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RESEARCH ARTICLE

STUDY OFZ = $(z-t)$ TYPE PLANE GRAVITATIONAL WAVES IN BIMETRIC RELATIVITY

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Abstract

In this paper, Z = $(z-t)$ type plane gravitational waves is studied with the source Cosmic cloud strings coupled with Electromagnetic fields in Rosen's bimetric relativity. It is shown that there is nil contribution either from Cosmic Cloud or from Maxwell's field and also for coupled cosmic cloud strings with Maxwell's field in this theory. Only vacuum model can be constructed.

INTRODUCTION

The plane gravitational waves $g_{ij}$are mathematically exposed by Takeno (1961) [3] in general relativity. He had studied (z - t) - type and(t /z)- type plane gravitational waves and obtained plane wave solutions of field equations ofgeneral relativity. Furthermore, Takeno [3] considered the Peres space-time metric, which does not conform to the above definition completely. Hence, using the generalized form of the definition of plane waves, he obtained plane wave-like solution ofthe field equations of general relativity in Peres space-time metric. Viewing the entire scenario, La land Ali [8] obtained the plane wave solutions of the field equations of general relativity in a generalized Peres space-time for Z = (z - t)- type plane gravitational waves.

Thus, we consider corresponding two line elements in bimetric relativity as –

$$ds^2 = g_{ij} dx^i dx^j \tag{1.1}$$

And $d\sigma^2 = \gamma_{ij} dx^i dx^j$ (1.2)

Where ds is the interval between two neighboring events as measured by means of a clock and a measuring rod. The interval $d\sigma$ is an abstract or geometrical quantity not directly measurable. One can regard it as describing the geometry that would exist if no matter were present.

Takeno [5] considered the generalized peres space-time whose metric is given by

$$ds^2 = -Adx^2 - Bdy^2 - (1-E)dz^2 - 2Edzdt + (1+E)dt^2 \quad (1.3)$$

In general relativity, plane and plane fronted gravitational waves have been studied by number of authors Peres [6] (1959), Zakhanov [9](1973) etc. obtained the plane fronted wave solutions in Peres space-time.

The theory of plane gravitational waves have been studied by many investigators, H Takeno [4][7]; Pandey et.al [10][12]; Lal and Shafiullah [11]; Hogan, P.A.[13];Lal K.B and PandeyS.N [14],Rane. R.S and Katore S. D (2009)[16], Bhoyar S.R and Deshmukh A.G [15][17], Deo and Ronghe [18],[19], Deo and Suple [20] and they obtained the various solutions.

In this paper, we will study Z = $(z-t)$ type plane gravitational waves with Cosmic cloud strings coupled with Maxwell's fields and will observe the result in the context of bimetric relativity.

2. FIELD EQUATIONS IN BIMETRIC RELATIVITY:

Rosen N.[1,2] has proposed the field equations of Bimetric Relativity from variation principle as

$$K_i^j = N_i^j - \frac{1}{2} N g_i^j = -8\pi\kappa T_i^j \quad (2.1)$$

Where $$N_i^j = \frac{1}{2} \gamma^{\alpha\beta} \left[g^{hj} g_{hi|\alpha} \right]_{|\beta} \quad (2.2)$$

$$N = N_\alpha^\alpha, \ \kappa = \sqrt{\frac{g}{\gamma}} \quad (2.3)$$

And $g = \det g_{ij}$, $\gamma = \det \gamma_{ij}$ (2.4)

Where a vertical bar (|) denotes a covariant differentiation with respect to γ_{ij}

3. Z = $(z-t)$ type plane gravitational waves with Cosmic Cloud strings:

For Z = $(z-t)$ plane gravitational waves, we have the line element as

$$ds^2 = -Adx^2 - Bdy^2 - (1-E)dz^2 - 2Edzdt + (1+E)dt^2 \quad (3.1)$$

where $A = A(Z)$, B = B(Z), E=E(x,y,Z) and Z = $(z-t)$

Corresponding to the equation (3.1), we consider the line element for background metric γ_{ij} as

$$d\sigma^2 = -\left(dx^2 + dy^2 + dz^2\right) + dt^2 \tag{3.2}$$

Here we consider the matter cosmic strings whose energy momentum tensor is given by

$$T_i^j = T_{i\ strings}^j = \rho v_i v^j - \lambda x_i x^j \tag{3.3}$$

together with $v_4 v^4 = -1$ and $x_3 x^3 = 1$ where v_i is the four-velocity of the cloud of particles, x^i is the four vector representing the direction of anisotropy (z-axis) and ρ is the rest energy density for a cloud of strings with particles attached along the extension.

Thus $\rho = \rho_p + \lambda$ (3.4)

where ρ_p is the particle energy density and λ is the tension density of the strings.

In co-moving coordinate system, we have

$T_{3\ strings}^3 = -\lambda$, $T_{4\ strings}^4 = -\rho$ and $T_{i\ strings}^j = 0$ for i, j=1, 2 and for $i \neq j$

Using equations (2.1) to (2.4) with (3.1) - (3.4),

We get the field equations as

$$\frac{1}{2}\left\{\left(E_x{}^2 + E_y{}^2 + E_z{}^2 - E_t{}^2\right) + E\left(E_{xx} + E_{yy} + E_{zz} - E_{tt}\right)\right\} = -8\pi\kappa\lambda \tag{3.5}$$

$$\frac{1}{2}\left\{\left(E_x{}^2 + E_y{}^2 + E_z{}^2 - E_t{}^2\right) + E\left(E_{xx} + E_{yy} + E_{zz} - E_{tt}\right)\right\} = 0 \tag{3.6}$$

$$\frac{1}{2}\left\{E_{xx} + E_{yy} + E_{zz} - E_{tt}\right\} = 0 \tag{3.7}$$

$$\frac{1}{2}\left\{E_{xx} + E_{yy} + E_{zz} - E_{tt}\right\} = -8\pi\kappa\rho \tag{3.8}$$

where

$$\overline{A} = \frac{\partial A}{\partial Z} \quad , \quad \overline{\overline{A}} = \frac{\partial^2 A}{\partial Z^2} \ , \quad \overline{B} = \frac{\partial B}{\partial Z} \quad , \quad \overline{\overline{B}} = \frac{\partial^2 B}{\partial Z^2}$$

$$E_x = \frac{\partial E}{\partial x} \quad , \quad E_{xx} = \frac{\partial^2 E}{\partial x^2} \ , \quad E_y = \frac{\partial E}{\partial y} \quad , \quad E_{yy} = \frac{\partial^2 E}{\partial y^2} \quad \text{etc}$$

Using equation (3.5) to (3.8), we get

$$\lambda = \rho = 0 \tag{3.9}$$

This equation of state is known as false vacuum.

Equation (3.9) immediately implies that cosmic cloud strings does not exist in Z = $(z - t)$ plane

gravitational waves in Rosen's Bimetric theory of relativity.
Hence for vacuum case $\lambda = 0 = \rho$, the field equation reduced to

$$E_{xx} + E_{yy} + E_{zz} - E_{tt} = 0 \tag{3.10}$$

and

$$E_x{}^2 + E_y{}^2 + E_z{}^2 - E_t{}^2 = 0 \tag{3.11}$$

Solving equation (3.10) using D'Alembert's method of wave equation, we obtain

$$E = f\left(u+v\right) + g\left(u-v\right) \tag{3.12}$$

where u and v are functions of x, y, Z, i.e. $u = u\left(x, y, Z\right) and\, v = v\left(x, y, Z\right)$

Substituting the value of E in (3.1) the metric takes the form

$$\begin{aligned} ds^2 = &-Adx^2 - Bdy^2 - \left(1 - f\left(u+v\right) - g\left(u-v\right)\right)dz^2 \\ &- 2\left(f\left(u+v\right) + g\left(u-v\right)\right)dzdt + \left(1 + f\left(u+v\right) + g\left(u-v\right)\right)dt^2 \end{aligned} \tag{3.13}$$

Thus, it is found that in plane gravitational wave Z = $\left(z-t\right)$, the Cosmic cloud strings does not survive in Bimetric theory of relativity and only vacuum model can be constructed.

4. Z = $\left(z-t\right)$ type plane gravitational wave with Maxwell's Field:

In this section, we consider the region of the space-time filled with electromagnetic field whose energy momentum tensor is given by

$$E_{i\ mag}^{j} = -F_{ir}F^{jr} + \frac{1}{4}F_{ab}F^{ab}g_i^j \tag{4.1}$$

where $E_{i\ mag}^{j}$ is the electromagnetic energy tensor, F_i^j is the electromagnetic field tensor.

As the electromagnetic field is moving along the z-direction alone, F_{12} is the only nonzero component of Maxwell's tensorF_{ij}. Maxwell's equation is given by

$F_{ij,k} + F_{jk,i} + F_{ki,j} = 0$ gives rise to $F_{12} = -F_{21} = F$ (constant)such that

$$E_1^1 = E_2^2 = \eta\,,\ E_3^3 = E_4^4 = -\eta \tag{4.2}$$

Where $\eta = \frac{1}{2}\frac{F^2}{AB}$

Using equations (2.1) to (2.4) for the metric (3.1)-(3.2) with (4.1) and (4.2), the field equations are

$$\frac{1}{2}\left\{\left(E_x{}^2 + E_y{}^2 + E_z{}^2 - E_t{}^2\right) + E\left(E_{xx} + E_{yy} + E_{zz} - E_{tt}\right)\right\} = 8\pi\kappa\eta \tag{4.3}$$

$$\frac{1}{2}\left\{E_{xx} + E_{yy} + E_{zz} - E_{tt}\right\} = 8\pi\kappa\eta \tag{4.4}$$

$$\frac{1}{2}\left\{E_{xx}+E_{yy}+E_{zz}-E_{tt}\right\}=-8\pi\kappa\eta \tag{4.5}$$

Solving (4.4) and (4.5), we get

$$\eta=0 \ i.e. \ \frac{F^2}{AB}=0 \tag{4.6}$$

This implies $F=F_{12}=0$ (4.7)

Thus for the space-time (3.1) Maxwell's field does not survive in Bimetric theory of relativity and only vacuum model exists and it is same as defined in equation (3.13).

5. Coupling of Cosmic cloud strings with Electromagnetic Field:

The energy momentum tensor for a mixture of cosmic cloud string and Electromagnetic field together is given by

$$T_i^j=T_i^j{}_{strings}+E_i^j{}_{mag} \tag{5.1}$$

Where $T_i^j{}_{strings}$ and $E_i^j{}_{mag}$ are defined in equations (3.3) and (4.1).

With the help of co-moving co-ordinate system, the field equation (2.1) to (2.4) for the metric (3.1) and (3.2) corresponding to the energy momentum tensor (5.1) can be written as

$$\frac{1}{2}\left\{\left(E_x{}^2+E_y{}^2+E_z{}^2-E_t{}^2\right)+E\left(E_{xx}+E_{yy}+E_{zz}-E_{tt}\right)\right\}=8\pi\kappa\eta \tag{5.2}$$

$$\frac{1}{2}\left\{E_{xx}+E_{yy}+E_{zz}-E_{tt}\right\}=8\pi\kappa\left(\lambda+\eta\right) \tag{5.3}$$

$$\frac{1}{2}\left\{E_{xx}+E_{yy}+E_{zz}-E_{tt}\right\}=-8\pi\kappa\left(\rho+\eta\right) \tag{5.4}$$

On solving (5.2) to (5.4) we get

$$\lambda+\rho+2\eta=0 \tag{5.5}$$

In view of the reality conditions (Hawking S.W. and Ellis G.F.R)[21] i.e. $\lambda>0, \rho>0$ and $\eta>0$ must hold.

The above conditions (5.5) is satisfied only when

$$\lambda=0, \rho=0 \ and \ \eta=0 \ . \tag{5.6}$$

This means that the physical parameters, viz tension density (λ), rest energy density (ρ) and the magnetic field along the z-axis (η) are identically zero.

Thus plane gravitational waves with cosmic cloud strings coupled with Maxwell's field in bimetric relativity does not survive and hence only vacuum model is obtained and it is already defined in equation (3.13).

6. Z = $\left(z-t\right)$ type plane gravitational wave with Strange quark matter:

In this present work we unable to find the metric potential A and B and hence reformulating the Takeno's [1] definition of plane wave in generalized peres space time as

$$ds^2=-Adx^2-Bdy^2-\left(1-E\right)dz^2-2Edzdt+\left(1+E\right)dt^2 \tag{6.1}$$

where $A=A(x,Z)$, $B=B(y,Z)$, $E=E(x,y,Z)$ and Z = $\left(z-t\right)$

The components of covariant tensor g_{ij} from the metric (6.1) are

$g_{11}=-A, g_{22}=-B, g_{33}=-\left(1-E\right), g_{44}=1+E, g_{34}=-E=g_{43}$ and their contra variant tensor as

$$g^{11}=-\frac{1}{A}, g^{22}=-\frac{1}{B}, g^{33}=-(1+E), g^{44}=1-E$$

As we know that string is free to vibrate, the different vibrating modes of the string represent a different type of particle because these different modes are seen as different masses. Therefore here we will take quarks instead of particles in the string cloud.

Hence we consider strange quark matter energy density instead of particle density in the string cloud.

Therefore in this case by virtue of equation (3.4),we get

$$\rho=\rho_q+\lambda+B_c \tag{6.2}$$

Where B_c is a vacuum energy density (called as bag constant.)

Using equation (3.3) and (6.2), we have energy momentum tensor for strange quark matter attached to the string cloud as

$$T_i^j = T_{i\ quarks}^j = \left(\rho_q+\lambda+B_c\right)v_i v^j-\lambda x_i x^j \tag{6.3}$$

Where u_i and x^i are already defined.

Corresponding to the equation (6.1) with (3.2) and using equations (2.1) to (2.4) with (6.1)-(6.3),We get the field equations as

$$-\frac{1}{4}\left(\frac{B_y{}^2}{B^2}-\frac{B_{yy}}{B}\right)-\frac{1}{2}\left(E_x{}^2+E_y{}^2+E_z{}^2-E_t{}^2\right) -\frac{E}{2}\left(E_{xx}+E_{yy}+E_{zz}-E_{tt}\right)=0 \tag{6.4}$$

$$-\frac{1}{4}\left(\frac{A_x{}^2}{A^2}-\frac{A_{xx}}{A}\right)-\frac{1}{2}\left(E_x{}^2+E_y{}^2+E_z{}^2-E_t{}^2\right) -\frac{E}{2}\left(E_{xx}+E_{yy}+E_{zz}-E_{tt}\right)=0 \tag{6.5}$$

$$-\frac{1}{4}\left(\frac{A_x{}^2}{A^2}-\frac{A_{xx}}{A}\right)-\frac{1}{4}\left(\frac{B_y{}^2}{B}-\frac{B_{yy}}{B}\right) +\frac{1}{2}\left(E_{xx}+E_{yy}+E_{zz}-E_{tt}\right)=8\pi\kappa\lambda \tag{6.6}$$

$$-\frac{1}{4}\left(\frac{A_x{}^2}{A^2}-\frac{A_{xx}}{A}\right)-\frac{1}{4}\left(\frac{B_y{}^2}{B}-\frac{B_{yy}}{B}\right) -\frac{1}{2}\left(E_{xx}+E_{yy}+E_{zz}-E_{tt}\right)=8\pi\kappa\rho \tag{6.7}$$

$$\frac{1}{2}\left\{\left(E_x{}^2+E_y{}^2+E_z{}^2-E_t{}^2\right)+E\left(E_{xx}+E_{yy}+E_{zz}-E_{tt}\right)\right\}=0 \tag{6.8}$$

where $\overline{A}, \overline{\overline{A}}, A_x, A_{xx}\ etc$ are already defined with their usual meanings.

Using equation (6.4) to (6.8), we get

$$\lambda=\rho=0 \tag{6.9}$$

This equation of state is known as false vacuum.

Using (6.9) in (6.2)

we obtain $\rho_q = B_c = 0$

Equation (6.9) immediately implies that strange quark does not exist in Z = $(z-t)$ plane gravitational waves in Rosen's Bimetric theory of relativity.

Hence for vacuum case $\lambda = 0 = \rho$, the field equation reduced to

$$\left(\frac{A_x^{\;2}}{A^2} - \frac{A_{xx}}{A}\right) = 0 \qquad (6.10)$$

$$\left(\frac{B_y^{\;2}}{B^2} - \frac{B_{yy}}{B}\right) = 0 \qquad (6.11)$$

$$E_{xx} + E_{yy} + E_{zz} - E_{tt} = 0 \qquad (6.12)$$

and

$$E_x^{\;2} + E_y^{\;2} + E_z^{\;2} - E_t^{\;2} = 0 \qquad (6.13)$$

On solving (6.10) and (6.11) we obtain

$$A = k_1 \exp(k_2 x) \qquad (6.14)$$

$$B = k_3 \exp(k_4 y) \qquad (6.15)$$

where k_1,k_2,k_3 and k_4 are constants of integration.

And on solving (6.12) using D'Alembert's method of wave equation we obtain the same results as defined in equation (3.12)

Substituting the value of A,B and E in (3.1) the metric takes the form

$$\begin{aligned} ds^2 = -k_1 \exp(k_2 x)\, dx^2 - k_3 \exp(k_4 y)\, dy^2 - \big(1 - f(u+v) - g(u-v)\big) dz^2 \\ -2\big(f(u+v) + g(u-v)\big) dz dt + \big(1 + f(u+v) + g(u-v)\big) dt^2 \end{aligned}$$

(6.16)

6. CONCLUSION:

In the study of **Z =** $(z-t)$ type plane gravitational wave, there is nil contribution of Cosmic cloud strings, Maxwell's field and string coupled with Maxwell's field in Bimetric theory of relativity respectively. After modification of the line element we have studied Strange Quark matter in this theory and hence obtained the solution.

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