a fine-maximal closed set if any fine-closed set which is containing F is X or F.

Remark 3.1

- Every minimal-open set and maximal –open set is fine –open.
- Every minimal-closed set and maximal –closed set is fine –open.

Example 3.1 Let $X = \{a, b, c\}$ be a topological space with the topology $\tau_f = \{\phi, X, \{a\}, \{a, b\}\}$, $F_f = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, c\}\}$. Here, the minimal open sets are ϕ, $\{a\}, \{ b\}$ which are fine-open and minimal-closed sets are X, $\{b, c\}, \{ a, c\}$ which are fine-closed. Similarly, it may be checked that, the only miximal open sets are $X, \{a, b\}, \{a, c\}, \{b, c\}$ which are fine-open and maximal closed sets are $\phi, \{a\}, \{b\}, \{c\}$ which are fine-closed.

Definition 3.5 Let X and Y be the topological spaces. A map $f: X \to Y$ is called

7) Fine-minimal continuous if $f^{-1}(M)$ is a fine-open set in X for every fine-minimal open set M in Y.
8) Fine-minimal irresolute if $f^{-1}(M)$ is fine-minimal open set in X for every fine-minimal open set M in Y.
9) Fine-minimal open if f(M) is an fine-open set in Y for every fine-minimal open set M in X.
10) Fine-strongly minimal open if f(M) is fine-minimal open in Y for every fine-minimal open set M in X.
11) Fine-minimal closed if f(F) is a fine-closed set in Y for every fine-minimal closed set F in X.

12) Fine-strongly minimal closed if f(F) is fine-minimal closed set in Y for every fine-minimal closed set F in X.

Remark 3.1 From Definition 2.6, Definition 2.14 and Remark 3.1, we conclude the following:

$\Rightarrow$ Minimal- continuous
$\Rightarrow$ Minimal-irresolute
$\Rightarrow$ Minimal-open
Fine-irresolute mapping $\Rightarrow$ Strongly minimal -open
$\Rightarrow$ Minimal-closed
$\Rightarrow$ Strongly minimal-closed

Example 3.2 Let X = {a, b, c} with the topology τ = { $\emptyset$, X, {a}, {a, b}}, τ_f = {$\emptyset$, X, {a}, {a, b}, {a, c}, {b}, {b, c}} and Y= {1, 2, 3} with the topology τ' = {$\emptyset$, Y, {1, 2}}, τ'_f= { $\emptyset$, {1}, {1, 2}, {1, 3}, {2}, {2, 3}}.

We define a mapping f: X → Y such that f(a)= 1, f(b)= 2 and f(c)=3. It may be checked that the pre-images of fine-open sets {1}, {1, 2}, {1, 3}, {2}, {2, 3} of Y are {a}, {a, b}, {a, c}, {b}, {b, c}, which are fine-open in X, therefore f is fine-irresolute. Also, it may be easily checked that f is fine-minimal continuous, fine-minimal irresolute, fine-minimal open, fine-strongly minimal open and fine-minimal closed.

4. Fine-minimal T_0 space9424659031

In this section, author has defined fine-minimal separation axioms in fine-topological space.

Definition 4.1 A topological space X is said to be fine-minimal T_0 (f-min-T_0) space if for any two distinct points x, y in X, there exists a fine-minimal open set containing one of them but not the other.

Theorem 4.1 Every fine min-T_0 space in a fine-T_0 space.

Proof. Let X be a fine- min-T_0 space. To prove X is a T_0 space. Let x, y be two distinct points in X. Since X is fine-min-T_0 space, there exists a fine-minimal open set M in X such $x \in M, y \notin M$. Since every fine-minimal open set is a fine-open set, M is a fine-open set in X. Thus, for any two distinct points x, y, in X, there exists a fine-open set M in X such that $x \in M, y \notin M$. Hence, X is a fine-T_0 space.

Theorem 4.2 If $f: X \to Y$ is a bijection, fine-strongly minimal open map and X is fine min-T_0 space then, Y is also fine-min-T_0space.

Proof. Let $f: X \to Y$ be a bijection, fine-strongly minimal open map and X is a fine-min-T_0space. Let $y_1, y_2 \in Y$ with $y_1 \neq y_2$. Since f is bijection, there exist $x_1, x_2 \in X$ with $x_1, \neq x_2$ such that $f(x_1) = y_1, f(x_2) = y_2$. Since X is fine-min- T_0 space, there exists a fine-minimal open set M in X such that $x_1 \in M, x_2 \notin M$. Since f is fine-strongly minimal open map, f(M) is a fine-minimal open set in Y. Now, we have $x_1 \in M \Rightarrow f(x_1) \in f(M) \Rightarrow y_1 \in f(M)$ and $x_2 \notin M \Rightarrow f(x_2) \notin f(M) \Rightarrow y_2 \notin f(M)$. Thus, for any two distinct points y_1, y_2 in Y, there exist a fine-minimal open set f(M) in Y such that $y_1 \in f(M), y_2 \notin f(M)$. Hence, Y is a fine min-T_0 space.

Theorem 4.3 If $f: X \to Y$ is a bijection, fine- minimal irresolute map and Y is fine-min-T_0 space then, X is also fine-min-T_0 space.

Proof. Let $f: X \to Y$ be a bijection, fine-minimal irresolute map and Y is a fine-min-T_0space. To prove X is fine-min- T_0 space. Let $x_1, x_2 \in X$ with $x_1 \neq x_2$. Since f is bijection, there exist $y_1, y_2 \in X$ with $y_1, \neq y_2$ such that$f(x_1) = y_1, f(x_2) = y_2 \Rightarrow x_1 = f^{-1}(y_1)$ and $x_2 = f^{-1}(y_2)$. Since Y is fine-min- T_0 space, there exists a fine-minimal open set M in Y such that $y_1 \in M, y_2 \notin M$. Since f is fine- minimal irresolute map, $f^{-1}(M)$ is a fine-minimal open set in X. Now, we have $y_1 \in M \Rightarrow f^{-1}(y_1) \in f^{-1}(M) \Rightarrow x_1 \in f^{-1}(M)$ and $y_2 \notin M \Rightarrow f^{-1}(y_2) \notin f^{-1}(M) \Rightarrow x_2 \notin f^{-1}(M)$. Thus, for any two distinct points x_1, x_2 in X, there exist a fine-minimal open set $f^{-1}(M)$ in X such that $x_1 \in f^{-1}(M),\ x_2 \notin f^{-1}(M)$. Hence, X is a fine min-T_0 space.

Theorem 4.4 If $f: X \to Y$ is a bijection, fine- minimal continuous map and Y is fine-min-T_0 space then, X is also fine-T_0 space.

Proof. Similar to that of Theorem 4.4.

5. Fine –minimal c-regular spaces and Minimal completely regular spaces.

In this section, we define Fine –minimal c-regular spaces and fine-minimal completely regular spaces.

Definition 5.1 A topological space X is said to be:

1) Fine- minimal c-regular (f-min-c-regular) if for every point $x \in X$ and each fine-minimal closed set F in X such that $x \notin F$, there exist fine-open sets U, V in X such that $x \in U, F \subset V$ and $U \cap V = \phi$.
2) Fine-minimal o-regular (min o-regular) if for every point $x \in X$ and each fine-closed set F in X such that $x \notin F$, there exist fine-minimal open sets M, N in X such that $x \in M, F \subset N,$ and $M \cap N = \phi$.

3) Fine-minimal completely regular (min –completely regular) if for every point $x \in X$ and each fine-minimal closed set F in X such that $x \notin F$, there exists a fine-continuous map $f: X \to [0,1]$ such f(x) = 0 and f(F) = {1}.

Theorem 5.2 Every fine-regular space is fine-min c-regular space.

Proof. Let X be a fine-regular space. To prove X is fine-min c-regular. Let $x \in X$ and F be any fine-minimal closed set in X such that x $\notin F$. Since, every fine-minimal closed set is a fine-closed set, F is fine-closed in X such that $x \notin F$. Since X is fine-regular space, there exists fine-open sets U, V in X such that $x \in U, F \subset V$ and $U \cap V = \phi$. Thus, for every point $x \in X$ and each fine-minimal closed set F in X such that $x \notin F$, there exist fine-open sets U, V in X such that $x \in U, F \subset V$ and $U \cap V = \phi$. Hence, X is a fine-min c-regular space.

Theorem 5.3 Every fine-completely regular space is fine-min completely-regular space.

Proof. Let X be a fine-completely regular space. To prove X is fine-min completely regular. Let $x \in X$ and F be any fine-minimal closed set in X such that x $\notin F$. Since, every fine-minimal closed set is a fine-closed set, F is fine-closed in X such that $x \notin F$. Since X is fine completely regular space, there exists fine-continuous map $f: X \to Y$ such that $f(x) = 0$ and $f(F) = \{1\}$. Thus, for every point $x \in X$ and each fine-minimal closed set F in X such that $x \notin F$, there exist fine-continuous map $f: X \to [0,1]$ such that $f(x) = 0$ and $f(F) = \{1\}$. Hence, X is a fine-min completely-regular space.

6. CONCLUSION

New generalizations of separation axioms called fine-minimal T_0, fine-minimal c-regular and fine minimal completely regular spaces in fine-topological spaces have investigated and studied some of their basic properties in fine-topological spaces. This concept may have an extensive applicational value in quantum physics.

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