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News Solution for Improved Korteweg de Vries and Korteweg-de Vries in two dimension by sub-ODE method

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Abstract

New exact solutions of some important nonlinear partial differential equations in one and two dimensions are obtained by using the sub-ODE method. The sub-ODE method is described to construct several kinds of exact travelling wave solutions for the improved Korteweg-de Vries (IKdV) equation and two dimension Korteweg-de Vries (2D KdV) equation.

INTRODUCTION

Many phenomena in physics and other fields are described by nonlinear partial differential equations (NLPDEs) and the investigation of exact solution of these (NLPDEs) play an essential role in the study the nonlinear science, especially they may provide much physical information and more inside to the physical aspect of the problem and help one to understand the mechanism that governs these physical models or to better provide knowledge of the physical problem and thus lead to further applications. In the past decades, much effort has been spent on the construction of exact solutions of NLPDEs and many powerful methods have been developed such as the new generalized algebraic method [1], the sub-ODE method [2-5], the auxiliary ordinary differential equation (ODE) method [6], the F-expansion method [7], the extended Fan sub-equation method [8,9] and so on. The sub-ODEs which were often used are Riccati equation, Jacobi elliptic equation and projective Riccati equations, etc. Recently, Sirendaoreji [10,11] proposed a new auxiliary equation method by introducing a new first order nonlinear differential equation with six degree nonlinear term and its solutions to construct exact travelling wave solutions of NLPDEs in a unified way. Later on Zhang and Xia [12] improved the ansatz in [11,12] so that it can be used to construct more general exact solutions. The rest of this paper is organized as follows: in section 2, we give Generalized new auxiliary equation method, in section 3 we give a brief description of this new auxiliary equation method and apply this method to the improved Korteweg de Vries (IKdV) equation and the two dimension Korteweg de Vries (2D KdV) equation, [30] in section 4, conclusions are given.

2- Generalized new auxiliary equation method

In order to solve NLPDEs involving higher order nonlinear terms, Li and Wang [4] proposed a suitable subsidiary higher ODEs including an arbitrary positive power of dependent variable

$$F'^2(\xi) = A F^2(\xi) + B F^{2+n}(\xi) + C F^{2+2n}(\xi), \quad n > 0 \quad (1)$$

where A, B and C are constants. This method has been used by several authors [4,5]. The ODE proposed by Sirendaoreji et al in [10,11] can be viewed a special case of (1) (when $n = 2$)

$$F(\xi)^2 = A F(\xi)^2 + B F(\xi)^4 + C F(\xi)^6 \tag{2}$$

and a special case of (2) (when $n = 1$)

$$F(\xi)^2 = A F(\xi)^2 + B F(\xi)^3 + C F(\xi)^4 \tag{3}$$

respectively. By using solutions of Eq.(3), novel nontrivial solutions for Eq. (1) can be derived, as [5]

3 – Outline of the sub-ODE method and its applications

3-1 Brief introduction of Sub-ODE method [5]

Considering a given NLPDE with two independent variable (x, t) and dependent variable u :

$$H(x, t, u, u_x, u_t, \dots) = 0, \tag{4}$$

Where the subscripts denote derivatives about x and t

Step1 Making the travelling wave transformation.

$$u(x, t) = u(\xi), \quad \xi = x - ct. \tag{5}$$

Using Eq.(5) to change Eq.(4) to the following ordinary differential equation(ODE)

$$Q(u, u_\xi, u_{\xi\xi}, \dots) = 0, \tag{6}$$

Step 2 Supposing Eq. (6) has solutions of the form

$$u(\xi) = \mu F(\xi)^m, \tag{7}$$

where $F(\xi)$ satisfies Eq. (3), μ is a constant, m is an integer determined by balancing the highest order derivative term and the highest order nonlinear term in Eq. (6).

Step 3 Substituting Eq.(7) into Eq. (6) along with Eq. (3) and setting the coefficients of $F(\xi)^0, F(\xi)$, and $F(\xi)^2$ to zero, we get a system of algebraic equations. Solving the system of algebraic equations for c, μ, A, B , and C .

Step 4 Taking advantage of the solutions of Eq. (3), one can get exact solutions of the NLPDE in concern.

Theorem

The more exact solution of the NLPDEs can be determine by the find more the new solution of Eq.(1).

3.2 Applications new results

3.2.1 The improved Korteweg de Vries (IKdV) equation

We Consider the IKdV equation in the form [13]

$$u_t + u u_x - u_{xxt} + u_{xxx} = 0, \tag{8}$$

Eq. (8) has been studied [13], in the following; we obtain some new nontrivial solutions for Eq. (8) by means of the sub-ODE method. To look for new explicit travelling wave solutions of Eq. (8), we make the transformation $u(x, t) = u(\xi), \xi = x - ct$, where c is the wave speed .Eq. (8) becomes

$$-c u' + u u' + c u''' + u''' = 0, \tag{9}$$

$u' = \frac{du}{d\xi}, u''' = \frac{d^3u}{d\xi^3}$, And integrating the above equation with respect to, we get,

$$-c u + \frac{1}{2} u^2 + (c + 1)u'' = 0, \tag{10}$$

According to step 2 in sub sec. 3.1, balancing u^2 with u'' gives $m = 2$. thus we suppose solutions of Eq. (10) can be expressed by

$$u(\xi) = \mu F(\xi)^2, \tag{11}$$

where $F(\xi)$ satisfies Eq. (3), μ is a constant. Substituting Eq. (11) into Eq. (10) along with Eq. (3) and solving the obtained system of algebraic equations, we have,

$$\mu = \frac{12c}{4A-1}, \quad c = -\frac{4c}{4A-1}, \quad B = 0, \tag{12}$$

taking advantage of all solutions of Eq. (3), which are listed in [5], we can get the following new explicit travelling wave solutions for Eq. (8).

(i) If $A > 0, C < 0$, Eq. (8) has two bell profile solutions

$$u_1 = \mu \left\{ \frac{A \operatorname{sech}^2 \left[\frac{\sqrt{A}}{2} (\xi + \alpha) \right]}{\sqrt{-4AC} - (\sqrt{-AC}) \operatorname{sech}^2 \left[\frac{\sqrt{A}}{2} (\xi + \alpha) \right]} \right\}^2, \quad u_2 = \mu \left\{ \frac{\pm A \operatorname{sech} \left[\frac{\sqrt{A}}{2} (\xi + \alpha) \right]}{\sqrt{-AC}} \right\}^2, \tag{13}$$

and two singular solutions

$$u_3 = \mu \left\{ \frac{A \operatorname{csch}^2 \left[\frac{\pm \sqrt{A}}{2} (\xi + \alpha) \right]}{\sqrt{-4AC} + (\sqrt{-AC}) \operatorname{csch}^2 \left[\frac{\pm \sqrt{A}}{2} (\xi + \alpha) \right]} \right\}^2, \quad u_4 = \mu \left\{ \frac{\pm A}{\sqrt{-AC} \cosh \left[\frac{\sqrt{A}}{2} (\xi + \alpha) \right]} \right\}^2 \tag{14}$$

(ii) If $A > 0, C > 0$, Eq. (8) has the following combined soliton-like solutions

$$u_5 = \mu \left\{ \frac{\pm A}{\sqrt{AC} \sinh \left[\frac{\sqrt{A}}{2} (\xi + \alpha) \right]} \right\}^2, \quad u_6 = \mu \left\{ \frac{\pm A \operatorname{csch} \left[\frac{\sqrt{A}}{2} (\xi + \alpha) \right]}{\sqrt{AC}} \right\}^2,$$

$$u_7 = \mu \left\{ \frac{-A \operatorname{sech}^2 \left[\frac{\sqrt{A}}{2} (\xi + \alpha) \right]}{\pm 2\sqrt{AC} \tanh \left[\frac{\sqrt{A}}{2} (\xi + \alpha) \right]} \right\}^2, \quad u_8 = \mu \left\{ \frac{-A \operatorname{csch}^2 \left[\frac{\sqrt{A}}{2} (\xi + \alpha) \right]}{\pm 2\sqrt{AC} \coth \left[\frac{\sqrt{A}}{2} (\xi + \alpha) \right]} \right\}^2, \tag{15}$$

(iii) If $A < 0, C > 0$, Eq. (8) has the following combined triangular periodic solutions

$$u_9 = \mu \left\{ \frac{-A \operatorname{sec}^2 \left[\frac{\sqrt{-A}}{2} (\xi + \alpha) \right]}{\sqrt{-4AC} - (\sqrt{-AC}) \operatorname{sec}^2 \left[\frac{\sqrt{-A}}{2} (\xi + \alpha) \right]} \right\}^2, \quad u_{10} = \mu \left\{ \frac{A \operatorname{csc}^2 \left[\frac{\pm\sqrt{-A}}{2} (\xi + \alpha) \right]}{\sqrt{-4AC} - (\sqrt{-AC}) \operatorname{csc}^2 \left[\frac{\pm\sqrt{-A}}{2} (\xi + \alpha) \right]} \right\}^2, \tag{16}$$

And the following singular triangular periodic solutions

$$u_{11} = \mu \left\{ \frac{A}{\pm\sqrt{-A} C \cos \left[\sqrt{-A} (\xi + \alpha) \right]} \right\}^2, \quad u_{12} = \mu \left\{ \frac{A}{\pm\sqrt{-AC} \sin \left[\sqrt{-A} (\xi + \alpha) \right]} \right\}^2, \tag{17}$$

(iv) Eq. (8) has the following rational function solutions

$$u_{13} = \mu \left\{ \frac{4Ae^{\pm\sqrt{A}(\xi+\alpha)}}{e^{\pm 2\sqrt{A}(\xi+\alpha)} - 4AC} \right\}^2, \quad A > 0, \quad u_{14} = \mu \left\{ \frac{4Ae^{\pm\sqrt{A}(\xi+\alpha)}}{-1 + 4ACe^{\pm 2\sqrt{A}(\xi+\alpha)}} \right\}^2, \quad A > 0, \\ u_{15} = \mu \left\{ \frac{1}{\sqrt{C} \xi} \right\}^2, \quad C > 0, A = 0, \tag{18}$$

Where α is an arbitrary constant, $x = x - ct$, c and μ are determined by Eq. (12)

Comment 1

If we take $\alpha = 0$ and $A = \frac{c}{4(c+1)}$, the solutions u_2, u_{13} and u_{14} coincide with u in, [13]. However, the other solutions of Eq. (8) obtained in this paper have not been found before.

3.2.2 Two dimensions solitary wave equation

We consider the two dimensions Koreweg-de Vries (2D KdV) equation in the form [13]

$$(u_t - 6u u_x + u_{xxx})_x + 3u_{yy} = 0, \tag{19}$$

We use the transformation $u(x, y, t) = u(\eta)$, $\eta = x + \beta y - \omega t$, and change Eq. (19) into the form

$$(3\beta^2 - \omega) u'' - 6(u u_x)_x + u'''' = 0, \tag{20}$$

Where β and ω are constants, $u' = \frac{d u}{d \eta}$, $u'''' = \frac{d^4 u}{d \eta^4}$, and integrating the above equation with respect to η , we get

$$(3\beta^2 - \omega) u - 3u^2 + u'' = 0, \tag{21}$$

Balancing u^2 with u'' gives, $m = 2$. Thus we suppose solutions of Eq. (21) can be expressed by

$$u(\eta) = \delta F(\eta)^2, \tag{22}$$

Where $F(\eta)$ satisfies Eq. (3), δ is a constant. Substituting Eq. (22) into Eq. (21) along with Eq. (3) and solving the obtained system of algebraic equations, we have

$$\omega = 4A + 3\beta^2, \quad \delta = 2C, \quad B = 0, \tag{23}$$

Taking advantage of all solutions of Eq.(3), which are listed [5], we can get the following new explicit travelling wave solutions for Eq. (19).

(i) If $A > 0, C < 0$, Eq. (19) has two bell profile solutions

$$u_1 = \delta \left\{ \frac{A \left[\operatorname{sech}^2 \left[\frac{\sqrt{A}}{2} (\eta + \alpha) \right] \right]}{\sqrt{-4AC} - (\sqrt{-AC}) \operatorname{sech}^2 \left[\frac{\sqrt{A}}{2} (\eta + \alpha) \right]} \right\}^2, \quad u_2 = \delta \left\{ \frac{\pm A \operatorname{sech} \left[\sqrt{A} (\eta + \alpha) \right]}{\sqrt{-AC}} \right\}^2, \tag{24}$$

and two singular solutions

$$u_3 = \delta \left\{ \frac{A \operatorname{csch}^2 \left[\frac{\pm\sqrt{A}}{2} (\eta + \alpha) \right]}{\sqrt{-4AC} + (\sqrt{-AC}) \operatorname{csch}^2 \left[\frac{\pm\sqrt{A}}{2} (\eta + \alpha) \right]} \right\}^2, \quad u_4 = \delta \left\{ \frac{\pm A}{\sqrt{-AC} \cosh \left[\sqrt{A} (\eta + \alpha) \right]} \right\}^2, \tag{25}$$

(ii) If $A > 0, C > 0$, Eq. (19) has the following combined soliton-like solutions

$$u_5 = \delta \left\{ \frac{\pm A}{\sqrt{AC} \sinh \left[\sqrt{A} (\eta + \alpha) \right]} \right\}^2, \quad u_6 = \delta \left\{ \frac{\pm A \operatorname{csch} \left[\sqrt{A} (\eta + \alpha) \right]}{\sqrt{AC}} \right\}^2, \\ u_7 = \delta \left\{ \frac{-A \operatorname{sech}^2 \left[\frac{\sqrt{A}}{2} (\xi + \alpha) \right]}{\pm 2\sqrt{AC} \tanh \left[\frac{\sqrt{A}}{2} (\xi + \alpha) \right]} \right\}^2, \quad u_8 = \delta \left\{ \frac{-A \operatorname{csch}^2 \left[\frac{\sqrt{A}}{2} (\eta + \alpha) \right]}{\pm 2\sqrt{AC} \coth \left[\frac{\sqrt{A}}{2} (\eta + \alpha) \right]} \right\}^2, \tag{26}$$

(iii) If $A < 0, C > 0$, Eq. (19) has the following combined triangular periodic solutions

$$u_9 = \delta \left\{ \frac{-A \operatorname{sec}^2 \left[\frac{\sqrt{-A}}{2} (\eta + \alpha) \right]}{\sqrt{-4AC} - (\sqrt{-AC}) \operatorname{sec}^2 \left[\frac{\sqrt{-A}}{2} (\eta + \alpha) \right]} \right\}^2, \quad u_{10} = \delta \left\{ \frac{A \operatorname{csc}^2 \left[\frac{\pm\sqrt{-A}}{2} (\eta + \alpha) \right]}{\sqrt{-4AC} - (\sqrt{-AC}) \operatorname{csc}^2 \left[\frac{\pm\sqrt{-A}}{2} (\eta + \alpha) \right]} \right\}^2, \tag{27}$$

and the following singular triangular periodic solutions

$$u_{11} = \delta \left\{ \frac{A}{\pm\sqrt{-A} C \cos \left[\sqrt{-A} (\eta + \alpha) \right]} \right\}^2, \quad u_{12} = \delta \left\{ \frac{A}{\pm\sqrt{-AC} \sin \left[\sqrt{-A} (\eta + \alpha) \right]} \right\}^2 \tag{28}$$

(iv) Eq.(19) has the following rational function solutions

$$\begin{aligned} u_{13} &= \delta \left\{ \frac{4Ae^{\pm\sqrt{A}(\eta+\alpha)}}{e^{\pm 2\sqrt{A}(\eta+\alpha)} - 4AC} \right\}^2, A > 0, & u_{14} &= \delta \left\{ \frac{4Ae^{\pm\sqrt{A}(\eta+\alpha)}}{-1+4ACe^{\pm 2\sqrt{A}(\eta+\alpha)}} \right\}^2, A > 0 \\ u_{15} &= \delta \left\{ \frac{1}{\sqrt{C}\eta} \right\}^2, C > 0, A = 0 \end{aligned} \quad (29)$$

where α is an arbitrary constant, $\eta = x + \beta y - \omega t$, ω and δ are determined by Eq. (23).

Also, if we consider the two dimensions Korteweg-de Vries in the form

$$(u_t - 6u u_x + u_{xxx})_x - 3u_{yy} = 0, \quad (30)$$

by using the same transformation Eq.(30) become

$$(3\beta^2 + \omega)u + 3u^2 - u'' = 0, \quad (31)$$

The solutions for the Eq.(31) is the same solution of Eq.(19) with different only the value of the constant ω , where ω in this case $\omega = (4A - 3\beta^2)$

Comment2

If we take $\alpha = 0$, and $\omega = c$, the solutions u_2, u_{13} and u_{14} coincide with u in [13]. However, the other solutions of Eq. (19) obtained in this paper have not been found before.

4. Conclusion

In this paper, the auxiliary equation and its solutions given by Sirendaoreji [6] have been successfully used to obtain some exact travelling wave solutions for the improved Korteweg- de Vries (IKdV) equation and the two dimension Korteweg-de Vries (2D KdV) equation. As a result, families of novel exact solutions, which contain soliton solutions, triangular periodic solutions, and rational solutions, for the two equations are obtained. It can be clearly seen that some solutions given in this paper are new solutions which have not been reported yet. The method can also be efficiently used to construct new and more exact travelling wave solutions for some other generalized nonlinear wave equations arising in mathematical physics.

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