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RESEARCH ARTICLE

Reliability Performances Based On Empirical Bayes Censored Gompertz Data

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Abstract

The main focus of present article is to study about the performance of reliability parameter under censored data. Properties are studied under Progressive Type-II and conventional Type-II censored data. An empirical Bayes estimation criterion under Gompertz distribution is addressed here. Performances of the procedures are illustrating by a simulation technique.

INTRODUCTION

Gompertz (1825) first introduced the underlying model and, its distribution function and probability density function are given for scale parameter θ as

$$F(x;\theta) = 1 - \exp\{-\theta e^x + \theta\}; \theta > 0, x > 0$$

and

$$f(x;\theta) = \theta e^x \exp\{-\theta e^x + \theta\}; \theta > 0, x > 0. \quad (1.1)$$

Gompertz distribution has many useful applications in areas of technology, medical, biological, and natural sciences (especially in failure and survival analysis). This distribution also widely used in model of human mortality and fit in the actuarial tables. The present article studied the reliability performances of the underlying model.

The reliability function $\Psi(t)$ and failure rate $\rho(t)$ of model (1.1), are given for some mission time $t(>0)$, as

$$\Psi(t) = \exp\{-\theta e^t + \theta\}; \theta > 0, t > 0 \quad (1.2)$$

and

$$\rho(t) = \theta e^t; \theta > 0, t > 0. \quad (1.3)$$

Some parameter estimation of Gompertz population based on human mortality model was discussed by Chen (1997). Wu, et al. (2003) discussed about point and interval estimations for Gompertz distribution under Progressive Type-II censoring. Bayes estimation for unknown parameters and acceleration factor under partially accelerated life tests based on Type-I censoring for Gompertz distribution was discussed by Ismail (2010). They applied a Bayesian approach for estimation problem in case of step stress partially accelerated life tests for two stress levels.

Under progressive first-failure censoring plan Soliman et al. (2012) studied Bayes and frequentist estimators for two-parameter Gompertz distribution. Recently, Prakash (2015 a) presents a comparative study under one-sample Bayes prediction bound length based on three different censoring plans for Gompertz model.

Bayesian analysts have pointed out that, if one has adequate information about unknown parameter, one should use informative prior; otherwise it is preferable to use non-informative prior. However, there is no clear-cut way from which one can conclude that one prior is better than the other. It is more frequently the case that, a prior is selected to restricts attention to a given natural family of priors, and one is chosen from that family, which seems to match best with one's personal beliefs.

In present case, the Gamma distribution is considered as a conjugate family of prior for unknown parameter θ, having the probability density function

$$\pi(\theta) = \frac{\alpha^{\nu}}{\Gamma\nu}\theta^{\nu-1}e^{-\alpha\theta}\;;\alpha>0,\;\nu>0,\;\theta\geq 0\,. \tag{1.4}$$

The selection of loss function may be crucial in Bayesian analysis also. If most commonly used loss function is taken as a measure of inaccuracy, then the resulting risk is often too sensitive to assumptions about behavior of tail of probability distribution. Also the use of symmetric loss function in Bayesian estimation may not be appropriate in case when positive and negative errors have different consequences and/or when overestimation is more serious than underestimation, or vice-versa. In present article, a useful and flexible class of asymmetric loss function known as invariant LINEX loss function (ILLF) have used. Following Singh et al (2007) ILLF is defined as

$$L\left(\partial^{*}\right)=e^{a\partial^{*}}-a\partial^{*}-1\;;\;a\neq 0,\;\partial^{*}=\left(\hat{\theta}-\theta\right)\theta^{-1}\,. \tag{1.5}$$

Here, $\hat{\theta}$ be any estimate corresponding to unknown parameter θ and $'a'$ is said to be shape parameter of ILLF. (See Prakash & Singh (2009) for detail).

This article presents a comparative study based on Progressive Type-II and conventional Type-II censoring plans under Bayes estimation for unknown reliability parameters. Empirical Bayes estimation has been addressed for Gompertz model under ILLF criterion. A simulation technique has applied for studying the performances of the procedures.

2.Empirical Bayes Estimation under Progressive Type-II Censoring

In many life testing experiments, the experimenter may not be observed the lifetimes of all inspected units in life test. This may be because of time limitation and/or cost or material resources on data collection. Also, the trimmed samples are widely utilized when some sample values at either or both extremes adulterated.

The progressive censoring appears to be a great importance in planned duration experiments in reliability studies. In many industrial experiments involving lifetimes of machines or units, experiments have to be terminated early and the number of failures must be limited for various reasons. Recently, Prakash (2015 b) studied the properties of Bayes estimator for unknown parameter of Rayleigh distribution under Progressively Type-II censored data.

Let us suppose an experiment in which n independent and identical units $x_1, x_2, ..., x_n$ are placed on a life test at beginning time and first $r\,;\,(1\leq r\leq n)$ failure times are observed. At the time of each failure occurring prior to the termination point, one or more surviving units are removed from the test. The experiment is terminated at time of r^{th} failure, and all remaining surviving units are removed from the test.

Let $x_{(1)}\leq x_{(2)}\leq ...\leq x_{(r)}$ are the lifetimes of completely observed units to fail and $R_1, R_2, ..., R_r\,;(r\leq n)$ are the numbers of units withdrawn at these failure times. Here, $R_1, R_2, ..., R_r\,;(r\leq n)$ all are predefined integers follows the relation

$$\sum_{j=1}^{r} R_j + r = n .$$

At first failure time $x_{(1)}$, withdraw R_1 items randomly from remaining $n-1$ surviving units. Immediately after the second observed failure time $x_{(2)}, R_2$ items are withdrawn from remaining $n-2-R_1$ surviving units at random, and so on. Experiments continue until at r^{th} failure time $x_{(r)}$, remaining items $R_r = n - r - \sum_{j=1}^{r-1} R_j$ are withdrawn. Here, $x_{1:r:n}^{(R_1,R_2,...,R_r)}, x_{2:r:n}^{(R_1,R_2,...,R_r)},..., x_{r:r:n}^{(R_1,R_2,...,R_r)}$ and $(R_1, R_2,..., R_r)$ be the r ordered failure times and progressive censoring scheme respectively.

Based on Progressively Type-II censoring scheme the joint probability density function of order statistics $x_{1:r:n}^{(R_1,R_2,...,R_r)}, x_{2:r:n}^{(R_1,R_2,...,R_r)},..., x_{r:r:n}^{(R_1,R_2,...,R_r)}$ is defined as

$$f_{\left(x_{(1:r:n)},x_{(2:r:n)},...,x_{(r:r:n)}\right)}\left(\underline{x} \mid \theta\right) = \xi_p \prod_{i=1}^{r} f\left(x_{(i)};\theta\right) \left(1 - F\left(x_{(i)};\theta\right)\right)^{R_i}; \tag{2.1}$$

Here ξ_p be the progressive normalizing constant and defined as $\xi_P = n\,(n - R_1 - 1)\,(n - R_1 - R_2 - 2)\,...$ $\left(n + 1 - \sum_{j=1}^{r-1} R_j - r\right)$. The Progressively Type-II censored sample is denoted by $\underline{x} \equiv \left(x_{(1)}, x_{(2)},..., x_{(r)}\right)$ and, $\left(R_1, R_2,..., R_r\right)$ being progressive censoring scheme for considered model. Simplifying (2.1) we have

$$f_{\left(x_{(1:r:n)},x_{(2:r:n)},...,x_{(r:r:n)}\right)}\left(\underline{x} \mid \theta\right) = \xi_p \prod_{i=1}^{r} \left(\theta\, e^{x_{(i)}} e^{-\theta e^{x_{(i)}} + \theta}\right)\left(1 - \left(1 - e^{-\theta e^{x_{(i)}} + \theta}\right)\right)^{R_i}$$

$$\Rightarrow f_{\left(x_{(1:r:n)},x_{(2:r:n)},...,x_{(r:r:n)}\right)}\left(\underline{x} \mid \theta\right) = \xi_P A_P^*\left(\underline{x}\right)\theta^r \exp\left(-\theta\, T_P^*\left(\underline{x}\right)\right); \tag{2.2}$$

where $A_p^*(\underline{x}) = \exp\left(\sum_{i=1}^{r} x_{(i)}\right)$, $T_p^*(\underline{x}) = \sum_{i=1}^{r} \left(1 + R_i\right)\left(e^{x_{(i)}} - 1\right)$ and the progressive Type-II censoring indicates by suffix P.

Based on Bayes theorem, the posterior density is now obtained as

$$\pi_P^*\left(\theta \mid \underline{x}\right) = \frac{\xi_P A_P^*\left(\underline{x}\right)\theta^r \exp\left(-\theta\, T_P^*\left(\underline{x}\right)\right)\cdot \frac{\alpha^{\nu}}{\Gamma\nu}\theta^{\nu-1} e^{-\alpha\theta}}{\int_{\theta} \xi_P A_P^*\left(\underline{x}\right)\theta^r \exp\left(-\theta\, T_P^*\left(\underline{x}\right)\right)\cdot \frac{\alpha^{\nu}}{\Gamma\nu}\theta^{\nu-1} e^{-\alpha\theta}\, d\theta}$$

$$\Rightarrow \pi_P^*\left(\theta \mid \underline{x}\right) = \frac{\theta^{r+\nu-1} \exp\left(-\theta\left(T_P^*(\underline{x}) + \alpha\right)\right)}{\Gamma\left(r+\nu\right)\left(T_P^*(\underline{x}) + \alpha\right)^{-r-\nu}} . \tag{2.3}$$

The unknown parameter θ is re-written as, in terms of reliability function by using equation (1.2):

$$\theta = \frac{\log \Psi}{1-e^{t}}; \Psi \cong \Psi(t). \tag{2.4}$$

Hence, the posterior density corresponding to reliability function is obtained by using equation (2.4) in equation (2.3):

$$\pi_P^*(\Psi \mid \underline{x}) = \frac{\left(T_P^*(\underline{x})+\alpha\right)^{r+\nu}}{\Gamma(r+\nu)} \left(\frac{\log \Psi}{1-e^{t}}\right)^{r+\nu-1} \exp\left(-\frac{\log \Psi}{1-e^{t}}\left(T_P^*(\underline{x})+\alpha\right)\right). \tag{2.5}$$

Similarly, the equation (1.3) is re-written in terms of failure rate as

$$\theta = \rho\, e^{-t}; \rho \cong \rho\,(t).$$

Using above relation in equation (2.3), the posterior density corresponding to failure rate is obtained as

$$\pi_P^*(\rho \mid \underline{x}) = \frac{\left(T_P^*(\underline{x})+\alpha\right)^{r+\nu}}{\Gamma(r+\nu)} \left(\frac{\rho}{e^{t}}\right)^{r+\nu-1} \exp\left(-\frac{\rho}{e^{t}}\left(T_P^*(\underline{x})+\alpha\right)\right). \tag{2.6}$$

The maximum likelihood (ML) estimate and method of moments are two best methods for estimating the hyper-parameter. In present case, method of ML estimate has used for estimating the unknown parameters of prior distribution. Since, Gamma distribution given in (1.4) is considered here as a conjugate family of prior for unknown parameter θ. Let us assume that the hyper parameter ν is known and parameter α be the unknown. Under, the empirical Bayesian approach, for estimating the unknown hyper prior parameter α, we begin with the Bayes model:

Since,

$$x_{(i)} \mid \theta \sim f\,(x;\theta),\ i = 1, 2, \ldots, r$$

and

$$\theta \mid \alpha \sim \pi(\theta).$$

As all failure units have identical Gompertz distribution, the marginal density of x, say $g\,(x)$, can be obtained as

$$g\,(x) = \int_{\theta} f_{\left(x_{(1:r:n)}, x_{(2:r:n)}, \ldots, x_{(r:r:n)}\right)}(\underline{x} \mid \theta) \cdot \pi(\theta)\, d\theta$$

$$\Rightarrow g\,(x) = \xi_P A_P^*(\underline{x}) \frac{\alpha^{\nu}}{\Gamma\nu} \int_{\theta} \theta^{r+\nu-1} \exp\left(-\theta\left(T_P^*(\underline{x})+\alpha\right)\right) d\theta$$

$$\Rightarrow g(x) = \xi_P A_P^*(\underline{x}) \frac{\alpha^{\nu}\,\Gamma(r+\nu)}{\Gamma(\nu)\left(T_P^*(\underline{x})+\alpha\right)^{r+\nu}}.$$

The maximum likelihood (ML) estimate of $\alpha \cong \hat{\alpha}_{ML}$ (say) based on $g(x)$ is obtain as

$$\hat{\alpha}_{ML} = \frac{\nu}{r} T_P^*(\underline{x}). \tag{2.7}$$

The empirical posterior distribution for unknown parameter θ is obtained by replacing parameter α with its ML estimate $\hat{\alpha}_{ML}$. Thus, using equation (2.7) in equation (2.3), we have

$$\pi^*_{EP}(\theta\,|\,\underline{x}) = \frac{\theta^{\,r+\nu-1}\exp\left(-\theta\,\frac{(r+\nu)}{r}T^*_P(\underline{x})\right)}{\Gamma(r+\nu)\left(\frac{r+\nu}{r}T^*_P(\underline{x})\right)^{-r-\nu}}\,. \tag{2.8}$$

The Bayes estimator $\hat{\theta}_{EP}$ (say) corresponding to parameter θ under ILLF is obtained by

$$E_\theta\left(\theta\, e^{a\,\hat{\theta}_{EP}\,\theta}\right) = e^a E_\theta(\theta)$$

$$\hat{\theta}_{EP} = \left(\frac{\nu+r}{r\,a}\right)\left\{1-\exp\left(-\frac{a}{\nu+r+1}\right)\right\}T^*_P(\underline{x})\,. \tag{2.9}$$

On similar lines the empirical posterior distribution corresponding to reliability function and their Bayes estimator under ILLF are given as

$$\pi^*_{EP}(\Psi\,|\,\underline{x}) = \frac{\left(T^*_P(\underline{x})\,\frac{r+\nu}{r}\right)^{r+\nu}}{\Gamma(r+\nu)}\left(\frac{\log\Psi}{1-e^t}\right)^{r+\nu-1}\exp\left\{-\left(\frac{\log\Psi}{1-e^t}\right)\left(\frac{r+\nu}{r}\right)T^*_P(\underline{x})\right\}$$

and

$$E_\Psi\left(\Psi\, e^{a\,\hat{\Psi}_{EP}\Psi}\right) = e^a E_\Psi(\Psi)$$

$$\Rightarrow e^a\,\frac{\Gamma(r+\nu)}{(\alpha^*-2)^{r+\nu}} = \int_0^\infty e^{-(\alpha^*-2)Z} z^{\nu+r-1} e^{a\,\hat{\Psi}_{EP}\,e^z}\,dz\ ;\ \alpha^* = \frac{T^*_P(\underline{x})}{1-e^t}\left(\frac{r+\nu}{r}\right). \tag{2.10}$$

A nice close form of Bayes estimator $\hat{\Psi}_{EP}$ does not exist. A numerical technique has applied here under simulation for study the properties.

Now, the empirical posterior distribution corresponding to failure rate and corresponding Bayes estimator under LLF are given as

$$\pi^*_{EP}(\rho\,|\,\underline{x}) = \frac{\left(\left(\frac{r+\nu}{r}\right)T^*_P(\underline{x})\right)^{r+\nu}}{\Gamma(r+\nu)}\left(\frac{\rho}{e^t}\right)^{r+\nu-1}\exp\left\{-\frac{\rho}{e^t}\left(\frac{r+\nu}{r}\right)T^*_P(\underline{x})\right\}$$

and

$$\hat{\rho}_{EP} = \frac{r+\nu}{a\,r\,e^t}\left(1-e^{-a/(\nu+r+1)}\right)T^*_P(\underline{x})\,. \tag{2.11}$$

3.Empirical Bayes Estimation under Conventional Type-II Censoring

In life testing, fatigue failures and other kinds of destructive test situations, the observations usually occurred in ordered manner such a way that weakest items failed first and then second one and so on. Let us suppose that n items are put to test under model (1.1) without replacement and test terminates as soon as first $r^{th} (r \le n)$ item fails. This censoring scheme is known as Type-II or better known as Item–Failure censoring scheme.

The progressively Type-II right censoring scheme reduces to conventional Type-II censoring scheme when

$$R_i = 0 \; \forall \, i = 1,2,...,r-1 \Rightarrow R_r = n - r. \tag{3.1}$$

Hence, the joint probability density function of order statistics $\underline{x} \equiv \left(x_{(1)}, x_{(2)},..., x_{(r)}\right)$ is defined as

$$f_{\left(x_{(1)},x_{(2)},...,x_{(r)}\right)}\left(\underline{x} \mid \theta\right) = \prod_{i=1}^{r} f\left(x_{(i)};\theta\right)\left(1 - F\left(x_{(r)};\theta\right)\right)^{(n-r)} \tag{3.2}$$

$$\Rightarrow f_{\left(x_{(1)},x_{(2)},...,x_{(r)}\right)}\left(\underline{x} \mid \theta\right) = A_P^*\left(\underline{x}\right)\theta^r \exp\left(-\theta\, T_I^*\left(\underline{x}\right)\right); \tag{3.3}$$

where $T_I^*(\underline{x}) = (n-r)\left(e^{x_{(r)}} - 1\right) + \sum_{i=1}^{r}\left(e^{x_{(i)}} - 1\right)$.

Hence, the marginal density of x, say $g'(x)$, can be obtained as

$$g'(x) = \int_\theta f_{\left(x_{(1)},x_{(2)},...,x_{(r)}\right)}\left(\underline{x} \mid \theta\right) \cdot \pi\left(\theta\right) d\theta = A_P^*\left(\underline{x}\right) \frac{\alpha^\nu \, \Gamma(r+\nu)}{\Gamma(\nu)\left(T_I^*\left(\underline{x}\right) + \alpha\right)^{r+\nu}}.$$

The maximum likelihood (ML) estimate of $\alpha \cong \hat{\alpha}'_{ML}$ (say) based on $g'(x)$ is

$$\hat{\alpha}'_{ML} = \frac{\nu}{r} T_I^*\left(\underline{x}\right). \tag{3.4}$$

Thus, the empirical posterior densities corresponding to unknown parameter θ, reliability function and failure rate are obtain respectively as

$$\pi_{EI}^*\left(\theta \mid \underline{x}\right) = \frac{\theta^{\,r+\nu-1} \exp\left(-\theta \frac{(r+\nu)}{r} T_I^*(\underline{x})\right)}{\Gamma\left(r+\nu\right)\left(\frac{r+\nu}{r} T_I^*(\underline{x})\right)^{-r-\nu}}, \tag{3.5}$$

$$\pi_{EI}^*\left(\Psi \mid \underline{x}\right) = \frac{\left(T_I^*(\underline{x}) \frac{r+\nu}{r}\right)^{r+\nu}}{\Gamma\left(r+\nu\right)} \left(\frac{\log \Psi}{1-e^t}\right)^{r+\nu-1} \exp\left\{-\left(\frac{\log \Psi}{1-e^t}\right)\left(\frac{r+\nu}{r}\right) T_I^*(\underline{x})\right\} \tag{3.6}$$

and

$$\pi_{EI}^*\left(\rho \mid \underline{x}\right) = \frac{\left(\left(\frac{r+\nu}{r}\right) T_I^*(\underline{x})\right)^{r+\nu}}{\Gamma\left(r+\nu\right)} \left(\frac{\rho}{e^t}\right)^{r+\nu-1} \exp\left\{-\frac{\rho}{e^t}\left(\frac{r+\nu}{r}\right) T_I^*(\underline{x})\right\}. \tag{3.7}$$

On similar line, empirical Bayes estimators corresponding to unknown parameter θ, reliability function and failure rate are obtain respectively as

$$\hat{\theta}_{EI} = \left(\frac{\nu + r}{r\,a}\right)\left\{1 - \exp\left(-\frac{a}{\nu + r + 1}\right)\right\} T_I^*(\underline{x}), \tag{3.8}$$

$$e^a \frac{\Gamma(r+\nu)}{(\alpha^{**}-2)^{r+\nu}} = \int_0^\infty e^{-(\alpha^{**}-2)Z} z^{\nu+r-1} e^{a\,\hat{\Psi}_{EI}\,e^z} dz \;;\; \alpha^{**} = \frac{T_I^*(\underline{x})}{1-e^t}\left(\frac{r+\nu}{r}\right) \tag{3.9}$$

and

$$\hat{\rho}_{EI} = \frac{r+\nu}{a\,r\,e^t}\left(1 - e^{-a/(\nu+r+1)}\right) T_I^*(\underline{x}). \tag{3.10}$$

Again close form for Bayes estimator $\hat{\Psi}_{EI}$ corresponding to the reliability function does not exist. A numerical technique has applied here under simulation for study the properties.

4.Simulation Study

In present section a complete numerical analysis based on simulated data has presented for Empirical Bayes estimation under Progressive Type-II and conventional Type-II censoring criterion.

The random samples are generated as follows:

The selected set of values for hyper parameters of prior density are $(\alpha, \nu) = (0.71, 0.50), (1.41, 2), (2, 4)$ $(3, 9)$. The selection of these values meets the criterion that prior variance should be unity. Generate the values of shape parameter θ through prior density $\pi(\theta)$ given in (1.4) for a given set of prior parameters $\nu\,(= 0.50, 2, 4$ $, 9)$ and, the value of prior parameter α is estimated by its ML estimate $\hat{\alpha}_{ML}$.

Using generated values of θ, we generate a progressively Type-II censored sample, of size r for a given values of censoring scheme $R_i; i = 1,2,...,r$. The censoring scheme for different values of r is presented in Table 1.

Here, 10,000 random samples of size $N = 20$ from model (1.1) have generated by using following relation with help of above selected values

$$x_i = \log\left\{1 - \frac{\log(1-\delta_i)}{\theta}\right\}.$$

Here, $\delta_i \;\forall\, i$ are independently distributed uniform distribution with parameter $U(0,1)$.

Table (1): Different Progressive Censoring Scheme

Case	r	$R_i\,; i = 1,2,...,r$
1	5	1 2 1 0 1
2	10	1 0 0 3 0 0 1 0 0 1
3	15	1 0 2 0 0 1 0 2 0 0 0 1 0 0 1

The estimated risk for Bayes estimators $\hat{\theta}_{EP}$, $\hat{\Psi}_{EP}$ and $\hat{\rho}_{EP}$ corresponding to parameter θ, reliability function and failure rate respectively are obtained by using following relation

$$ER\left(\hat{\lambda}\right)=\frac{1}{10000}\sum_{i=1}^{10000}\left(\hat{\lambda}_i-\lambda\right)^2$$

Table (2): Estimated Risk of $\hat{\theta}_{EP}$ Under Progressive Censoring

$n=20$		$(\alpha,\nu)\downarrow$			
$a\downarrow$	$r\downarrow$	0.71, 0.5	1.41, 2.00	2.00, 4.00	3.00, 9.00
0.25	5	1.282	1.286	1.289	1.292
	10	1.275	1.279	1.282	1.284
	15	1.222	1.225	1.227	1.234
0.50	5	1.312	1.316	1.325	1.328
	10	1.306	1.314	1.321	1.324
	15	1.278	1.282	1.285	1.287
1.00	5	1.341	1.344	1.347	1.359
	10	1.311	1.324	1.333	1.361
	15	1.366	1.372	1.373	1.376

Table (3): Estimated Risk of $\hat{\Psi}_{EP}$ Under Progressive Censoring

$n=20, t=1.00$		$(\alpha,\nu)\downarrow$			
$a\downarrow$	$r\downarrow$	0.71, 0.5	1.41, 2.00	2.00, 4.00	3.00, 9.00
0.25	5	1.299	1.302	1.305	1.308
	10	1.286	1.293	1.295	1.299
	15	1.257	1.261	1.263	1.266
0.50	5	1.385	1.389	1.392	1.395
	10	1.372	1.376	1.379	1.382
	15	1.341	1.345	1.347	1.351
1.00	5	1.541	1.546	1.549	1.552
	10	1.506	1.511	1.514	1.517
	15	1.316	1.322	1.326	1.328

and presented in Tables 2-4 under progressive Type-II censoring respectively. The other considered parametric values are $t=1.00$ and the values of shape parameter of LLF are $a=0.25, 0.50, 1.00$.

It is observed from above tables that, the estimated risks increases when prior parameter ν increases. Similar trend also has seen when shape parameter of LLF increases. Except for Bayes estimator $\hat{\Psi}_{EP}$ when $r = 15$. An increasing trend in estimated risk have also seen when progressive sample size r increases, except when $a = 1.00$ for Bayes estimators $\hat{\theta}_{EP}$ and $\hat{\rho}_{EP}$. However, the magnitude of gain or loss in the estimated risks are nominal.

Table (4): Estimated Risk of $\hat{\rho}_{EP}$ Under Progressive Censoring

$n = 20, t = 1.00$		$(\alpha, \nu)\downarrow$			
$a\downarrow$	$r\downarrow$	0.71, 0.5	1.41, 2.00	2.00, 4.00	3.00, 9.00
0.25	5	1.359	1.363	1.366	1.369
	10	1.243	1.247	1.252	1.258
	15	1.069	1.072	1.074	1.076
0.50	5	1.364	1.368	1.371	1.374
	10	1.358	1.363	1.365	1.368
	15	1.183	1.187	1.189	1.192
1.00	5	1.537	1.542	1.545	1.548
	10	1.397	1.401	1.414	1.441
	15	1.441	1.445	1.448	1.451

Based on similar set of considered parametric values, the estimated risks of the Bayes estimators under conventional Type-II censored data are obtained and presented in Tables 5-7 respectively for estimators $\hat{\theta}_{EI}$, $\hat{\Psi}_{EI}$ and $\hat{\rho}_{EI}$.

Table (5): Estimated Risk of $\hat{\theta}_{EI}$ Under Type-II Censoring

$n = 20$		$(\alpha, \nu)\downarrow$			
$a\downarrow$	$r\downarrow$	0.71, 0.5	1.41, 2.00	2.00, 4.00	3.00, 9.00
0.25	5	1.333	1.335	1.336	1.339
	10	1.283	1.287	1.294	1.297
	15	1.249	1.252	1.255	1.258
0.50	5	1.401	1.406	1.409	1.411
	10	1.353	1.357	1.366	1.376
	15	1.288	1.292	1.294	1.297
1.00	5	1.403	1.408	1.417	1.423
	10	1.364	1.368	1.371	1.377
	15	1.424	1.469	1.492	1.501

All properties have seen similar as stated above for progressive Type-II censoring criterion. One remarkable point is observed, the magnitude of estimated risk of Bayes estimators under progressive censoring is least as compared to

Type-II censoring when other parametric values are considered to be fixed. Hence, one may prefer empirical Bayes estimator under progressive Type-II censoring over Type-II censoring.

Table (6): Estimated Risk of $\hat{\Psi}_{EI}$ Under Type-II Censoring

$n = 20, t = 1.00$		$(\alpha, \nu)\downarrow$			
$a\downarrow$	$r\downarrow$	0.71, 0.5	1.41, 2.00	2.00, 4.00	3.00, 9.00
0.25	5	1.424	1.428	1.431	1.434
	10	1.418	1.422	1.425	1.431
	15	1.413	1.417	1.421	1.428
0.50	5	1.518	1.522	1.525	1.528
	10	1.468	1.472	1.478	1.481
	15	1.457	1.459	1.464	1.478
1.00	5	1.581	1.585	1.588	1.591
	10	1.552	1.556	1.562	1.563
	15	1.424	1.428	1.431	1.434

Table (7): Estimated Risk of $\hat{\rho}_{EI}$ Under Type-II Censoring

$n = 20, t = 1.00$		$(\alpha, \nu)\downarrow$			
$a\downarrow$	$r\downarrow$	0.71, 0.5	1.41, 2.00	2.00, 4.00	3.00, 9.00
0.25	5	1.369	1.373	1.376	1.378
	10	1.348	1.352	1.354	1.357
	15	1.214	1.218	1.223	1.228
0.50	5	1.449	1.453	1.456	1.459
	10	1.385	1.389	1.392	1.394
	15	1.358	1.362	1.365	1.368
1.00	5	1.568	1.573	1.576	1.579
	10	1.504	1.508	1.511	1.514
	15	1.531	1.535	1.538	1.541

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