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RESEARCH ARTICLE

STABILIZATION OF UNCERTAIN TIME-VARYING BILINEAR DESCRIPTOR DYNAMICAL CONTROL SYSTEM

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Abstract

The descriptor system application plays an important role in modern science and mathematics. This paper focuses on studying the stabilization of time varying bilinear descriptor system Some theoretical results supported have been adopted .

1. INTRODUCTION

Descriptor systems are those the dynamics of which are governed by a mixture of algebraic and differential equations .During the past decades descriptor systems have attracted much attention due the comprehensive applications in chemical engineering, control theory [3], electrical [1] and mechanical models[5] atc.

Bilinear systems are important sub classes of nonlinear systems with many applications in engineering, biology and economics. The control of bilinear systems has been extensively studied in the 70s and at the beginning of the 80s of the last century and remarkable by [4].

2. PROBLEM FORMULATION

Consider the following uncertain time-varying bilinear descriptor system which is described by:

$$\dot{Z}_1(t) = (A + \delta A)(t)Z_1(t) + (B + \delta B)u(t)Z_1(t) \quad ...(1)$$

$$0 = (C + \delta C)Z_1(t) + (D + \delta D)Z_2(t) \quad ... (2)$$

Where $Z_1 \in \mathbb{R}^r, Z_2 \in \mathbb{R}^{n-r}, A(t) \in \mathbb{R}^{r\times r}, B \in \mathbb{R}^{r\times 1}, C \in \mathbb{R}^{(n-r)\times r}, D \in \mathbb{R}^{(n-r)\times(n-r)}$.

And $\delta B, \delta C, \delta D$ are constant perturbations matrices with appropriate dimensions and which are bounded i.e.$\|\delta B\| \le b$, $\|\delta C\| \le C$, $\|\delta D\| \le d$. and $\|\delta A(t)\| \le a$, for some constant a , b , c , d ,

3. BASIC CONCEPT

Some basic definitions, lemmas and theorems as well as necessary requirements for the solvability of time varying uncertain linear descriptor system have been presented and as follows:

3.1 **Definition** [6]

Consider the time – varying linear descriptor system

$$E(t)X'(t) = A(t)X(t) \quad \ldots(3)$$

Where $(E(t), A(t)) \in C(I, \mathbb{R}^{n\times n})^2$, $n \in \mathbb{N}$ then a function $X: \mathcal{J} \to \mathbb{R}^n$ is called solution of (3) if and only if X is a continuously differentiable function on open interval $\mathcal{J} \leq \mathbb{R} \leq I$ and solve (3) for $t \in \mathcal{J}$;it called global solution if $\mathcal{J} = I \leq \mathbb{R}$ for $t \in \mathcal{J}$.

3.2 Remark [6]

If $(S(t), T(t)) \in C(I; GL_n(\mathbb{R}) \times C^1(I; GL_n(\mathbb{R})), then\ X: \mathcal{J} \to \mathbb{R}^n$ solves (3) if and only if $w(t) = T^{-1}(t)x(t)$ solves

$$S(t)E(t)\dot{w} = \left[S(t)A(t)T(t) - S(t)E(t)\dot{T}(t)\right]w$$

$$\tilde{E}(t)\dot{w} = \tilde{A}(t)w \quad \ldots(4)$$

Where

$$\tilde{E}(t) = S(t)E(t)T(t), \tilde{A}(t) = S(t)A(t)T(t) - S(t)E(t)\dot{T}(t)$$

This new system is equivalent to the system (3) where

$$\frac{d}{dt}T^{-1}(t) = -T^{-1}(t)\dot{T}(t)T^{-1}(t)$$

3.3 Definition [6]

The descriptor system$(E(t), A(t)) \in C(I, \mathbb{R}^{n\times n})^2$is called transferable into standard Canonical form (SCF) if and only if there exist

$$(\mathrm{S}(t), \mathrm{T}(t)) \in C\big(I; GL_n(\mathbb{R})\big) \times C^1\big(I; GL_n(\mathbb{R})\big)$$

such that

$$E(t) = diag\left(I_{n_1}, N(t)\right), A(t) = diag\big(J(t), I_{n_2}\big) \quad \ldots(5)$$

Where $N(t): I \to \mathbb{R}^{n_2\times n_2}$ is a pointwise strictly lower triangular and $J(t): I \to \mathbb{R}^{n_1\times n_1}$.

3.4 Proposition [2]

The system (2) is transferable into standard canonical form if and only if $(E(t), A(t))$ is regular.

3.5 Definition [7]

The set of all pairs of consistent initial values of (3) is denoted by

$$V_{E,A} = \{(t_0, x_0) \in \mathbb{R} \times \mathbb{R}^n \,|\, \exists\ (local) solution\ x(t) of\ (3): t_0 \in dom\ x(t), x(t_0) = x_0\}$$

and the linear subspace of initial values which are consistent at time $t_0 \in I$ is denoted by

$V_{E,A}(t_0) = \{x_0 \in \mathbb{R}^n | (t_0, x_0) \in V_{E,A}\}$ and since $X: \mathcal{J} \to \mathbb{R}^n$ is a solution of (3), then $x(t) \in V_{E,A}(t)$ for all $t \in \mathcal{J}$.

3.6 Proposition [7]

suppose that the Descriptor system (3) is transferable into standard Canonical form as in (5), then

1)$(t_0, x_0) \in V_{E,A} \leftrightarrow x_0 \in im\, T(t_0)\begin{bmatrix} I_{n1} \\ O \end{bmatrix}$

2) Any solution of the initial values problem (3) , $X(t_0) = x_0$, $where(t_0, x_0) \in V_{E,A}$, extends uniquely to a global solution $X(t)$, and this solution satisfies

$x(t) = U(t, t_0)x_0$, $U(t, t_0) = T(t)\begin{bmatrix} \phi_J(t, t_0) & 0 \\ 0 & 0 \end{bmatrix} T^{-1}$, where $t \in I, \phi_J(t, t_0)$denotes the transition matrix of $\dot{w} = J(t)w$

3.7 Definition [7]

Suppose (3) is tranferable into (SCF) as in (4) for some $\big(S(t), T(t)\big) \in C\big(I; GL_n(\mathbb{R})\big) \times C^1\big(I; GL_n(\mathbb{R})\big)$, then the generalized transition matrix $U(.,.)$ of system (1) is defined by.

$$U(t, s) = T(t)\begin{bmatrix} \emptyset_J(t, s) & 0 \\ 0 & 0 \end{bmatrix} T^{-1}(S), t, s \in I.$$

3.8 Proposition [7]

Let the system (3) be transferable into standard canonical form with generalized transition matrix $U(.,.)$ then we have, $for\ all\ t, r, s \in I$

1) $E(t)\frac{d}{dt}U(t,s) = A(t)U(t,s)$

2) $im\ U(t,s) = V_{E,A}(t)$

3) $U(t,r)U(r,s) = U(t,s)$

4) $U(t,t)^2 = U(t,t)$

5) $\forall x \in V_{E,A}(t): U(t,t)x = x$

6) $\frac{d}{dt}U(s,t) = -U(s,t)T(t)S(t)A(t)$

3.9 Definition [6]

A global solution $X: (a,\infty) \to \mathbb{R}^n$ of the time varying system

$$E(t)X'(t) = A(t)X(t) + f(t)$$

Where $(E(t), A(t), f(t)) \in C\big((\mathcal{J},\infty), \mathbb{R}^{n\times n}\big)^2 \times C\big((\mathcal{J},\infty); \mathbb{R}^n\big), n \in \mathbb{N}, \mathcal{J} \in [-\infty,\infty), a \geq \mathcal{J}$ is said to be Exponentially stable if and only if $\exists \alpha, \beta > 0\ \forall\ t_0 > a\ \exists \eta > 0\ \forall y_0 \in \mathcal{B}_\eta\big(x(t_0)\big) \forall\ y(t)$ solution $||y(t) - x(t)|| \leq \alpha e^{-\beta(t-t_0)}\ ||y(t_0) - x(t_0)||, \forall t \geq t_0$.

3.10 Theorem [7]

Suppose the time varying system $(E(t), A(t)) \in C((\mathcal{J},\infty); \mathbb{R}^{n\times n})^2$ is transferable into standard canonical form and let $U(.,.)$ denotes the generalized transition matrix of $\big(E(t), A(t)\big)$, then the following characterizations hold:

1) $(E(t), A(t))$ *is stable if and only if* $\forall\ t_0 > \mathcal{J}\ \exists \mu \geq 0$

$$\forall x_0 \in V_{E,A}(t_0) \forall t \geq t_0: ||\ U(t,t_0)x_0\ || \leq \mu ||x_0||$$

2) $(E(t), A(t))$ is exponentially stable if and only if $\exists \alpha, \beta > 0$,

$$\forall (t_0, x_0) \in V_{E,A}, \forall t > t_0:\ \|U(t,t_0)x_0\| \leq \alpha \bar{e}^{\beta(t-t_0)}\|x_0\|.$$

Notice that $V_{E,A}$ is the set of all consistent initial condition.

3.11 Lemma (Generalization of Gronwell's lemma) [3]

Let $a, b, n, k \in \mathbb{R}$, $a < b$, $n > 1$ and $K > 0, f : [a,b] \to \mathbb{R}^+$ an integral function such that

$\forall\ a, \beta \in [a, b]\ (\propto < b)$: $\int_\propto^\beta f(s)\, ds > 0$ and $X: [a,b] \to \mathbb{R}^+$

If $X(t) \leq K + \int_0^t f(s)[X(s)]^n$ ds and $1 - (n-1)K^{n-1}\int_\propto^t f(s)\, ds > 0$

Then $X(t) \leq \frac{K}{\left[1-(n-1)K^{n-1}\int_a^t f(s)ds\right]^{\frac{1}{n-1}}}$

4. STABILITY OF UNCERTAIN TIME VARYING DESCRIPTOR SYSTEM

The main theoretical requirements (theorem) of the solvability and then finding the solution / approximate solution are the main theme of this section.

4.1 Theorem

For the descriptor system with parametric uncertainty (1), (2) if :

1- Let $\delta A(t)$ be a perturbation matrix such that $(A + \delta A)(t)$ is stable.

2- The matrix D be nonsingular such that $\|D^{-1}\delta D\| < 1$.

3- There exists an integral function $f: \mathbb{R}^+ \to \mathbb{R}^+$ and positive integer number α , μ s.t. $\forall t \geq t_0$, $1 - 2\mu^{-2}(\|B\| + b)\int_{to}^t e^{2\alpha(t_0-s)}$

4- consider the control is given by $u = \frac{f(Z_1(t))}{\|Z_1(t_0)\|}$, $Z_1(t_0) \neq 0$, and $f(Z_1(t))$is vector of nonlinear functions with$\|f(Z_1(t))\| \leq f(t)\|(Z_1(t))\|^2$. Is positive and bounded

and $u(t) = \frac{f(T^{-1}(t)X_1(t))}{\|T^{-1}(t_0)\,X_1(t_0)\|^2} = \frac{f(T^{-1}(t)X_1(t))}{\|T(t_0)\|^{-2}\|X_1(t_0)\|^2}$, $\|f(T^{-1}X_1)\| \leq f(t)\|T\|^{-2}\|X_1\|^2$, …(6)

where $Z_1 = T^{-1}X_1$ where T is nonsingular matrix
Then the nonlinear system (1), (2) is asymptotically stable.

Proof:

Let $Z(t_0) = [Z_1(t_0), Z_2(t_0)]$ initial condition in (1), (2) and
$Z(t_0) \in N([C + \delta C \quad D + \delta D])$.
From (1)

$$Z_1' = (A + \delta A)(t)Z_1(t) + (B + \delta B)u(t)Z_1(t)$$
$$Z_1' - (A + \delta A)(t)Z_1(t) = (B + \delta B)u(t)Z_1(t)$$
$$Z_1(t) = Z_1(t_0)\emptyset_{A+\delta A}(t, t_0) + \int_{t_0}^{t}(B + \delta B)u(s)Z_1(s)\,\emptyset_{A+\delta A}(t, s)ds$$

Let $Z_1 = T^{-1}X_1$, T nonsingular matrix.

$$T^{-1}(t)X_1(t) = T^{-1}(t_0)X_1(t_0)\emptyset_{A+\delta A}(t, t_0) + \int_{t_0}^{t}(B + \delta B)u(s)T^{-1}(s)\,X_1(s)\,\emptyset_{A+\delta A}(t, s)ds$$
$$X_1(t) = T(t)\emptyset_{A+\delta A}(t, t_0)T^{-1}(t_0)X_1(t_0) + T(t)\int_{t_0}^{t}(B + \delta B)u(s)T^{-1}(s)\,\emptyset_{A+\delta A}(t, s)X_1(s)ds$$

Since $U(t, s) = T(t)\emptyset(t, s)T^{-1}(s)$
Then $X_1(t) = U(t, t_0)X_1(t_0) + T(t)\int_{t_0}^{t}(B + \delta B)u(s)T^{-1}(s)\,U(t, s)X_1(s)ds$

$$= U(t, t_0)X_1(t_0) + \int_{t_0}^{t}(B + \delta B)u(s)U(t, s)X_1(s)ds$$

But $u(t) = \frac{f(T^{-1}(t)X_1(t))}{\|T^{-1}(t_0)\,.X_1(t_0)\|^2} = \frac{f(T^{-1}(t)X_1(t))}{\|T(t_0)\|^{-2}\|X_1(t_0)\|^2}$

Implies

$$\|X_1(t)\| \leq \|U(t, t_0)\|\|X_1(t_0)\|\left[1 + \int_{t_0}^{t}\|U(t_0, s)\|\,\|B + \delta B\|\|f(T^{-1}X_1)\|\|T(t_0)\|^{-2}\|X_1(t_0)\|^{-3}\|X_1(s)\|ds\right]$$

Divide the inequality by $\|U(t, t_0)\|\|X_1(t_0)\|$ and using (6)

$$\frac{\|X_1(t)\|}{\|U(t,t_0)\|\|X_1(t_0)\|} \leq 1 + \int_{t_0}^{t}\|U(t_0, s)\|\,\|B + \delta B\|\|f(s)\|\|T(s)\|^{-2}\|X_1(s)\|^2\|X_1(s)\|\,\|X_1(t_0)\|^{-3}$$

$$\frac{\|X_1(t)\|}{\|U(t,t_0)\|\|X_1(t_0)\|} \leq 1 + \int_{t_0}^{t}\|U_{A+\delta A}(t_0, s)\|^{-2}\|B + \delta B\|\|f(s)\|\|T(s)\|^{-2}.\left[\frac{\|X_1(s)\|}{\|U_{A+\delta A}(t_0,s)\|\|X_1(t_0)\|}\right]^3$$

Since

1- δA is chosen such that $\delta + \delta A$ stable matrix
2- Using theorem (3.10)

We get

$\|U_{A+\delta A}(t, t_0)\| \leq \mu e^{-\alpha(t-t_0)}$

Now using Gronwall lemma with $n = 3$ and $k = 1$ weget

$$\frac{\|X_1(t)\|}{\|U(t, t_0)\|\|X_1(t_0)\|} \leq \frac{1}{\sqrt{1 - 2\mu^{-2}(\|B\| + b)\int_{t_0}^{t} e^{2\alpha(t_0-s)}\|f(s)\|\|T(s)\|^{-2}}\,ds}$$

$$\|X_1(t)\| \leq \frac{\mu e^{-\alpha(t-t_0)}\|X_1(t_0)\|}{\sqrt{1 - 2\mu^{-2}(\|B\| + b)\int_{t_0}^{t} e^{2\alpha(t_0-s)}\|f(s)\|\|T(s)\|^{-2}}\,\,ds}$$

Hence $lim_{t\to\infty}\|X_1(t)\| \to 0$ and the nonlinear control $u(t)$ verify :

$$\|U(t)\| \leq f(t)\|T(t)\|^{-2}\frac{\mu^2 e^{-2\alpha(t-t_0)}}{\sqrt{1 - 2\mu^{-2}(\|B\| + b)\int_{t_0}^{t} e^{2\alpha(t_0-s)}\|f(s)\|\|T(s)\|^{-2}}\,\,ds}$$

Since D is nonsingular matrix $D + \delta D = D(I + D^{-1}\delta D)$
And $(I + D^{-1}\delta D)$ invertable
Then $Z_2 = -D^{-1}(I + D^{-1}\delta D)^{-1}(C + \delta C)Z_{1(t)}$

But $Z_2 = T^{-1}X_2$ and $Z_1 = T^{-1}X_1$ we get

$$T^{-1}(t)X_2(t) = -D^{-1}(I + D^{-1}\delta D)^{-1}(C + \delta C)T^{-1}(t)X_1(t)$$

$$X_2(t) = -D^{-1}(I + D^{-1}\delta D)^{-1}(C + \delta C)X_1(t)$$

$$\lim_{t\to\infty}\|X_2(t)\| \le \frac{\|D^{-1}\|}{I - \|D^{-1}SD\|}(\|C\| + C)\lim_{t\to\infty}\|X_1(t)\| \to 0.$$

$Z_2 = T^{-1}X_2$ and $Z_1 = T^{-1}X_1$ then solution $Z(t) = \begin{bmatrix} Z_1(t) \\ Z_2(t) \end{bmatrix}$ is stable.

5. CONCLUSION

In this paper survey was presented of time invariant stability concept and using this concept to stabilizing the time –varying uncertain bilinear descriptor systems.

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