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RESEARCH ARTICLE

fpg^* −Closed sets in Fine-Topological space

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Abstract

The aim of this paper is to introduce a new class of closed sets called fine-pre-g*-closed (fpg*-closed sets) using $f\omega\alpha -$ open sets in fine-topological space. Also we study the some property of fpg*-closed sets in fine-topological space.

1. INTRODUCTION

In 1969, Levine [10] gives the concept and properties of generalized closed (briey g-closed) set and the complement of g-closed set is said to be g-open set. Later, Noiri et al. [11], Dontechev [5], Gnanambal [6], Arya and Nour [1], Bhattacharya and Lahiri [4], Maki et al. [12, 13] and Sundaram and Sheik John [21] introduced and studied the concept of gp-closed, gsp-closed, gpr-closed, gs-closed, sg-closed, αg-closed, gα-closed and ω-closed sets and their complements are respectively open sets. Moreover, the notion of ω-closed [3] sets was introduced by the present authors.

Powar P. L. and Rajak K. [20], have investigated a special case of generalized topological space called fine topological space. In this space, they have defined a new class of open sets namely fine-open sets which contains all $\alpha -$open sets, $\beta -$open sets, semi-open sets, pre-open sets, regular open sets etc. By using these fine-open sets they have define fine-irresolute mappings which include pre-continuous functions, semi-continuous function, $\alpha -$continuouos function, $\beta -$continuous functions, $\alpha -$irresolute functions, $\beta -$irresolute functions, etc (cf. [12]-[16]).

In the present paper, the author have introduce a new class of closed sets called fine-pre-g*-closed (fpg*-closed sets) using $f\omega\alpha -$ open sets in fine-topological space. Also we study the some property of fpg*-closed sets in fine-topological space.

2. Preliminaries

Throughout this paper, (X,τ) and (Y,σ) means topological spaces on which no separation axioms are assumed. For a subset A of a space X the closure and interior of A with respect to τ are denoted by $cl(A)$ and $int\ (A)$. We use the following definitions:

Definition 2.1 A subset A of a space (X,τ) is called

1) Semi-open if $A \subseteq cl(int\ (A))$ (cf. [11]).
2) α −open if $\subseteq int(cl(int(A)))$ (cf. [8]).
3) β −open if $A \subseteq cl(int(cl(A)))$ (cf. [3]).
4) Pre-open if $A \subseteq int(cl(A))$ (cf. [1]).

The complements of α −open sets, β −open sets, semi-open sets, pre-open sets are called α −closed sets, β −closed sets, semi-closed sets, pre-closed sets

Definition 2.2 A subset A of (X,τ) is said to be generalized closed (briefly g−closed) if cl(A) ⊆ U whenever A ⊆ U and U is open in (X, τ) (cf. [7]).

Definition 2.3 A subset A of (X,τ) is said to be weakly closed set (ω−closed) set if cl(A) ⊆ U whenever A ⊆ U and U is semi-open in (X, τ). The complement of ω −closed set is called ω −open in (X,τ) (cf. [7]).

Definition 2.4 A subset A of (X,τ) is said to be $\omega\alpha -$ closed if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is ω −open set in (X,τ) (cf. [7]).

Definition 2.5 A subset A of (X,τ) is said to be pre-g*− closed (briefly pg^* −closed) if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is $\omega\alpha$ −open set in (X,τ). The complement of pg^* −closed set is called pg^* −open in (X,τ) (cf. [7]).

Definition 2.6 Let (X, τ) be a topological space we define

$\tau(A_\alpha) = \tau_\alpha$ (*say*) = $\{G_\alpha(\neq X) : G_\alpha \cap A_\alpha = \phi$, for $A_\alpha \in \tau$ and $A_\alpha = \phi, X$, for some $\alpha \in J$, where J is the index set.$\}$

Now, we define

$$\tau_f = \{\phi, X, \cup_{\{\alpha \in J\}} \{\tau_\alpha\}\}$$

The above collection τ_f of subsets of X is called the fine collection of subsets of X and (X, τ, τ_f) is said to be the fine space X generated by the topology τ on X (cf. [20]).

Definition 2.7 A subset U of a fine space X is said to be a fine-open set of X, if U belongs to the collection τ_f and the complement of every fine-open sets of X is called the fine-closed sets of X and we denote the collection by F_f (cf. [20]).

Definition 2.8 Let A be a subset of a fine space X, we say that a point $x \in X$ is a fine limit point of A if every fine-open set of X containing x must contains at least one point of A other than x (cf. [20]).

Definition 2.9 Let A be the subset of a fine space X, the fine interior of A is defined as the union of all fine-open sets contained in the set A i.e. the largest fine-open set contained in the set A and is denoted by f_{Int} (cf. [20]).

Definition 2.10 Let A be the subset of a fine space X, the fine closure of A is defined as the intersection of all fine-closed sets containing the set A i.e. the smallest fine-closed set containing the set A and is denoted by f_{cl} (cf. [20]).

Definition 2.11 A function $f : (X, \tau, \tau_f) \rightarrow (Y, \tau', \tau'_f)$ is called fine-irresolute (or f-irresolute) if $f^{-1}(V)$ is fine-open in X for every fine-open set V of Y (cf. [20]).

3. fpg^* −closed sets in fine-topological spaces

In this section, we introduce fpg^* −closed sets in fine-topological space and study some of their properties.

Definition 3.1 A subset A of fine-topological space (X,τ,τ_f) is said to be fine-generalized closed (briefly fg−closed) if $f_{cl}(A) \subseteq U$ whenever A ⊆ U and U is fine-open in (X, τ,τ_f).

Example 3.2 Let $X = \{a,b,c\}$ and $\tau = \{\phi, X, \{a\},\{a,b\}\}$, $\tau_f = \{\phi, X, \{a\},\{b\},\{a,b\},$ $\{a,c\},\{b,c\}\}$, $F_f = \{\phi, X, \{b,c\},\{a,c\},\{c\},\{b\},\{a\}\}$. It may be easily check that, the set $\{a\}$ is fg-closed.

Definition 3.3 A subset A of fine-topological (X,τ,τ_f) is said to be fine-weakly closed set ($f\omega$−closed) set if $f_{cl}(A) \subseteq U$ whenever A ⊆ U and U is fine-semi-open in (X, τ,τ_f). The complement of fω −closed set is called fω −open in (X,τ,τ_f).

Example 3.4 Let $X = \{a,b,c\}$ and $\tau = \{\phi, X, \{a,b\}\}$, $\tau_f = \{\phi, X, \{a\},\{b\},\{a,b\},$ $\{a,c\},\{b,c\}\}$, $F_f = \{\phi, X, \{b,c\},\{a,c\},\{c\},\{b\},\{a\}\}$. It may be easily check that, the set $\{a,c\}$ is $f\omega$-closed.

Definition 3.5 A subset A of fine-topological (X,τ,τ_f) is said to be $f\omega\alpha -$ closed if $f_{\alpha cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is fω −open set in (X,τ,τ_f).

Example 3.6 Let $X = \{a,b,c\}$ and $\tau = \{\phi, X, \{a,b\}\}$, $\tau_f = \{\phi, X, \{a\},\{b\},\{a,b\},$ $\{a,c\},\{b,c\}\}$, $F_f = \{\phi, X, \{b,c\},\{a,c\},\{c\},\{b\},\{a\}\}$. It may be easily check that, the set $\{b,c\}$ is $f\omega\alpha$-closed.

Definition 3.7 A subset A of fine-topological $(X,\tau,\tau_f)$is said to be fine pre-g*− closed (briefly fpg^* −closed) if $f_{pcl}(A) \subseteq U$ whenever $A \subseteq U$ and U is $f\omega\alpha$ −open set in (X,τ,τ_f). The complement of fpg^* −closed set is called fpg^* −open in (X,τ).

Example 3.8 Let $X = \{a,b,c\}$ and $\tau = \{\phi, X, \{a,b\}\}$, $\tau_f = \{\phi, X, \{a\},\{b\},\{a,b\},$ $\{a,c\},\{b,c\}\}$, $F_f = \{\phi, X, \{b,c\},\{a,c\},\{c\},\{b\},\{a\}\}$. It may be easily check that, the sets $\{a\},\{b,c\}$ is fpg^*-closed.

Remark 3.9: Every fω −closed set, $f\omega -$ closed set, $f\omega\alpha$ −closed set and fpg^* −closed sets are fine-closed.
Theorem 3.10 Every fine-closed (resp. fine-pre-closed) set is fpg*-closed.
Proof: Let A be any fine-closed and G be any $f\omega\alpha -$ open set containing A in (X, τ, τ_f). Since, A is fine-closed $f_{cl}(A) = A$ (resp. $f_{pcl}(A) = A$), so $f_{cl}(A) \subseteq U$. Hence, A is fpg*-closed in (X, τ, τ_f).
Theorem 3.11 Every f_α −closed set is fpg*-closed set.
Proof: Let A be f_α closed and G be ω −open set in (X, τ, τ_f) such that $A \subseteq G$. Since A is f_α-closed,
$f_{\alpha_{cl}}(A) = A$. But $f_{p_{Cl}}A \subseteq f_{Cl}(A)$ is always true. Thus, $f_{p_{cl}}(A) \subseteq G$. Hence A is fp ∗-closed in (X,τ, τ_f), the proof follows.
Theorem 3.12 A fine-regular open fpg*-closed set is f-pre-closed and hence f-clopen.
Proof. Let A be regular open, fpg*-closed. Since fine-regular open set is fine-open , $f_{pCl}(A) \subseteq A$. This implies A is f-pre-closed. Since every f-pre-closed (fine-regular) open set is (fine-regular) fine-closed, A is fine-clopen.

4. fpg*-Open Sets

In this section, the notion of fpg*-open set is defined and some of its basic properties are studied.
Definition 4.1 A subset A in X is called fpg*-open in (X,τ, τ_f) if $X - A$ is fpg^*-closed (X, τ, τ_f)
Example 4.2 Let $X = \{a, b, c\}$ and $\tau = \{\phi, X, \{a, b\}\}$, $\tau_f = \{\phi, X, \{a\}, \{b\}, \{a, b\},$ $\{a, c\}, \{b, c\}\}$, $F_f = \{\phi, X, \{b, c\}, \{a, c\}, \{c\}, \{b\}, \{a\}\}$. It may be easily check that, the sets $\{b, c\}$ is fpg^*-open.
Theorem 4.3 A set A is fpg*-open in (X,τ, τ_f) if and only if $F \subseteq f_{pInt}(A)$ whenever F is $f\omega\alpha$- closed in (X,τ, τ_f) and $F \subseteq A$.
Proof. Suppose $F \subseteq f_{pInt}(A)$ where F is f$\omega\alpha$-closed and $F \subseteq A$. Let $X - A \subseteq G$ where G is f$\omega\alpha$- open in (X,τ, τ_f). Then $G \subseteq X - G$ and $X - G \subseteq f_{pInt}(A)$. Thus $X - A$ is fpg^*-closed in (X,τ, τ_f). Hence A is fpg^* −open in (X,τ, τ_f).
Conversely, suppose that A is fpg^*-open, $F \subseteq A$ and F is f$\omega\alpha$ −closed in (X,τ, τ_f). Then $X - F$ is $f\omega\alpha$-open and $X - A \subseteq X - F$. Therefore $f_{pCl}(X - A) \subseteq X - F$. But $f_{pCl}(X - A) = X - f_{pInt}(A)$. Hence $F \subseteq f_{Int}(A)$.

5. Conclusion

By using the concepts of fpg*-closed sets on fine-topological space, we may define a generalized form of continuity in terms of fpg*-closed sets. Also, by defining some irresolute maps, the more general form of homeomorphism can be studied which is widely used in quantum physics.

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