ISSN NO. 2320-5407

Journal homepage: http://www.journalijar.com

INTERNATIONAL JOURNAL OF ADVANCED RESEARCH

RESEARCH ARTICLE

Positivity and Convexity Preserving Interpolation Using GC^1 Quartic Trigonometric Spline

[1] R.K. Dubey, [2] S.S.Thakur and [3] S.S.Rana

1. Department of Engineering Mathematics, Global Nature Care Sangathans Group of Institutions Jabalpur India
2. Department of Mathematics, Jabalpur Engineering College, Jabalpur (M.P.) India.
3. Department of Mathematics and Computer Science, R. D. University, Jabalpur, (M.P.) India.

Manuscript Info

Manuscript History:

Received: 15 September 2015
Final Accepted: 22 October 2015
Published Online: November 2015

Key words:

Corresponding Author

R.K. Dubey

Abstract

A GC^1 quartic trigonometric spline interpolation has been developed, which can produce a positive and convex interpolant to given positive and convex data respectively. The explicit representation is easily constructed and numerical examples indicate that the method produces visually pleasing curves. The degree of smoothness is GC^1 continuity.

INTRODUCTION

It is well known that problems concerning positivity, convexity preserving interpolation have received considerable attention, because of their interest in computer aided design and in other practical applications [9]. Problem of construction of interpolating curve which preserves the convexity of the initial discrete set of data still remains in the focus of investigators [3]. In what follows an algorithm preserving the convexity of given set of data is presented. In computer graphics, a designer in industries needs to generate splines which can interpolate the data points in such a way that they conserve positivity and convexity of data. Among the properties that the spline for curves need to satisfy smoothness and shape preservation of given data are mostly needed by all the designers. Convexity is a substantial shape characteristic of the data. The significance of the convexity-preserving interpolation problems in industry cannot be denied. A number of examples can be quoted in this regard, like the modelling of cars in automobile industry, aeroplane, and ship design. A crumpled curve is an unwanted characteristic. Human aesthetic sense demands convexity-preserving nice and smooth curves without wiggles [10]. A shape-preserving interpolating splines have been developed in [5] and [6]. Convexity should also be upheld in many applications including nonlinear programming problems occurring in engineering, telecommunication system design, approximation of functions, parameter estimation, and optimal control. The traditional cubic spline schemes have been used for quite a long time to deal with the problems of constructing smooth curves that passes through given data points. However, these splines sometimes fail to conserve the inherited shape characteristics because of unwanted oscillations that are not suitable for design purpose.

Abbas et al. [1] discussed the problem of local convexity-preserving data visualization using piecewise rational cubic and bicubic function with three shape parameters. Brodlie and Butt [4] solved the problem of shape preserving of convex data by using the cubic Hermite interpolation. The authors inserted one or two extra knots in the interval where the shape of data was not conserved. Bao et al. [2] developed rational interpolation based on function values and also discussed constrained control of the interpolating curves. They obtained conditions on function values for constraining the interpolating curves to lie above, below, or between the given straight lines.

Sarfraz et al. [13] addressed the problem of shape preserving rational cubic spline for positive and convex data. Simple data dependent constraints were derived for free parameters used in the description of rational cubic function to achieve the desired shape of the data. The scheme provided a limited freedom to designer to obtain a visually pleasing display of the data. Sarfraz et al. [12] developed a rational cubic spline with two families of free parameters for positive, monotone and convex curve. Sufficient data dependent constraints were made for free parameters to maintain the shape of data. Dube and Rana [8] solved the problem of positive data by rational quadratic trigonometric spline. Rana et al [11] has been developped rational cubi trigonometric spline with two shape parameters.

This paper is organized as follows. A piecewise GC^1 quartic trigonometric function function with three shape parameters has been developped in Section 2. The determination of derivatives has been explained in section 3. The positivity problem and its illustrations have been discussed in Section 4 for the generation of a GC^1 quartic trigonometric spline which can preserve the shape of positive data. Section 5 is talking about convex interpolation and its examples. Conclusion of the paper is discussed in section 6.

2. GC^1 quartic trigonometric function

Let (t_i, f_i), $i = 0, 1, 2, \dots, n$, be a given set of data points where $t_0 < t_1 < \cdots < t_n$. $h_i = t_{i+1} - t_i$, $\nabla_i = \frac{f_{i+1} - f_i}{h_i}$, $i = 0, 1, 2, \dots, n-1$. For $t \in [t_i, t_{i+1}]$, let $\theta = \frac{\pi(t - t_i)}{2h_i}$, $0 \le \theta \le \frac{\pi}{2}$. Now consider the following piecewise GC^1 quartic trigonometric function :

$$\begin{aligned} P(t) &= P_i(t) \\ &= [(1-sin\theta)^3\{1+(2-\delta_i)sin\theta\} + sin\theta(1-sin\theta)^2\{1+(2-\alpha_i)sin\theta\}]A_0 \\ &+ [\delta_i(1-sin\theta)^3 sin\theta + \alpha_i(1-sin\theta)^2 sin^2\theta]A_1 \\ &+ [\delta_i(1-cos\theta)^3 cos\theta + \beta_i(1-cos\theta)^2 cos^2\theta]A_2 \\ &+ [(1-cos\theta)^3\{1+(2-\delta_i)cos\theta\} + cos\theta(1-cos\theta)^2\{1+(2-\beta_i)cos\theta\}]A_3 \end{aligned} \quad (2.1)$$

where

$A_0 = f_i$, $A_1 = \frac{2h_i d_i}{\pi\delta_i r_i} + f_i$, $A_2 = f_{i+1} - \frac{2h_i d_{i+1}}{\pi\delta_i r_{i+1}}$ and $A_3 = f_{i+1}$ are control points. $\alpha_i, \beta_i, \delta_i$ and r_i are positive shape paraeters and d_i denotes a given derivative value at knot t_i. The quartic trigonometric interpolant (2.1) satisfies the following properties.

$$P(t_i) = f_i, \quad P(t_{i+1}) = f_{i+1},$$
$$P'(t_i) = \frac{d_i}{r_i}, \; P'(t_{i+1}) = \frac{d_{i+1}}{r_{i+1}}. \quad (2.2)$$

3. Determination of derivatives

In most applicaton, derivative parameters d_i are not given and hence must be determined from the data (x_i, f_i). In this paper the derivative values will be computed by arithmetic mean (see [6]) as follows.

$$d_i = \frac{h_i\Delta_{i-1} + h_{i-1}\Delta_i}{h_i + h_{i-1}}, \; i = 1,2..,n-1, \quad (3.1)$$

$$d_1 = \Delta_1 + \frac{(\Delta_1 - \Delta_2)h_1}{(h_1 + h_2)}, \quad (3.2)$$

$$d_n = \Delta_{n-1} + \frac{(\Delta_{n-1} - \Delta_{n-2})h_{n-1}}{(h_{n-2} + h_{n-1})}. \quad (3.3)$$

These arithmetic mean approximation are suitable for convex interpolation because it is satisfying the inequality $d_1 < \Delta_1 < \cdots \Delta_{i-1} < d_i < \Delta_{i-1} \ldots < \Delta_{n-1} < d_n$.This method is computationally more economical.

4. Positivity preserving spline interpolation

The GC^1 quartic trigonometric spline interpolant (2.1) does not always preserves the positivity of the given data sets. The GC^1 quartic trigonometric spline does not guarantee to preserve the positivity of the data sets listed in table 1. Figure 1 shows this fact clearly. We may find suitable value of shape parameters $\alpha_i, \beta_i, \delta_i$ and r_i for i = 1, 2,...., n-1 and error strategy. But, this is not an efficient method and not easy to be implemented. Thus, the best possible ways to overcome those problems is to derive the sufficient condition for positivity on one of the shape parameter while the other parameter can be used to refine the final shape of the curves. Our main strategy is to impose certain conditions on a rational cubic defined in (2.1) to ensure that it will preserve the positivity of a data set. Let us assume a strictly positive data set is given $\{(t_0, f_0), (t_1, f_1), \dots, (t_n, f_n)\}$ for :

$$t_1 < t_2 < \cdots < t_n \quad (4.1)$$

and

$f_0 > 0, f_1 > 0, ..., f_n > 0$ (4.2)

In this paper, we would find sufficient and simple constraints for single shape parameter in order to preserve the positive curve of positive data in each sub interval if

$$P_i(t) > 0, i = 1, 2, \dots n \quad . \qquad (4.3)$$

The key idea is to determine constraints for $\alpha_i, \beta_i, \delta_i$ and r_i which guarantee to preserve the shape of positive data. The equation (4.3) is satisfied if A_1 and A_2 are positive.

Therefore, the necessary condition for free parameters to preserve shape of positive data are

$$\alpha_i > 0, \beta_i > 0, \delta_i > 0, \qquad (4.4)$$

and

$$r_i > \frac{-2h_i d_i}{\pi \alpha_i f_i} \qquad (4.5)$$

$$r_{i+1} > \frac{2h_i d_{i+1}}{\pi \beta_i f_{i+1}} \qquad (4.6)$$

Thus, we prove the following when we appeal to (4.4) – (4.6).

Theorem 1. For a strictly positive data, the GC^1 quartic trigonometric interpolant (2.1) defined over the interval $[t_0, t_n]$ is positive in each subinterval $[t_i, t_{i+1}]$, i =0,1,2,..., n −1 if the following sufficient conditions are satisfied: $\alpha_i > 0, \beta_i > 0, \delta_i > 0$, $r_i > \frac{-2h_i d_i}{\pi \alpha_i f_i}$ and $r_{i+1} > \frac{2h_i d_{i+1}}{\pi \beta_i f_{i+1}}$.

Algorithm for positivity-preserving using GC^1 quartic spline

1. Input the number of data points, n , and data points (t_i, f_i), i =0, 1, 2,...., n.
2. For i = 1,2,..., n , estimate d_i using arithmetic mean method (AMM).
3. Calculate h_i and Δ_i for i = 1, 2,...,n
4. Calculate $r_i > \frac{-2h_i d_i}{\pi \alpha_i f_i}$ and $r_{i+1} > \frac{2h_i d_{i+1}}{\pi \beta_i f_{i+1}}$ for i =0, 1, 2,...,n-1
5. Calculate the inner control ordinates A_0 and A_1 in each subinterval and generate the piecewise positive interpolating curves using (2.1).

4.1. Example and discussion

According to table 1, for positive data, we will use the derivative approximation as mentioned in section 3. Theorem 1 will be used and the result is followed by table 1.

Table1: Positive data

I	1	2	3	4	5	6	7
t_i	0	2	4	10	28	30	32
f_i	20.8	8.8	4.2	.5	3.9	6.2	9.6

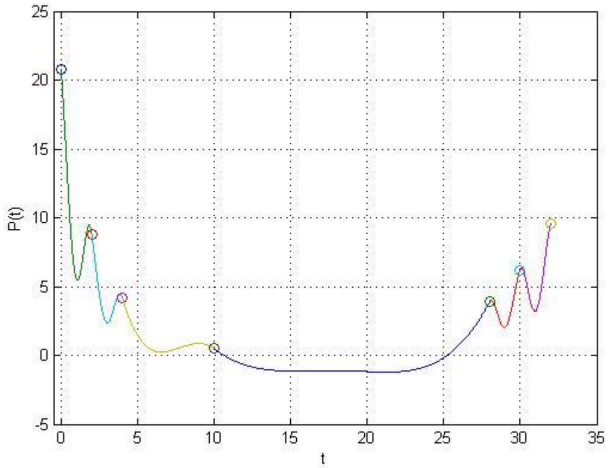

Figure1: Nonpositivity preserving GC[1] quartic trigonometric curve without using theorem 1

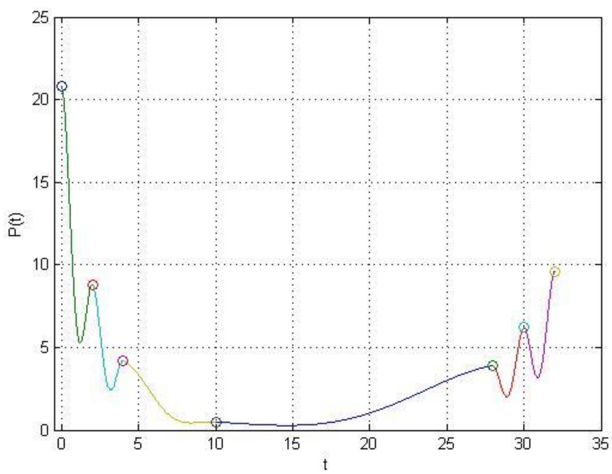

Figure2: Positivity preserving GC[1] quartic trigonometric curve using theorem 1

The developed scheme is used to demonstrate the positivity preserving of positive data. Random values to the shape parameters are assigned and it is clearly visible that the resulting curve do not preserve the positivity see figure 1. On the other hand, the positivity preserving curve in figure 2 is generated by the theorem 1.

5. Convex interpolation

Engineering practice usually requires that the interpolating function retains the shape of the given data. For a convex interpolant P(t), it is very necessary that the derivative parameters to be

$$d_1 < \Delta_1 < \cdots < \Delta_{i-1} < d_i < \Delta_i < \cdots < \Delta_{n-1} < d_n \qquad (5.1)$$

P(t) is convex if and only if

$$P^{(2)}(t) \geq 0 \qquad (5.2)$$

For all $t \in [t_0, t_n]$, the second derivative of P(t) for $t \in [t_i, t_{i+1}]$ is given by

$P^{(2)}(t) = (\frac{\pi}{2h_i})^2[2\, sin\theta(1-sin\theta)^3X_0 + 2cos^2\theta(1-sin\theta)^2X_1 + sin\theta(1-sin\theta)cos^2\theta X_2$
$+2sin^2\theta(1-sin\theta)^2X_3 + 4sin^3\theta(1-sin\theta)X_4 + 2f_i(cos^2\theta + sin\theta(1-sin\theta))$
$+4sin^2\theta cos^2\theta X_5 + 2f_{i+1}(sin^2\theta + cos\theta(1-cos\theta)) + 4cos^3\theta(1-cos\theta)X_6$ $+ cos^2\theta(1-$
cosθ)2X7+2cosθ1−cosθsin2θX8+2sin2θ(1−cosθ)2X9 +2 cosθ(1−cosθ)3X10]

where

$$X_0 = -f_i - \frac{h_i d_i}{\pi r_i},$$

$$X_1 = -4f_i + \frac{(\alpha_i - 3\delta_i)2h_i d_i}{\pi r_i \delta_i},$$
$$X_2 = -2(1+\alpha_i)f_i + \frac{(3\delta_i - 4\alpha_i)2h_i d_i}{\pi r_i \delta_i},$$
$$X_3 = f_i + \frac{(3\delta_i - 2\alpha_i)h_i d_i}{\pi r_i \delta_i},$$
$$X_4 = f_i + \frac{h_i d_i}{\pi r_i \delta_i},$$
$$X_5 = \left(f_i + \frac{\alpha_i h_i d_i}{\pi r_i \delta_i}\right) + \left(f_{i+1} + \frac{\beta_i h_i d_{i+1}}{\pi r_{i+1} \delta_i}\right),$$
$$X_6 = f_{i+1} - \frac{h_i d_{i+1}}{\pi r_{i+1} \delta_i},$$
$$X_7 = f_{i+1} - \frac{(3\delta_i - 2\beta_i)h_i d_{i+1}}{\pi r_{i+1} \delta_i},$$
$$X_8 = -2(1+\beta_i)f_{i+1} + \frac{(3\delta_i - 4\beta_i)h_i d_{i+1}}{\pi r_{i+1} \delta_i},$$
$$X_9 = -4f_{i+1} + \frac{(\beta_i - 3\delta_i)2h_i d_{i+1}}{\pi r_{i+1} \delta_i},$$
$$X_{10} = -f_{i+1} + \frac{h_i d_{i+1}}{\pi r_{i+1}}.$$

We set the parameters $X_0, X_1, X_2, X_3, X_4, X_5, X_6, X_7, X_8, X_9, X_{10} \geq 0$ then we find that

$$r_i \geq \max\left\{-\frac{h_i d_i}{\pi f_i}, \frac{(\alpha_i - 3\delta_i)h_i d_i}{2\pi f_i \delta_i}, \frac{(3\delta_i - 4\alpha_i)h_i d_i}{(1+\alpha_i)\pi f_i \delta_i}, \frac{-(3\delta_i - 2\alpha_i)h_i d_i}{\pi f_i \delta_i}, -\frac{h_i \alpha_i d_i}{\pi f_i \delta_i}\right\} \quad (5.2)$$

$$r_{i+1} \geq \max\left\{\frac{h_i d_{i+1}}{\pi f_{i+1}}, -\frac{(\beta_i - 3\delta_i)h_i d_{i+1}}{2\pi f_{i+1} \delta_i}, -\frac{(3\delta_i - 4\beta_i)h_i d_{i+1}}{(1+\beta_i)\pi f_{i+1} \delta_i}, \frac{(3\delta_i - 2\beta_i)h_i d_i}{\pi f_{i+1} \delta_i}, -\frac{h_i \beta_i d_{i+1}}{\pi f_{i+1} \delta_i}\right\} \quad (5.3)$$

Thus, we have the following theorem.

Theorem 2. Given $\{(t_i, f_i, d_i)$, i = 0,1,...n}, the sufficient and condition for the interpolation defined by (2.1) to be convex on $[t_i, t_{i+1}]$ is that the given data and positive parameters $\alpha_i, \beta_i, \delta_i$ and r_i satisfy both

$$r_i \geq \max\left\{-\frac{h_i d_i}{\pi f_i}, \frac{(\alpha_i - 3\delta_i)h_i d_i}{2\pi f_i \delta_i}, \frac{(3\delta_i - 4\alpha_i)h_i d_i}{(1+\alpha_i)\pi f_i \delta_i}, \frac{-(3\delta_i - 2\alpha_i)h_i d_i}{\pi f_i \delta_i}, -\frac{h_i \alpha_i d_i}{\pi f_i \delta_i}\right\}$$
$$r_{i+1} \geq \max\left\{\frac{h_i d_{i+1}}{\pi f_{i+1}}, -\frac{(\beta_i - 3\delta_i)h_i d_{i+1}}{2\pi f_{i+1} \delta_i}, -\frac{(3\delta_i - 4\beta_i)h_i d_{i+1}}{(1+\beta_i)\pi f_{i+1} \delta_i}, \frac{(3\delta_i - 2\beta_i)h_i d_i}{\pi f_{i+1} \delta_i}, -\frac{h_i \beta_i d_{i+1}}{\pi f_{i+1} \delta_i}\right\}$$

5.1. Example and discussion

Table 2 shows the convex data set

Table 2; convex data

I	1	2	3	4	5
t_i	1	2	4	5	10
f_i	10	2.5	.625	.4	.1

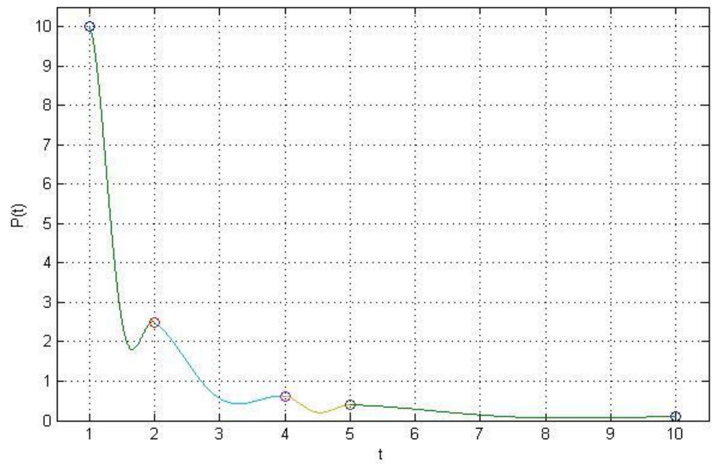

Figure 2: Convexity preserving GC^1 quartic trigonometric curve using theorem 2.

The developed scheme is used to demonstrate the convexity preserving of convex data. The convexity preserving curve in figure 3 is generated by the theorem 2.

6. Conclusion

The main goal of this paper is the preservation of the positive and convex data using GC^1 quartic trigonometric interpolation scheme. The result in this paper shows that GC^1 quartic trigonometric spline works well. The advantageous features on the given scheme are: no extra knots are inserted to preserve the positivity of positive data and convexity of convex data, it works well for equally spaced data, and in this scheme there is no constraints on derivatives. Thus there is no need to modify the derivative. This scheme is computationally economical and time saving.

Refrences

[1] Abbas, M., Majid, A. A. , Awang, M. N. H., and Ali, J. M. Local convexity shape-preserving data visualization by spline function, ISRN Mathematical Analysis, Article ID 174048, 14 pages, 2012.

[2] Bao, F. , Sun, Q. and Duan, Q. Point control of the interpolating curve with a rational cubic spline, Journal of Visual Communication and Image Representation,vol.20(2009),275–280.

[3] Bogdanov, V. V. Volkov, Yu. S. Selection of parameters of generalized cubic splines with convexity preserving interpolation, Sib. Zh. Vychisl. Mat., 9:1 (2006), 5–22.

[4] Brodlie K. W. and Butt, S. Preserving convexity using piecewise cubic interpolation, Computers & Graphics, 15(1991), 15–23.

[5] Costantini, P. Boundary-valued shape-preserving interpolating splines, ACM Transactions on Mathematical Software, 23(1997), 229–251.

[6] Delbourgo, R. And Gregory, J.A., The determination of derivative parameters for a monotonic rational quadratic interpolant. IMA Journal of Numerical Analysis, 5(1985

[7] Dube, M and Rana, P.S., GC^2 Rational cubic spline with tension parameters. IJRDET, 3(2014), 6, 1-4.

[8] Fiorot, J.C. and Tabka, J. Shape-preserving C^2 cubic polynomial interpolating splines, Mathematics of Computation, 57(1991), 195, 291–298,

[9] Kuijt, F. Convexity preserving interpolation stationary nonlinear subdivision and splines ,Ph.D. thesis, University of Twente, Enschede, The Netherlands, 1998.

[10] Kvasov. B. I., Methods of shape-preserving spline approximation, World Scientific, Singapore, 2000.

[11] Rana, S.S., Dube, M. And Tiwari, P., Rational cubic trigonometric spline with two shape parameters, IJETAE 3(2013), 144-149.

[12] Sarfraz, M., Hussain, M. Z. and M. Hussain, Shape-preserving curve interpolation, International Journal of Computer Mathematics, 89(2012), 1, 35–53.

[13] Sarfraz, M., Hussain, M. Z., Shaikh, T. S. and Iqbal, R. Data visualization using shape preserving C^2 rational spline, in Proceedings of the 15th International Conference on Information Visualisation (IV 11), 528–533, London, UK, July 2011, 528–533.