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RESEARCH ARTICLE

Chaos and Bifurcation Analysis of Plant-Herbivore System with Intra-Specific Competitions

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Abstract

We formulate a simple plant-herbivore type model to study the interaction of certain plants and herbivores. In this paper, a discrete-time plant-herbivore model with intra specific competitions is proposed. Nicholson Bailey model with Hassel's function is applied to know the plant-herbivore dynamical system. Stability natures of the fixed points of the model were determined analytically. Bifurcation diagrams were obtained, which exhibits the co-existence of stability of the system.

I.INTRODUCTION

Plant-herbivore interactions are important in all ecosystems. Mathematical modeling has provided a very useful tool for understanding complex dynamics of ecosystem [18]. Over the past few decades the ecological research detected chaos and other forms of complex dynamics in simple population models which are continuously motivating Biologists, Physicists, Engineers and Applied Mathematicians to study ecological models [2]. In 1929, Thompson introduced a host-parasitoid model with a set of biological assumption. In 1935, Nicholson and Bailey modified the Thompson's model for better prediction of real life situations [1, 15]. After that, Rogers [6, 13] applied the model of Holling to host-parasitoid system by assuming two kinds of limitation on Thompson's model and Nicholson-Bailey's model. Many researchers produced many discrete type host-parasitoid models with different ecological factors [3, 8, 9]. Sanyi Tang and Lansun Chen modified the Nicholson- Bailey's model by introducing Holling type II and type III interactions [11, 14].

Competition among members of same species for the same resources in an ecosystem is known as Intra specific competition. Intra specific resource competition in plant-herbivore system is widely considered to be more intense than inter specific competition because the genetically similar plants make the same resource demands. Nicholson's terms "scramble and contest", distinguish between two quite different kinds of intra-specific competition when resource is limited. Scramble competition involves the exactly equal partitioning of the resource such that too large population density this season results in very low population density next season due to insufficient resource to maintain any individual. In contest competition, each successful individual gets all it requires and unsuccessful animals get insufficient for survival and reproduction [12].

In this paper, we shall focus our attention on analyzing how the Nicholson-Bailey model with Hassel's function affects the dynamic complexities of plant-herbivore interactions.

II. The Model

We consider discrete-time dynamics of plant-herbivore system. We define P_n to be the number of plants at generation n and H_n to be the number of herbivores at generation n and r to be the plant's basic reproductive rate. The general Nicholson-Bailey model is of the following form:

$$P_{n+1} = r\, P_n\, f(P_n, H_n)$$
$$H_{n+1} = P_n\,[1 - f(P_n, H_n)] \quad \text{............ (1)}$$

The model of Tang and Chen with Holling type II functional response is of the following form:

$$P_{t+1} = P_t\, e^{r\left(1-\frac{P_t}{k}\right)-\frac{aTH_t}{1+aT_hP_t}}\ ;\quad H_{t+1} = P_t\left[1 - e^{\frac{-aTH_t}{1+aT_hP_t}}\right] \quad \text{................ (2)}$$

Using the concept of Hassel, the proposed model contains the natural death rate of the herbivore [7, 10]. We formulate the model of plant-herbivore populations such that herbivores are regulated by intra specific competition. We shall assume the competition affects the function f (P_t ,H_t) and we assume that the system is independent of the time variable.

The proposed model is

$$P_{t+1} = P_t\, e^{r\left(1-\frac{P_t}{k}\right)-\frac{\alpha H_t}{(1+\alpha P_t)^{\beta}}}$$
$$H_{t+1} = P_t\left[1 - e^{\frac{-\alpha H_t}{(1+\alpha P_t)^{\beta}}}\right] - \gamma H_t \quad \text{............... (3)}$$

The parameters are described as below:

k	carrying capacity of the environment
r	intrinsic growth rate
α	instantaneous search rate
γ	natural death rate of the herbivore
β	intra specific competition coefficient
$\beta = 0$	no competition
$0 < \beta < 1$	under-compensation
$\beta = 1$	exact compensation
$\beta > 1$	over-compensation

III. The fixed points and local stability

Fixed points of the system are determined by solving the following non-linear system of equations:

$$P = P\, e^{r\left(1-\frac{P}{k}\right)-\frac{\alpha H}{(1+\alpha P)^{\beta}}}$$
$$H = P\left[1 - e^{\frac{-\alpha H}{(1+\alpha P)^{\beta}}}\right] - \gamma H$$

By simple calculation, we get three fixed points as follows:

i) $E_0 = (0, 0)$ is the origin.

ii) $E_1 = (k, 0)$ is the axial fixed point in the absence of herbivore (H=0) exists for $k > 0$.

iii) $E_2 = (\dot{P}, \dot{H})$

Where

$$\dot{P} = k\left(1 - \frac{P^*}{r}\right)$$
$$\dot{H} = \frac{1}{1+\gamma}\, k\left(1 - \frac{P^*}{r}\right)\left(1 - e^{-P^*}\right) \quad \text{................ (4)}$$
$$P^* = \frac{\alpha H}{(1+\alpha P)^{\beta}}$$

IV. The Dynamical behavior of the model

In this section, we investigated the local behavior of the model (3) around each fixed point. The local stability analysis of the model (3) can be studied by computing the variation matrix corresponding to each fixed point. The variation matrix of the model at the state variable is given by

$$J(P, H) = \begin{bmatrix} \frac{\partial F}{\partial P} & \frac{\partial F}{\partial H} \\ \frac{\partial G}{\partial P} & \frac{\partial G}{\partial H} \end{bmatrix} \quad \ldots\ldots\ldots\ldots\ldots\ldots (5)$$

Where

$$F = Pe^{r\left(1-\frac{P}{k}\right)-\frac{\alpha H}{(1+\alpha P)^{\beta}}};\ G = P\left[1 - e^{\frac{-\alpha H}{(1+\alpha P)^{\beta}}}\right] - \gamma H.$$

$$\frac{\partial F}{\partial P} = e^{r\left(1-\frac{P}{k}\right)-\frac{\alpha H}{(1+\alpha P)^{\beta}}}\left[1 + P\left(\frac{\alpha^2\beta H}{(1+\alpha P)^{\beta+1}} - \frac{r}{k}\right)\right]$$

$$\frac{\partial F}{\partial H} = \frac{-\alpha H}{(1+\alpha P)^{\beta}}e^{r\left(1-\frac{P}{k}\right)-\frac{\alpha H}{(1+\alpha P)^{\beta}}}$$

$$\frac{\partial G}{\partial P} = \left(1 - e^{\frac{-\alpha H}{(1+\alpha P)^{\beta}}}\right) + \frac{P\alpha^2\beta H}{(1+\alpha P)^{\beta+1}}e^{\frac{-\alpha H}{(1+\alpha P)^{\beta}}}$$

$$\frac{\partial G}{\partial H} = \frac{\alpha P}{(1+\alpha P)^{\beta}}e^{\frac{-\alpha H}{(1+\alpha P)^{\beta}}} - \gamma$$

The characteristic equation of Jacobian matrix is written as $\lambda^2 - Tr\lambda + Det = 0$ where Tr is the trace and Det is the Determinant of the Jacobian matrix J (P, H). Hence the model (3) is a Dissipative dynamical system if $|Det| < 1$ and conservative dynamical one if and only if $|Det| = 0$.[17]

Theorem 1:

If $P(\lambda) = \lambda^3 + B\lambda^2 + C\lambda + D = 0$ is the characteristic equation of the matrix, then the following statements are true:

a) If every root of the equation has absolute value less than one, then the fixed point of the system is locally asymptotically stable and the fixed point is called a sink.
b) If at least one of the roots of the equation has absolute value greater than one, then the fixed point of the system is unstable and the fixed point is called a saddle.
c) If every root of the equation has absolute value greater than one, then the fixed point is a source.
d) If no root of the equation has absolute value equal to one, then the fixed point of the system is called hyperbolic. If there exists a root of the equation with absolute value equal to one, then the fixed point is called non-hyperbolic.[5]

Proposition 1:

The Eigen values of the trivial fixed point $E_0 = (0, 0)$ is locally asymptotically stable if $r < 0$ and $\gamma < 1$.

Proof:

In order to prove the result, we estimate the Eigen values of Jacobian matrix J at $E_0 = (0, 0)$. The Jacobian matrix at E is J_0.

$$J_0 = \begin{bmatrix} e^r & 0 \\ 0 & -\gamma \end{bmatrix}$$

Hence the Eigen values of the matrix are $\lambda_1 = e^r$ and $\lambda_2 = -\gamma$. By Theorem1, if $e^r < 1$ and $\gamma < 1$ which implies that $r < 0$ and $\gamma < 1$ then E_o is asymptotically stable otherwise unstable fixed point.

Proposition 2:

The fixed point $E_1 = (k, 0)$ is locally asymptotically stable if $r > 0$ and $\gamma > \frac{\alpha k}{(1+\alpha k)^{\beta}} - 1$.

Proof:

The Jacobian matrix evaluated at E_1 gives

$$J(E_1) = J_1 = \begin{bmatrix} 1 - r & \frac{-\alpha k}{(1+\alpha k)^{b}} \\ 0 & \frac{\alpha k}{(1+\alpha k)^{\beta}} - \gamma \end{bmatrix}$$

The Eigen values of the matrix J_1 are $\lambda_1 = 1 - r$ and $\lambda_2 = \frac{\alpha k}{(1+\alpha k)^{\beta}} - \gamma$.

By theorem 1, if $(1 - r) < 1$ and $\frac{\alpha k}{(1+\alpha k)^{\beta}} - \gamma < 1$ which implies $r > 0$ and $\frac{\alpha k}{(1+\alpha k)^{\beta}} - \gamma < 1$ which implies

$r > 0 \; and \; \frac{\alpha k}{(1+\alpha k)^{\beta}} - 1 < \gamma$, then, E_1 is asymptotically stable otherwise unstable.

Theorem 2:

If the Eigen values of the Jacobian matrix of the fixed point are inside the unit circle of the complex plane, the fixed point is locally stable. Using Jury's condition [4] we have necessary and sufficient condition for local stability of interior fixed point which are necessary and sufficient conditions for $|\lambda_{1,2}| < 1$

(i) 1+Tr(J)+Det(J) > 0

(ii) 1-Tr(J)+Det(J)> 0

(iii) $|Det J| < 1$

V. Local Stability and Dynamical behavior around interior fixed point E_2

We now investigate the local stability of interior fixed point E_2. The Jacobian matrix at E_2 is of the form

$$J\,[E_2] = J_2 = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

Where

$$B_{11} = e^{r\left(1-\frac{\hat{P}}{k}\right)-\frac{\alpha \hat{H}}{(1+\alpha \hat{P})^{\beta}}}\left[1+\hat{P}\left(\frac{\alpha^2 \beta \hat{H}}{(1+\alpha \hat{P})^{\beta+1}} - \frac{r}{k}\right)\right];$$

$$B_{12} = \frac{-\alpha \hat{P}}{(1+\alpha \hat{P})^{\beta}} e^{r\left(1-\frac{\hat{P}}{k}\right)-\frac{\alpha \hat{H}}{(1+\alpha \hat{P})^{\beta}}};$$

$$B_{21} = \left(1 - e^{\frac{-\alpha \hat{H}}{(1+\alpha \hat{P})^{\beta}}}\right) + \frac{\hat{P}\alpha^2 \beta \hat{H}}{(1+\alpha \hat{P})^{\beta+1}} e^{\frac{-\alpha \hat{H}}{(1+\alpha \hat{P})^{\beta}}}; \quad B_{22} = \frac{\alpha \hat{P}}{(1+\alpha \hat{P})^{\beta}} e^{\frac{-\alpha \hat{H}}{(1+\alpha \hat{P})^{\beta}}} - \gamma$$

Its characteristic equation is $\lambda^2 - Tr(J_2)\lambda + Det\,(J_2) = 0$ …………………....................... (7)

Tr (J_2)= $B_{11} + B_{22} = -D_1$

Det(J_2)=$B_{11}B_{22} - B_{21}B_{12} = D_2$

By theorem, using formulas for Tr and Det, the interior fixed point is locally stable if we find the inequality (i) in theorem 2 is $1 - D_1 + D_2 > 0$ which implies that $D_1 - D_2 < 1$, the inequality (ii) is equivalent to $D_1 + D_2 > -1$,and the inequality (iii) is equivalent to $|D_2| < 1$.[16]

For the interior fixed point E_2 the roots of the equation (7) are

$$\lambda_{1,2} = \frac{D_1 \pm \sqrt{{D_1}^2 - 4{D_2}^2}}{2}$$

Both of the Eigen values are locally asymptotically stable if $D_1 + \sqrt{{D_1}^2 - 4{D_2}^2} < 2$ and $D_1 - \sqrt{{D_1}^2 - 4{D_2}^2} < 2$.

VI. Numerical simulation

In this section, we give the numerical simulations to verify theoretical results proved in the previous sections by using MATLAB (8.1) programming. We also confirmed the results by visual representation of the system for some values of the parameters. Bifurcation diagrams with respect to k, β and γ were presented in the appendix.

Figure: 1 is a bifurcation diagram for model (3) for k =10.17. A stable coexistence is observed for $0 \leq r < 2$, bifurcates 2-cycles when $r = 2$ and bifurcates 4-cycles at $r = 2.5$ and chaos after $r = 2.5$.

Figure: 2 shows when intrinsic growth rate $r = 0$ to 2 with one herbivore, the plant population bifurcate 2-cycles at $r = 2$, 4-cycles at $r = 2.3$, 8-cycles at $r = 2.5$ and chaos after $r = 2.5$.

Figure: 3 is a bifurcation diagram for β=0.15. A stable coexistence is observed for $r = 0$ to 4. There is no period doubling bifurcation for r = 0 to 4. Figure: 4 and Figure: 5 are the bifurcation diagrams for the intraspecific competition coefficient β = 0.35 and β = 0.55 respectively. Figure: 6, 7 and 8 are the bifurcation diagrams with respect to the natural death rate of the herbivore γ = 0.45, 0.65 and 0.75 respectively.

VII. Conclusion

In this paper, we analyzed dynamics of a discrete-time plant-herbivore system with intra specific competition involving natural death rate of the herbivore. The study of plant-herbivore with the help of modified NB model with Hassel's function yields interesting results. In the present study we discussed the local stability of plant-herbivore model by using modified NB model with Hassel's function. We have introduced some new parameters to discrete-time plant-herbivore model and obtain equilibrium points. The effect of intra specific competition on the model depends on the value of the intrinsic growth rate and natural death rate of the herbivore. Bifurcation diagrams were obtained for different values of the parameters of the system to ensure the analytical results..

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Appendix:

BIFURCATION DIAGRAM WITH RESPECT TO k

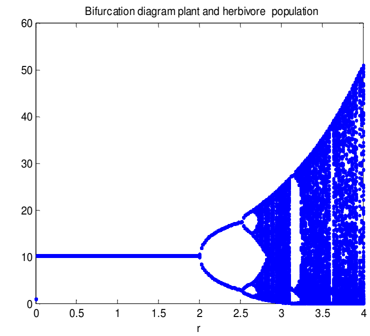

Figure: 1
α=1.5,k=10.17, β=0.65,γ=0.25

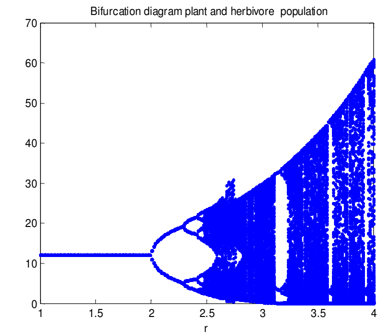

Figure: 2
α =1.5,k=12.17,β=0.65,γ=0.25

BIFURCATION DIAGRAM WITH RESPECT TO β

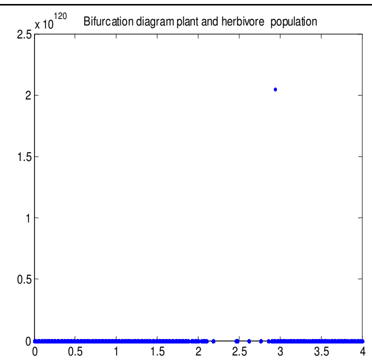

Figure: 3
α=1.5,k=12.17, β=0.15, γ=0.15

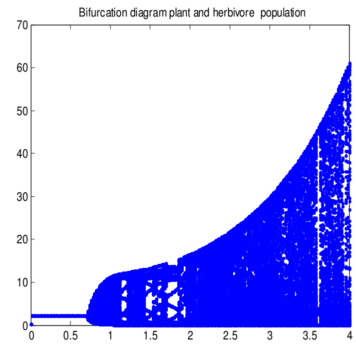

Figure: 4
α=1.5,k=12.17, β=0.35, γ=0.15

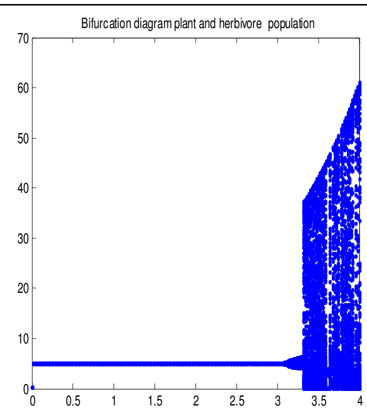

Figure: 5
α=1.5,k=12.17, β=0.55,γ=0.15

BIFURCATION DIAGRAM WITH RESPECT TO γ

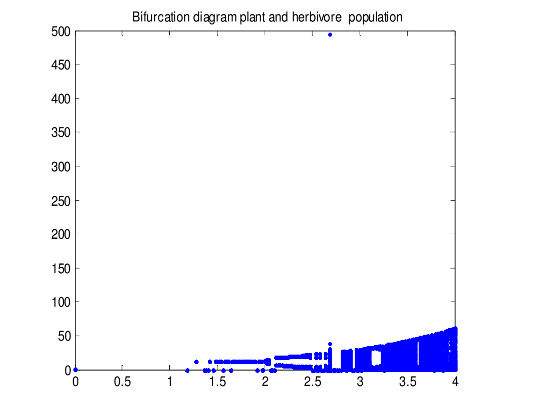

Figure: 6
α=1.5;k=12.17;β=0.55;γ=0.45

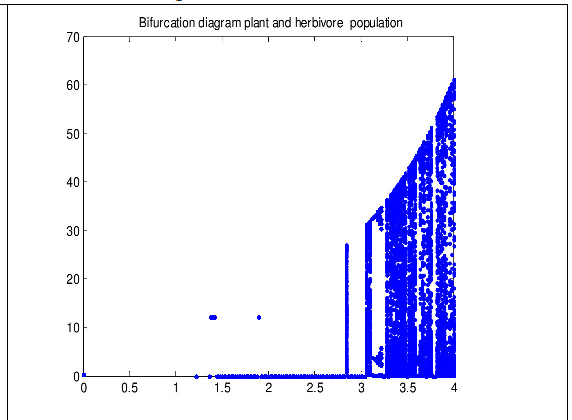

Figure: 7
α=1.5;k=12.17;β=0.55;γ=0.65

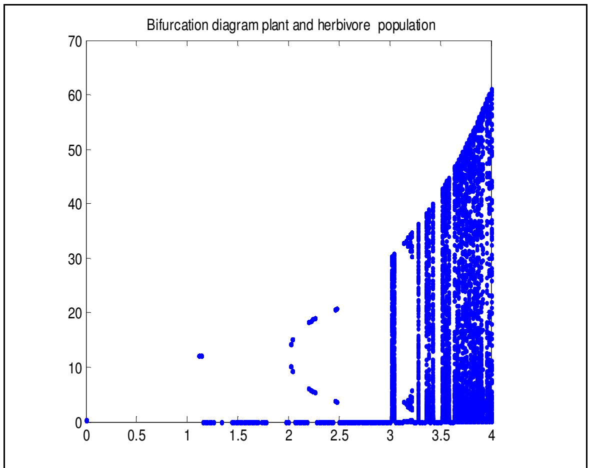

Figure: 8
α=1.5;k=12.17;β=0.55;γ=0.75