ISSN NO. 2320-5407

Journal homepage: http://www.journalijar.com

INTERNATIONAL JOURNAL OF ADVANCED RESEARCH

RESEARCH ARTICLE

Estimation of Impulsive Velocity for Nanosatellite Formation Keeping under Perturbation

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Manuscript Info

Manuscript History:

Received: 18 April 2015
Final Accepted: 28 May 2015
Published Online: June 2015

Key words:

Nanosatellite, Formation Flying, Fuel Consumption,Hill's Equations,Perturbing Acceleration,

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Abstract

In nanosatellite formation flying, the second geopotential harmonic gravitational acceleration is the dominant perturbation. In order to take this perturbation into account a set of linearized differential equations in Hill frame is presented. Based on these linearized equations, the estimation technique of the impulsive velocity to correct the relative position is newly presented, which only needs a set of the initial conditions. The estimated results are compared to those obtained by the integration of differential equations for the cases in which a corrective displacement exists either in the radial, in-track or cross track direction.

Introduction

Recently, the formation flying by nanosatellites gathers an attraction of engineers and researchers in field of space technologies. It can be applied for space and atmospheric observation, data communication, and high-resolution geographic imaging. For an example, CanX-4 and CanX-5 nanosatellites have a mission of the dual-nanosatellite formation proposed by UTIAS/SFL, University of Toronto, Institute for Aerospace Studies/Space Flight Laboratory, Canada. The goal is to prove that the nanosatellite formation flying can be accomplished with sub-meter tracking error accuracy for low ΔV requirements. In order to achieve this, the development of control algorithms for autonomous formation maintenance is necessary in the presence of orbital perturbations (Nathan et al., 2007).

The nanosatellites in a formation can be either in the same or in parallel orbits. Meanwhile, the amount of fuel consumption for keeping the formation under the existence of perturbations is limited. Nagai and Nakasuka, (2008), of University of Tokyo, had studied how much fuel is needed to keep a satellite formation. They discussed theoretically the formation parameterization and provided a convenient way to find the minimum fuel consumption for nanosatellite formation flying. They considered the J_2 perturbing force and established an essential ΔV using the *Gauss' Variation Equations* (GVE). Another work, Matko et al. (2011) analyzed an optimal fuel consumption including suitable manoeuvres with respect to orbital requirements for high resolution remote sensing by satellite constellation. They studied the relative motion of two satellites in a formation under no perturbing acceleration.

This paper describes a set of expanded linearized differential equations of relative motion under J_2 gravitational perturbation in Hill frame. First, the differential equations of relative motion that includes J_2 effect are integrated by Laplace techniques. Then, the expanded linearized equations will be used to show the estimation technique of required ΔV for formation keeping. After that, the estimated results will be compared with the integrated ones for the cases in which a corrective displacement exits either in radial, in-track or cross track direction. In addition, the formulas of estimation technique will be shown and can see that they are more convenient than the integration technique of differential equations, because they are short formulas and ready to use by input the desired displacement and some needed parameters, and then we can easily get the ΔV required for corrective displacement.

Coordinate System

Fig. 1 depicts the Hill frame that is used in the relative motion analysis. The main satellite, MS, is the origin of the frame, and the position of the sub satellite, SS, is expressed by the x, y, z axes which are pointing to the radial, in-track and cross track directions, respectively.

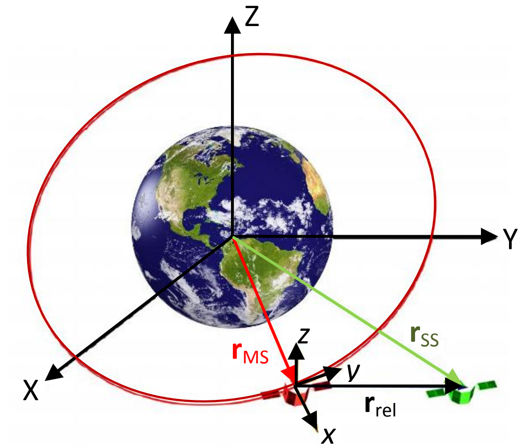

Fig. 1 The Hill coordinate system

Relative Motion expressed in Hill Frame

Equation with J_2 Force

The effect of the second geopotential harmonic, J_2, is taken into accounts in the equation of the relative motion of SS by Schweighart and Sedwick, (2002). They presented a set of ordinary differential equations including J_2 gravitational perturbation. The equations are as follows:

$$\begin{aligned} &\ddot{x} - 2(nc)\dot{y} - (5c^2 - 2)n^2 x = 0 \\ &\ddot{y} + 2(nc)\dot{x} = 0 \\ &\ddot{z} + q^2 z = 2lq\cos(qt + \phi) \end{aligned} \quad (1)$$

where n is angular velocity, $c = \sqrt{1+s}$, and l, q, s are J_2 parameters as follows:

$$l = -r_{ref} \frac{\sin i_{MS} \sin i_{SS} \sin \Delta\Omega_0}{\sin \Phi_0} \left(\dot{\Omega}_{SS} - \dot{\Omega}_{MS}\right)$$

$$\dot{\Omega}_{SS} = -\frac{3nJ_2 R_E^2}{2r_{ref}^2} \cos i_{SS}$$

$$\dot{\Omega}_{MS} = -\frac{3nJ_2 R_E^2}{2r_{ref}^2} \cos i_{MS}$$

$$\Phi_0 = \cos^{-1}\left[\cos i_{MS} \cos i_{SS} + \sin i_{MS} \sin i_{SS} \cos \Delta\Omega_0\right]$$

$$\Delta\Omega_0 = \frac{\Delta z_0}{r_{ref} \sin i_{ref}}$$

$$m = \frac{z_0}{\sin \phi}$$

$$q = nc - \left(\cos\gamma_0 \sin\gamma_0 \cot\Delta\Omega_0 - \sin^2\gamma_0 \cos i_{SS}\right)\left(\dot{\Omega}_{SS} - \dot{\Omega}_{MS}\right) - \dot{\Omega}_{SS} \cos i_{SS}$$

$$\gamma_0 = \cot^{-1}\left[\frac{\cot i_{MS}\sin_{SS} - \cos i_{SS}\cos\Delta\Omega_0}{\sin\Delta\Omega_0}\right]$$

$$s = \frac{3}{8}\frac{J_2 R_E^2}{r_{ref}^2}\left(1 + 3\cos 2i_{ref}\right)$$

In order to solve these equations, the initial conditions should be considered. Thus, the transformation by Laplace operators is applied on the equations, then we have:

$$x(t) = x(0) + \frac{2(nc)\dot{y}(0) + (5c^2 - 2)n^2 x(0)}{n^2 c_2^2} + \left(\frac{\dot{x}(0)}{nc_2}\right)\sin(nc_2 t) - \frac{2(nc)\dot{y}(0) + (5c^2 - 2)n^2 x(0)}{n^2 c_2^2}\cos(nc_2 t)$$

$$y(t) = y(0) - \frac{2c\dot{x}(0)}{nc_2^2} + \dot{y}(0)t - \frac{4(nc)^2\dot{y}(0) + 2c(5c^2 - 2)n^3 x(0)}{n^2 c_2^2}t$$

$$+ \frac{4(nc)^2\dot{y}(0) + 2c(5c^2 - 2)n^3 x(0)}{n^3 c_2^3}\sin(nc_2 t) + \frac{2c\dot{x}(0)}{nc_2^2}\cos(nc_2 t) \quad (2)$$

$$z(t) = \frac{\dot{z}(0)}{q}\sin(qt) + z(0)\cos(qt) - \frac{l}{q}\left[1 - qt\left(\cos(\phi) + \cot(qt)\right)\right]\sin(qt)\sin(\phi)$$

Eq. (2) give the relative position of SS to MS which is in a circular orbit under J_2 perturbing force. The relative velocities of SS are:

$$\dot{x}(t) = \frac{2(nc)\dot{y}(0) + (5c^2 - 2)n^2 x(0)}{nc_2}\sin(nc_2 t) + \dot{x}(0)\cos(nc_2 t)$$

$$\dot{y}(t) = \dot{y}(0) - \frac{4(nc)^2\dot{y}(0) + 2c(5c^2 - 2)n^3 x(0)}{n^2 c_2^2} - \frac{2c\dot{x}(0)}{c_2}\sin(nc_2 t) + \frac{4(nc)^2\dot{y}(0) + 2c(5c^2 - 2)n^3 x(0)}{n^2 c_2^2}\cos(nc_2 t) \quad (3)$$

$$\dot{z}(t) = -qz(0)\sin(qt) + \dot{z}(0)\cos(qt) + \frac{l}{q}\left[\left(q^2 t\cos(qt) + q\sin(qt)\right)\cos(\phi) - q^2 t\sin(qt)\sin(\phi)\right]$$

Eq. (3) is the essential *ΔV* of nanosatellite formation flying under J_2 perturbing acceleration. Thus, the integration of these equations under a certain initial condition provides the information of *ΔV* required in the radial, in-track and cross track directions. But there are amplitudes of displacement increasing when time varies as shown in Fig. 2.

Fig. 2 shows the error displacement in 2 days, therefore we need to decrease the amplitude displacement by *ΔV* required. Particularly, the integration technique of differential equations is quite long formulas and not ready to find the *ΔV* requirement when we need to correct the SS displacement. Therefore the estimation technique will be presented and applied to find the *ΔV* required in the next section.

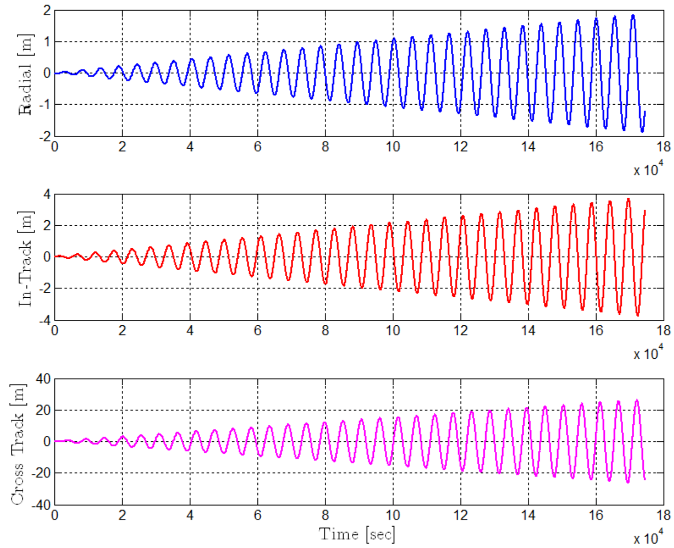

Fig. 2 Simulation of the amplitudes of displacement in 2 days

Estimation of *ΔV* Required

In this section, the estimation technique will be described. This technique is operated by two stages for the minimum *ΔV* required. The initial stage is to find an impulsive manoeuvre that minimizes the velocity. This process is referred targeting or guidance. At the second stage will be the additional impulsive manoeuvre required to cancel the initial velocity when the transition finished.

Correction in the In-Track Direction

The correction is made against the amplitude displacement in in-track direction. The initial conditions are $x(0) = X$, $y(0) = Y + D_Y$ and $z(0) = Z$, and the target conditions are $x(t_f) = X$, $y(t_f) = Y$ and $z(t_f) = Z$, therefore the intention of the manoeuvre is to shift SS with $-D_Y$ in *y*-direction.

In order to achieve this correction with minimized *ΔV* required, the time of manoeuvre can be defined to choose the timing with nearly zero velocity as follows:

$$t_f = t_0 + N\frac{2\pi}{nc_2}, \quad N = 1, 2, 3, ... \tag{4}$$

Substituting into Eq. (2) in *x* and *y* directions and taking the subtraction between the initial and final positions, they should be:

$$x(t_f) - x(t_0) = \frac{2(nc)\dot{y}(0)}{n^2c_2^2} - \frac{2(nc)\dot{y}(0)}{n^2c_2^2} = 0$$

$$y(t) - y(0) = \left(1 - \frac{4c^2}{c_2^2}\right)N\frac{2\pi}{nc_2}\dot{y}(0) - \frac{2c(5c^2-2)nX}{c_2^2}N\frac{2\pi}{nc_2} = -D_y$$

From the first equation the impulsive velocity at the initial stage should be:

$$\dot{y}(0) = \left(\frac{(1-s)^{\frac{3}{2}}}{(5s+3)}\frac{D_Y}{NT} - 2\sqrt{1+s}nX\right) \tag{5}$$

where *T* is orbital period. When the transition is finished, the relative velocities in *x*-*y* motions are:

$$\dot{x}(t_f) = \dot{x}(0)$$

$$\dot{y}(t_f) = \dot{y}(0) - \frac{4(nc)^2}{n^2c_2^2}\dot{y}(0) + \frac{4(nc)^2}{n^2c_2^2}\dot{y}(0) = \dot{y}(0)$$

The velocities both in *x* and *y* directions do not change, this means that the final stage requires the same velocities in opposite direction for cancellation of the initial velocity when the nanosatellite achieved the desired distance. This means that the total *ΔV* required in in-track direction is:

$$Total\ \Delta V_{required} = 2\left|\frac{(1-s)^{\frac{3}{2}}}{(5s+3)}\frac{D_Y}{NT} - 2\sqrt{1+s}nX\right|, \quad N = 1,2,3,... \tag{6}$$

Fig. 2 compares the estimated result with the integrated result for the case of D_Y = -20 m (*a* = 6678.136 km, *e* = 0.001, *incl* = 35°, $x_0 = 100$ m, $y_0 = 120$ m, $z_0 = 100$ m, and $x_f = 100$ m, $y_f = 100$ m, $z_f = 100$ m).

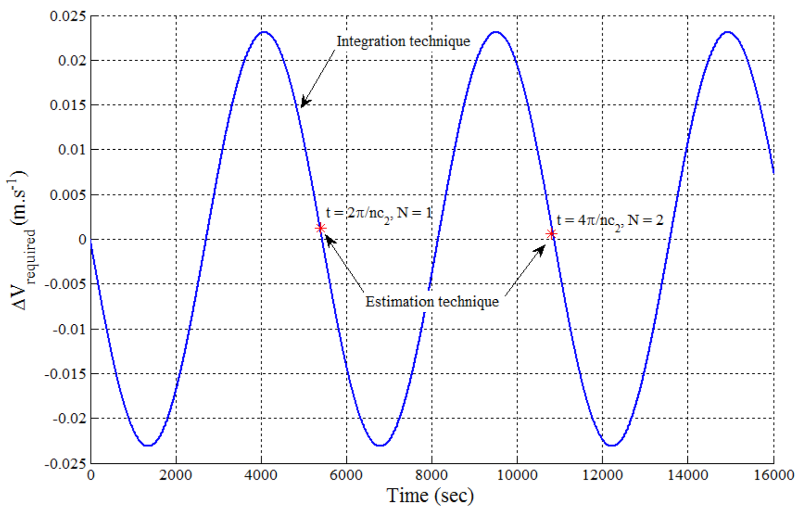

Fig. 2 Comparison of integration and estimation technique in in-track

The results of integration and estimation technique are identical and the insensitive time gives the minimum *ΔV* required as expected.

Correction in Radial Direction

This correction can be operated by two cases. Due to we need to minimize the velocity, in Eq. (2) if we select the timing with $2\pi/nc_2$, it will duplicate the last case and cannot find $x(t_f)$ or specify the corrective displacement in *x*-direction. Therefore, this case needs to estimate the *ΔV* required by timing with at $\pi/2nc_2$ and π/nc_2. The technique of $\pi/2nc_2$ is called impulsive velocity in *x*-direction because $x(t_f)$ in Eq. (2) will have only $\dot{x}(0)$, and the technique of π/nc_2 is called impulsive velocity in *y*-direction because $x(t_f)$ in Eq. (2) will have only $\dot{y}(0)$. This two cases show hereafter.

The Impulsive Velocity in x direction

With this scenario, the correction is made for the corrective displacement in the radial directions. The initial conditions are $x(0) = X + D_X^x$, $y(0) = Y$ and $z(0) = Z$. The target conditions are $x(t_f) = X$, $y(t_f) = Y$ and $z(t_f) = Z$, therefore, the intention of the manoeuvre is to shift $-D_X^x$ in *x*-direction.

Eq. (2) becomes:

$$x(t_f)=x(0)+\frac{2(nc)\dot{y}(0)+(5c^2-2)n^2x(0)}{n^2c_2^2}+\left(\frac{\dot{x}(0)}{nc_2}\right)\sin\left[nc_2(t_f-t_0)\right]-\frac{2(nc)\dot{y}(0)+(5c^2-2)n^2x(0)}{n^2c_2^2}\cos\left[nc_2(t_f-t_0)\right]$$
$$y(t)=y(0)-\frac{2c\dot{x}(0)}{nc_2^2}+\dot{y}(0)(t_f-t_0)-\frac{4(nc)^2\dot{y}(0)+2c(5c^2-2)n^3x(0)}{n^2c_2^2}(t_f-t_0)$$
$$+\frac{4(nc)^2\dot{y}(0)+2c(5c^2-2)n^3x(0)}{n^3c_2^3}\sin\left[nc_2(t_f-t_0)\right]+\frac{2c\dot{x}(0)}{nc_2^2}\cos\left[nc_2(t_f-t_0)\right] \tag{7}$$

As mentioned earlier, this scenario needs to keep the equation of $x(t_f)$. Therefore, the final time which is expected the minimum *ΔV* required, should be:

$$t_f=t_0+N\frac{\pi}{2nc_2},\quad N=1,3,5,... \tag{8}$$

Substitute into Eq. (7), it becomes:

$$x(t_f)-x(0)=\frac{2(nc)\dot{y}(0)+(5c^2-2)n^2(X+D_X^x)}{n^2c_2^2}+\frac{\dot{x}(0)}{nc_2}=-D_X^x \tag{9}$$

$$y(t_f)-y(0)=-\frac{2c\dot{x}(0)}{nc_2^2}+\left[\frac{4c^2}{nc_2^3}+\left(1-\frac{4c^2}{c_2^2}\right)N\frac{\pi}{2nc_2}\right]\dot{y}(0)+\left(\frac{1}{nc_2}-N\frac{\pi}{2nc_2}\right)\left(\frac{2c(5c^2-2)n(X+D_X^x)}{c_2^2}\right)=0 \tag{10}$$

Then, Eq. (9) should be:

$$\dot{x}(0)=-nc_2D_X^x-n\frac{(5c^2-2)}{c_2}(X+D_X^x)-\frac{2c}{c_2}\dot{y}(0) \tag{11}$$

And then Eq. (10) becomes:

$$\dot{y}_1(0)=\frac{\frac{2c\dot{x}(0)}{nc_2^2}-\left(\frac{1}{nc_2}-N\frac{\pi}{2nc_2}\right)\left(\frac{2c(5c^2-2)n}{c_2^2}\right)(X+D_X^x)}{\left[\frac{4c^2}{nc_2^3}+\left(1-\frac{4c^2}{c_2^2}\right)N\frac{\pi}{2nc_2}\right]} \tag{12}$$

Substitute into Eq. (11), it becomes:

$$\dot{x}_1(0)=-\frac{nc_2D_X^x+n\frac{(5s+3)}{c_2}(X+D_X^x)-\frac{\left(\frac{2-\pi N}{2nc_2}\right)\left(\frac{4c^2(5s+3)n}{c_2^3}\right)(X+D_X^x)}{\left[\frac{4c^2}{nc_2^3}-\left(\frac{5s+3}{1-s}\right)\frac{NT}{4c_2}\right]}}{\left(\frac{8c^2-(5s+3)\frac{\pi N}{2}}{4c^2-(5s+3)\frac{\pi N}{2}}\right)} \tag{13}$$

where $\dot{x}_1(0)$ and $\dot{y}_1(0)$ are the initial stage of velocity. Explicitly, the impulsive velocity in *x*-direction also affects the motion in *y*-direction. Note that Eq. (11) or (13) and (15) show the compensation of impulsive velocity because *x*-*y* motions are coupling, when increases the orbit by $\dot{x}_1(0)$, the altitude will be changed then the speed will also change, therefore we need to compensate by $\dot{y}_1(0)$. Taking Eq. (12) and (13) into Eq. (3) when the transition is finished, then we obtain:

$$\dot{x}(t_f) = \frac{(5c^2-2)n}{c_2}(X + D_X^x) + \frac{2c}{c_2}\dot{y}_1(0) \tag{14}$$

$$\dot{y}(t_f) = -\frac{2c(5c^2-2)n}{c_2^2}(X + D_X^x) - \frac{2c}{c_2}\dot{x}_1(0) + \left(1 - \frac{4c^2}{c_2^2}\right)\dot{y}_1(0) \tag{15}$$

The total *ΔV* required is expressed below:

$$Total\ \Delta V_{required} = \sqrt{\dot{x}_1(0)^2 + \dot{y}_1(0)^2} + \sqrt{\dot{x}(t_f)^2 + \dot{y}(t_f)^2} \tag{16}$$

The Impulsive Velocity in y direction

The initial conditions are set as same, but the intention of the reduction in *x*-direction is $-D_X^y$. As mentioned earlier, this technique selects the timing with π / nc_2, which is operated by $\dot{y}(0)$ of $x(t_f)$ in Eq. (2), therefore the final time must be as follows:

$$t_f = t_0 + N\frac{\pi}{nc_2}, \quad N = 1,3,5... \tag{17}$$

Substitute into Eq. (2), it becomes:

$$x(t_f) - x(0) = \frac{4c}{nc_2^2}\dot{y}(0) + \frac{2(5c^2-2)}{c_2^2}(X + D_X^y) = -D_X^y \tag{18}$$

$$y(t) - y(0) = -\frac{4c}{nc_2^2}\dot{x}(0) + \left(1 - \frac{4c^2}{c_2^2}\right)\frac{\pi N}{nc_2}\dot{y}(0) - \frac{2c\pi N(5c^2-2)}{c_2^3}(X + D_X^y) = 0 \tag{19}$$

where D_X^y is the drift displacement of radial. Then, Eq. (18) and (19) are solved, they become:

$$\dot{y}_1(0) = -\frac{c_2^2}{4c}nD_X^y - \frac{(5c^2-2)}{2c}n(X + D_X^y) \tag{20}$$

$$\dot{x}_1(0) = -\frac{2nc(5s+3)}{4c_2}\pi N(X + D_X) - \left(\frac{(5s+3)}{4c}\right)\frac{\pi N}{c_2}\dot{y}_1(0) \tag{21}$$

where $\dot{x}_1(0)$ and $\dot{y}_1(0)$ are the initial stage of velocity. Eq. (20) and (21) show the compensation in *x*-*y* motions because the movement of nanosatellite by $\dot{y}(0)$ affects $\dot{x}(0)$, when the speed in *y*-direction increases, altitude also changes as well as speed in *x*-direction. When the transition is finished, the relative velocities in *x*-*y* motions in Eq. (3) are expressed below:

$$\dot{x}(t_f) = -\dot{x}_1(0) \tag{22}$$

$$\dot{y}(t_f) = -\frac{4c(5s+3)n}{1-s}(X + D_X) + \left(1 - \frac{(1+s)}{1-s}8\right)\dot{y}_1(0) \tag{23}$$

Eq. (22) decreases the periodic amplitude with the same impulsive velocity in *x*-direction. Eq. (20) and (23) compensates in *y*-direction. The total *ΔV* required is expressed below:

$$Total\ \Delta V_{required} = \sqrt{\dot{x}_1(0)^2 + \dot{y}_1(0)^2} + \sqrt{\dot{x}(t_f)^2 + \dot{y}(t_f)^2} \tag{24}$$

Fig. 3 shows examples of the identically numerical results of the integration and estimation technique by the corrective displacement D_X^y = -20 m (a = 6678.136 km, e = 0.001, $incl$ = 35°, $x_0 = 120$ m, $y_0 = 100$ m, $z_0 = 100$ m, and $x_f = 100$ m, $y_f = 100$ m, $z_f = 100$ m).

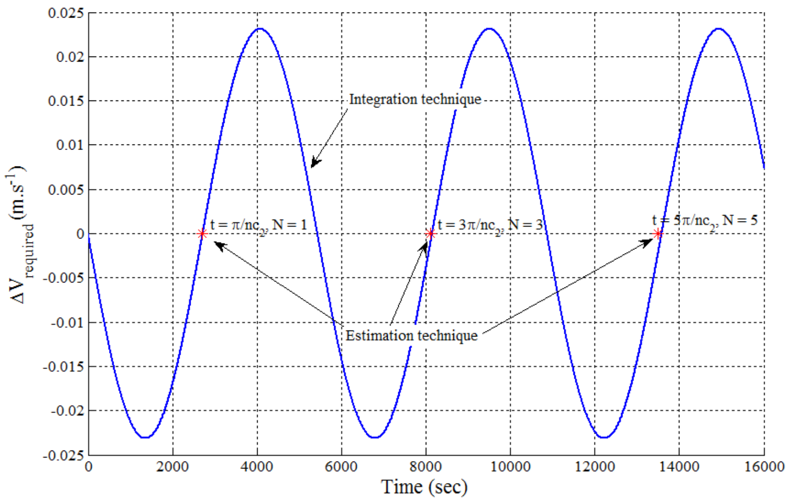

Fig. 3 Comparison of integration and estimation technique in radial by *y*-direction correction

Fig. 3 shows the identical results and the minimum ΔV required by the insensitive time which is selected for the correction in radial by *y*-direction. Thus, the appropriate time is very important to manoeuvre.

Correction of Cross Track Direction

The motion in this direction is independent from those in the radial and in-track directions. The velocity correction is made to decrease the amplitude displacement D_Z. The initial conditions are $x(0) = X$, $y(0) = Y$ and $z(0) = Z+D_Z$. The final conditions are $x(t_f) = X$, $y(t_f) = Y$ and $z(t_f) = Z$, therefore the intention of the manoeuvre is to shift $-D_Z$.

Therefore, Eq. (2) (take only *z*-direction), it becomes:

$$z(t_f) = \frac{\dot{z}(0)}{q}\sin(q(t_f - t_0)) - \frac{l}{q}\left[1 - q(t_f - t_0)\left(\cos(\phi) + \cot(q(t_f - t_0))\right)\right]\sin(q(t_f - t_0))\sin(\phi) \qquad (25)$$

Due to we need to decrease the periodic amplitude, but Eq. (25) cannot use timing with π/q or $2\pi/q$ because we will not have the initial stage of velocity, $\dot{z}(0)$. Therefore, final time should be as follows:

$$t_f = t_0 + N\frac{\pi}{2q}, \quad N = 1,3,5,... \qquad (26)$$

Substitute into Eq. (25), it becomes:

$$z(t_f) = \frac{\dot{z}(0)}{q} - \frac{l}{q}\left[1 - N\frac{\pi}{2}\cos(\phi)\right]\sin(\phi)$$

$$z(t_0) = Z + D_Z$$

$$z(t_f) - z(t_0) = \frac{\dot{z}(0)}{q} - (Z + D_Z) - \frac{l}{q}\left[1 - N\frac{\pi}{2}\cos(\phi)\right]\sin(\phi) = -D_Z$$

where Z is the displacement of $z(t_f)$. Therefore, the initial velocity is:

$$\dot{z}(0) = qZ + l\left[1 - N\frac{\pi}{2}\cos(\phi)\right]\sin(\phi) \qquad (27)$$

The second order can be neglected due to very small because the l is very small in term of $(\dot{\Omega}_{SS}-\dot{\Omega}_{MS})$ as shown earlier, and then the initial stage velocity is as follow:

$$\dot{z}(0)=qZ \tag{28}$$

The final stage is expressed below:

$$\dot{z}(t_f)=-q(Z+D_Z)+l\left[\cos(\phi)-N\frac{\pi}{2}\sin(\phi)\right] \tag{29}$$

With the same reason, the second order can be neglected because the l is very small as described, therefore Eq. (29) becomes:

$$\dot{z}(t_f)=-q(Z+D_Z) \tag{30}$$

Thus, the total ΔV required of cross track correction is expressed below:

$$Total\ \Delta V_{required}=\left|2qZ+qD_Z\right| \tag{30}$$

Fig. 4 shows examples of the identically numerical results of the integration and estimation technique by the corrective displacement D_Z = -20 m (a = 6678.136 km, e = 0.001, $incl$ = 35°, $x_0=100$ m, $y_0=100$ m, $z_0=120$ m, and $x_f=100$ m, $y_f=100$ m, $z_f=100$ m).

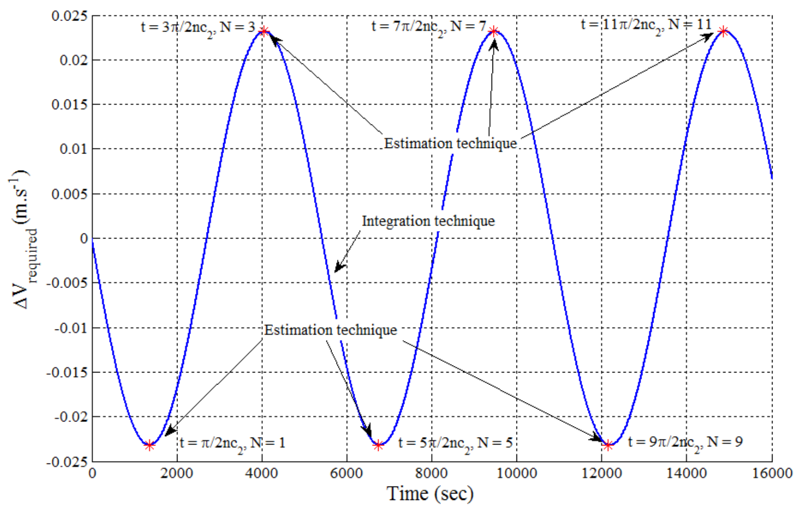

Fig. 4 Comparison of integration and estimation technique in cross track

In Fig. 4, the results of the integration and estimation technique are identical as same, but the insensitive time, $N\pi/2q$ gives the maximum ΔV required. Although, the appropriate time should be $N\pi/q$ which we can get the minimum ΔV required, but as mentioned, if we use the timing with $N\pi/q$, we cannot get the solution. Thus, the ΔV required in cross track is largest.

The results of estimation and integration technique are identical. But the formulas of the estimation technique are ready to use for calculation of ΔV required. Moreover, this ΔV required can be applied to estimate the amount of fuel needed, can see summary in Table 1 as follows:

Table 1 Summary of the estimation technique

Direction	Total *ΔV* required
Radial *x* (adjustment by *y*-direction)	$\sqrt{\left[\frac{2\pi ncN(5s+3)}{4c_2}(X+D_X)+\frac{5s+3}{4c}\left(\frac{c_2\pi N}{4c}nD_X+\frac{n\pi N}{c_2}\frac{5s+3}{2c}(X+D_X)\right)\right]^2+\left[\frac{nc_2^2}{4c}D_X+\frac{n(5s+3)}{2c}(X+D_X)\right]^2}$ $+\sqrt{\left[\frac{\pi ncN(5s+3)}{2c_2}(X+D_X)+\frac{\pi N}{c_2}\frac{5s+3}{4c}\left(\frac{c_2\pi N}{4c}nD_X+\frac{n\pi N}{c_2}\frac{5s+3}{2c}(X+D_X)\right)\right]^2 +\left[-\frac{4nc(5s+3)}{1-s}(X+D_X)+\left(1-\frac{(1+s)}{1-s}8\right)\left(\frac{nc_2\pi N}{4c}D_X+\frac{n\pi N}{c_2}\frac{5s+3}{2c}(X+D_X)\right)\right]^2}$
In-track *y*	$2\left\|\frac{(1-s)^{\frac{3}{2}}}{(5s+3)}\frac{D_y}{NT}-2\sqrt{1+s}nX\right\|$
Cross track *z*	$\left\|2qZ+qD_Z\right\|$

where $q = nc - \dot{\Omega}_{SS}\cos i_{SS}$. This section has already presented the estimation of *ΔV* required based on the relative dynamics equations under J_2 gravitational perturbation. Table 2 summarizes the results of the estimation technique comparing between the corrective displacement and the value of *ΔV* required for each case (a = 6678.136 km, e = 0.001, *incl* = 35°).

Table 2 Results of *ΔV* required

Corrective Displacement [m]	*ΔV* required in direction [m/s]		
	x	*y*	*z*
1	6.8154E-4	6.1582E-5	0.0012
5	0.0034	3.0791E-4	0.0058
10	0.0068	6.1582E-4	0.0116
20	0.0136	0.0012	0.0232
50	0.0341	0.0031	0.0579
100	0.0682	0.0062	0.1158
200	0.1363	0.0123	0.2317

Conclusions

The set of linearized differential equations of relative dynamics motion that captures the effect of J_2 gravitational perturbation is proposed which includes explicitly the initial conditions. In this paper, the way of estimation of fuel consumption is shown by the *ΔV* required that focuses on the corrective displacement in each direction. The numerical results of integration and estimation technique are identical but the estimation technique is more convenient because the formulas are ready to use to calculate the *ΔV* required by corrective displacement and not so long equations (especially in the correction of in-track and cross track) as well as it can be used to estimate the amount of fuel needed. In Fig. 2 and 3, the estimated *ΔV* are nearly zero, which need little fuel. Meanwhile in Fig. 4, the estimated *ΔV* is at the peak, which needs a lot of fuel. The Table 1 shows the summary of the estimation technique formulas. Although, the correction in radial by *x*-direction gives the peak of *ΔV* required, but the correction in radial by *y*-direction can be replaced with smaller *ΔV* required properly. Table 2 shows the summaries of *ΔV* required which cross track corrective displacement requires more *ΔV* than radial and in-track. This paper has already provided a convenient way to estimate the requirement of *ΔV* as well as can be applied to find the fuel needed in nanosatellite formation flying.

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