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RESEARCH ARTICLE

A STUDY OF INTERPOLATION THROUGH g-SPLINES

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Abstract

In this paper, we consider a new technique of spline methods is used for (0, 2, 5)- lacunary interpolation by g-splines with functions belonging to $C^{(5)}$(I) and (0, 1 ,2) - interpolation with functions belonging to $C^{(4)}$(I) using piecewise polynomials with certain specific properties. Our methods are of lower degree having better convergence property than the earlier investigations .

INTRODUCTION

Spline functions are used in many areas such as interpolation, datafitting, numerical solution of ordinary partial differential equation and also numerical solution of integral equations Lacunary interpolation by splines appears function about a function and its derivatives but without Hermite condition in which consecutive derivatives are used at each nodes, Several researchers have studied the use of spline to solve such interpolation [5 ,8 , 9 ,10 , 11] One uses polynomial for approximation because they can be evaluated. cubic spline interpolation is the most common piecewise polynomial method and is referred as "piecewise" since a unique polynomial is fitted between each pair of data points. The collection of polynomials that form the curve of polynomials that form the curve is collectively referred to as " the spline". The traditional and constrained cubic splines are few different groups of the same family. The group of traditional cubic splines can furthermore be divided into sub group natural, parabolic, runout, cubic run-out and damped cubic splines. The natural cubic spline is by far the most popular and widely used version of the cubic splines family.

The spline interpolation is based on the following principle: The interpolation interval is divided into small subintervals. The polynomial coefficients are chosen to satisfy certain conditions. A "spline" was a common drafting tool a flexible rod, that was used to help draw smooth curves connecting widely spaced points. Spline interpolation method, as applied to the solution of differential equation employ some from approximating function such as polynomials to approximate the solution by evaluating the function for sufficient number of points in the domain of the solution.

Th Fawzy ([3] [4]) constructed special kinds of lacunary quintic g-splines and proved that for functions $f \in C^{(4)}$ the method converges faster that investigated by A.K. Verma[1] and for functions $f \in C^{(5)}$ the order of approximation is the same as the best order of approximation using quintic g- splines. Saxena and Tripathi [7] have studied splines methods for solving the (0,1,3) interpolation problem. They have used spline interpolants of degree six for functions $\mathbf{f} \in \boldsymbol{C}^{(6)}$ to solved the problem ..R.S.Misra and K.K. Mathur [2] solved lacunary interpolation by splines (0;0 2,3) and(0;0,2,4) cases. During the past twentieth both theories of splines and experiences with their use in numerical analysis have undergone a considerable degree of development. According to Fawzy [3] the interest in spline function is due to the fact that spline function are a good tool for the numerical approximation of functions.

In addition to the paper mentioned above dealing with best interpolation on approximation by splines there were also few papers that deal with constructive properties of space of splines interpolation. In my earlier work [6] [12] [13] some kinds of lacunary interpolation by g-splines have been investigated. In this paper we will continue to discuss the problem.

This paper is organized as follows- In Section 2, we construct a spline function of degree five which interpolates the lacunary data (0, 2, 5) In section 3 we construct (0,1,2)-interpolation of degee 4. Some theorems about spline functions are also studied, by using some specific conditions , the method converges faster than the earlier investigations .

II. SPLINE INTERPOLANT (0, 2, 5) FOR f $\in C^{(5)}$

Let

Δ: $0 = x_0 < x_1 < \ldots < x_{n-1} < x_n = 1$

be a partition of the interval I = $[0,1]$ with $X_{k+1} - x_k = h_k$, $k = o(1)n - 1$.

and

$s_{1,\Delta}$ be a piecewise polynomial of degree ≤ 5. The spline interpolation(0, 2, 5) for $f \in C^{(5)}$ (I) is given by :

(2.1) $S_{1,\Delta}(x) = s_{1,k}(x) = \Sigma_{j=0}^{5} \frac{s_{k,j}^{(1)}}{j!}(x - x_k)^j$, $x_k \leq x \leq x_{k+1}$, $k = 0(1)n - 1$,

Where $s_{k,j}^{(1)}$'s are explicitly given below in terms of the prescribed data

$\{f_k^{(j)}\}$, j = 0,2,5 ; k = 0(1)n.

In particular for j = 0,2,5

(2.2) $s_{k,j}^{(1)} = f_k^{(j)}$, j = 0,2,5 and for j = 1, 3, 4 , we have

(2.3) $s_{k,3}^{(1)} = \frac{12}{h^3}$ $[\mathrm{h}\ \{f_{k+1}^{(1)} - \mathrm{h}\ f_k^{(2)} - \frac{h^4}{4!} f_k^{(5)}\} - \left\{f_{k+1} - f_k - \frac{h^2}{2!} f_k^{(2)} - \frac{h^5}{5!} f_k^{(5)}\right\} - \frac{h^2}{4}\{f_{k+1}^{(2)} - f_k^{(2)} - \frac{h^3}{3!} f_k^{(5)}\}]$

(2.4) $s_{k,4}^{(1)} = \frac{24}{h^4}[\{f_{k+1} - f_k - \frac{h^2}{2!} f_k^{(2)} - \frac{h^5}{5!} f_k^{(5)}\} - \mathrm{h}\left\{f_{k+1}^{(1)} - h f_k^{(2)} - \frac{h^4}{4!} f_k^{(5)}\right\} + \frac{h^2}{3}\{f_{k+1}^{(2)} - f_k^{(2)} - \frac{h^3}{3!} f_k^{(5)}\}]$

and

(2.5) $s_{k,1}^{(1)} = \frac{1}{\mathrm{h}}[f_{k+1} - f_k - \frac{h^2}{2!} f_k^{(2)} - \frac{h^3}{3!} s_{k,3}^{(1)} - \frac{h^4}{4!} s_{k,4}^{(1)} - \frac{h^5}{5!} f_k^{(5)}]$

The coefficients $s_{k,j}^{(1)}$ j = 1, 3, 4, have been choosen such that

$D_{\mathrm{L}}^{(p)}\ S_{1,k}\ (x_{k+1}) = D_{\mathrm{R}}^{(p)}\ S_{1,k+1}\ (x_{k+1})$ p = 0, 2, 5, k = 0(1) n-1

Thus

$S_{1,\Delta} \in C^{(0,2,5)}$ $[\mathrm{I}] = \{ \mathrm{f} : f^{(p)} \in \mathrm{C(I)}, \mathrm{p} = 0,2,5 \}$ is a unique quintic piecewise polynomial satisfying interpolatory conditions (2.2). If $f \in C^{(5)}$ $(\boldsymbol{I})$ than owing to (2.3)-(2.5) and using Taylor's expansion, we have

(2.6) $|s_{k,1}^{(1)} - f_k^{(1)}| \leq C_{k,1}^{(1)} h^4\ \omega(f^{(5)}; \mathrm{h})$,

(2.7) $|s_{k,3}^{(1)} - f_k^{(3)}| \leq C_{k,3}^{(1)} h^2\ \omega(f^{(5)}; \mathrm{h})$,

and

(2.8) $|s_{k,4}^{(1)} - f_k^{(4)}| \leq C_{k,4}^{(1)}\ \mathrm{h}\ \omega(f^{(5)}; \mathrm{h})$,

Where the constants $C_{k,j}^{(1)}$ are given by

For j = 1, 3, 4, k = 0(1) n-1

$C_{k,1}^{(1)} = \frac{31}{180}$, $C_{k,3}^{(1)} = \frac{3}{5}$, and $C_{k,4}^{(1)} = \frac{23}{15}$

Using (2.1) - (2.8) and a little computation gives :

Theorem 2.1

Let $f \in C^{(5)}(\boldsymbol{I})$ and $S_{1,\Delta} \in C^{(0,2,5)}$ [I] be the unique spline interpolant (0, 2, 5) given in (2.1) - (2.5),
then

(2.9) $|| D^{(j)}$ (f- $S_{1,\Delta}$) || $L_\infty [x_k, x_{k+1}] \le c_{1,k}^{j} h^{5-j} \omega(f^{(5)}, h)$, j =0(1) 5; k=0 (1) n-1

Where the constants $c_{1,k}^{(j)}$ are given by :

$c_{1,k}^{0} = \frac{41}{120}$, $c_{1,k}^{(1)} = \frac{277}{360}$, $c_{1,k}^{2} = \frac{23}{15}$, $c_{1,k}^{(3)} = \frac{79}{30}$, $c_{1,k}^{(4)} = \frac{38}{15}$, $c_{1,k}^{(5)} = 1$.

III. SPLINE INTERPOLANT (0, 1, 2) FOR $f \in C^{(4)}$

Let $S_{2,\Delta}$ be a piecewise polynomial of degree ≤ 4 which is a solution of (0,1,2)- interpolation for functions $f \in C^{(4)} [x_0\ x_n]$ in form :

(3.1) $S_{2,\Delta} = S_{2,k}(x) = \sum_{j=0}^{4} \frac{S_{k,j}^{(2)}}{j!}(x - x_k)^j$, $x_k \le x \le x_{k+1}$, $for \quad k = 0(1)n-1$

Where $S_{k,j}^{(2)}$, s are explicitly given below in terms of the prescribed data

$\{f_k^{(j)}\}$, j = 0,1,2 ; k = 0(1)n.

In particular for j = 0,1,2

(3.2) $S_{k,j}^{(2)} = f_k^{(j)}$, k = 0(1) n-1

and for j = 3, 4 , we have

(3.3) $S_{k,3}^{(2)} = \frac{4}{h^3}$ [(f_{k+1} - f_k - h$f_k^{(1)}$-$\frac{h^2}{2!}$ $f_k^{(2)}$) – h ($f_{k+1}^{(1)} - f_k^{(1)} -$ h$f_k^{(2)}$) - $\frac{5}{12}$ h^2($f_{k+1}^{(2)}$ - $f_k^{(2)}$)]

and

(3.4) $S_{k,4}^{(2)} = \frac{8}{h^4}$ [$\frac{2}{3}$ h^2($f_{k+1}^{(2)}$- $f_k^{(2)}$)] - h ($f_{k+1}^{(1)} - f_k^{(1)} -$ h$f_k^{(2)}$) – (f_{k+1} - f_k - h$f_k^{(1)}$- $\frac{h^2}{2!}$ $f_k^{(2)}$)]

The coefficients $S_{k,j}^{(2)}$, j = 0,3,4 have been determined

(3.5) $D_L^{(p)} S_{2,k}$ (x_{k+1}) = $D_R^{(p)} S_{2,k+1}$ (x_{k+1}) p = 0, 1, 2, k = 0(1) n-1

Thus

$S_{2,\Delta} \in C^{(0,1,2)}$ [I] = { f : $f^{(p)} \in$ C(I)}, p = 0,1,2 is a unique quartc piecewise polynomial satisfying interpolatory conditions (3.2). If $f \in C^{(4)}(\boldsymbol{I})$ than owing to (3.3)-(3.5) and Taylor's expansion, we have

(3.6) $\left| s_{k,j}^{(2)} - f_k^{(j)} \right| \le C_{k,j}^{(2)} h^{4-j} \omega(f^{(4)}; h)$ j = 0, 1, 2, k = 0(1) n-1

Where the constants $C_{k,j}^{(2)}$ are given by

$C_{k,3}^{(2)} = \frac{5}{6}$ and $C_{k,4}^{(2)} = 2$

Using (3.1) - (3.6) and a little computation gives:

Finally we have

Theorem 3.1

Let $f \in C^{(4)}(\boldsymbol{I})$ and $S_{2,\Delta} \in C^{(0,1,2)}$ [I] be the unique spline interpolant (0, 1, 2) given in (3.1) - (3.4); then we have

(3.7) $|| D^{(j)}$ (f- $S_{2,\Delta}$) || $L_\infty [x_k, x_{k+1}] \le c_{2,k}^{j} h^{4-j} \omega(f^{(4)}, h)$, j =0(1) 4, k=0 (1) n-1

Where the constants $c_{2,k}^{(j)}$ are given by :

$c_{2,k}^{(0)} = \frac{2}{9}$, $c_{2,k}^{(1)} = \frac{3}{4}$, $c_{2,k}^{(2)} = \frac{11}{6}$, $c_{2,k}^{(3)} = \frac{17}{6}$, $c_{2,k}^{(4)} = 2$.

IV.CONCLUSION

In this paper, we have studied the existence and uniqueness of (0 ,2 , 5) for functions belonging to $C^{(5)}$(I). and (0, 1 ,2) – interpolation for functions belonging to $C^{(4)}$(I) through g-splines and their local approximation with functions belonging $C^{(4)}$(I) and $C^{(5)}$(I) . Our methods are of lower degree having better convergence property

Also we conclude that this new technique we used in proving of the two important theorems by spline function is very easy than the earlier investigations.

REFERENCES

(1) A.K. VARMA : Lacunary interpolation by splines-II Acta Math. Acad. Sci. Hungar., 31(1978), pp. 193-203.

(2) R. S. MISRA & K.K. MATHUR : Lacunary interpolation by splines (0; 0, 2, 3) and (0; 0, 2, 4) cases, Acta Math. Acad. Sci. Hungar, 36 (3-4) (1980), pp. 251-260.

(3) Fawzy Th. Notes on Lacunary interpolation with splines-III, (0, 2) – interpolation with quintic g-splines Acta Math. Hung. 50 (1-2) (1987) pp.33-37.

(4) Th. FAWZY : (0, 1, 3) Lacunary interpolation by G-splines, Annales Univ. Sci., Budapest, Section Maths. XXXIX (1986), pp.63-67.

(5) J. GYORVARI : Lacunary interpolation spline functionen, Acta Math. Acad. Sci. Hungar, 42(1-2) (1983) , pp. 25-33.

(6) R. SRIVASTAVA lacunary interpolation by g-splines: International journal of Mathematics and Computer Research, Vol. 2, Issue 12 Dec.2014.

(7) R.B. SAXENA & H.C. TRIPATHI : (0, 2, 3) and (0, 1, 3)- interpolation by six degree splines, Jour. Of computational and applied Maths., 18 (1987), pp. 395-101.

(8) AMBRISH KUMAR PANDEY, Q S Ahmad, Kulbhushan Singh : Lacunary Interpolation (0, 2; 3) problem and some comparison from Quartic splines: American journal of Applied Mathematics and statistics 2013, 1(6), pp- 117-120.

(9) F. Lang and X. Xu: “A new cubic B-spline method for linear fifth order boundary value problems”. Journal of Applied Mathematics and computing, vol. 36, no. 1-2, pp-110-116, 2011.

(10) ABBAS Y. ALBAYATI, ROSTAM K.S., FARAIDUN K. HAMASALH: Consturction of Lacunary Sixtic spline function Interpolation and their Applications. Mosul University, J. Edu. And Sci., 23(3)(2010).

(11) JWAMER K.H. and RADHA G.K.: Generalization of (0, 4) lacunary interpolation by quantic spline . J. of Mathematics and Statistics; New York 6 (1) (2010) 72-78.

(12) R. SRIVASTAVA On lacunary Interpolation through g-splines: Inter national journal of Innovative Research in Science, Engineering and Technology. Vol. 4 Issue 6 June 2015.

(13) R. SRIVASTAVA A new kind of Lacunary Interpolation through g-Spines: International journal of Innovative Research in Science, Engineering and Technology. Vol. 4 Issue 8 August 2015.