ISSN 2320-5407 International Journal of Advanced Research (2015), Volume 3, Issue 12, 1578 – 1583

ISSN NO. 2320-5407

Journal homepage: http://www.journalijar.com

INTERNATIONAL JOURNAL OF ADVANCED RESEARCH

RESEARCH ARTICLE

A New Multiplication Algorithm for Higher-radix Signed Numbers with Multiple-valued Logic Operations

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Manuscript Info

Manuscript History:

Received: 19 October 2015
Final Accepted: 26 November 2015
Published Online: December 2015

Key words:

Multi-valued logic;
Higher-radix algebra;
Signed-number arithmetic;
Multiplication algorithms

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Abstract

A new higher-radix multiplication algorithm is presented based on multi-valued logic max, min, cyclic (modular sum), literal and threshold operations. The algorithm computes overall multiplication with considerably small number of intermediate operations by employing a transformation on operands by simultaneously relocating them into lower portion of the number set they belong to.

Introduction

Multiplication is a crucial operation which mainly affects precision, capability and speed of arithmetic systems in processing information in almost all major applications, e.g. neural network algorithms in pattern recognition, [1]-[3], and control, [4]-[5]. The major realization approach is to attain fewer partial products to be implemented in simpler arithmetic structures, i.e. adders. For example, well-known Booth's coding algorithm is an elegant method for reducing the partial product terms where the domain of operators and operands is binary logic, i.e. radix-2 or two-valued [6]. Use of higher radices has been considered as a major approach in implementing high-throughput multipliers for improved accuracy and precision. Higher-radix multiplication algorithms, [7]-[10], map a group of two-valued operands into a different number system, e.g. signed-digit, redundant higher-radix algebra. Commonly chosen radix is generally a 2's power by using a suitable encoding scheme to increase the arithmetical and operational yield and reduce the overhead. On the other hand, the multiplication algorithms in [11]-[13] map higher-radix operands into two-valued bit set associated with binary logic operators. In studies cited above, algorithmic complexity and regularity for individual digits are heavily dependent on the chosen radix, number system and digit positions. Therefore, in order to fully benefit from use of higher-radix, multiplication digits should be obtained without encoding operand digits between different number systems. A possible solution can be given by developing suitable algebraic forms in multi-valued logic (MVL) owing to diverse algebraic forms outlined in [14]. Despite considerable studies toward realizing MVL switching functions, higher-radix arithmetic operations with MVL has been a neglected area to our knowledge, which is partly due to lack of suitable algorithms and respective efficient implementation structures. However, a recent study in [15]-[16] reveals that use of MVL operators can be utilized as a direct means for implementing current-mode higher-radix signed-adders having much better design and performance characteristics compared to binary logic counterparts.

In this study, we present an algorithm based on MVL operators such as max, min, literal, cyclic and threshold for multiplication of higher-radix signed numbers where two digits of the result are identified in considerably small number of intermediate operations. We apply a transformation that relocates input operands into lower portion of the number system in multiplication table they belong to. In case of a multi-digit multiplication, partial product terms resulting from the proposed algorithm can be accumulated and results can be moved toward higher digit positions by using the addition algorithm in [16]. The study in [16] also provides the relevant current-mode MVL sub-circuits, which would be considered for a possible implementation of the algorithm proposed in this study.

Background

Given a set $R = \{0, 1, \dots, r\text{-}1\}$, where r is the radix, and variables to be used are members which take values from R, binary (two-operand) MVL operators are min(x,y) and max(x,y) which choose the minimum and maximum of their operands, respectively. A k_l-valued literal (L) and k_c-valued complementary literal (CL) operations are defined as

$$L(k_l,\ \overset{a\ b}{x}\ ) = \begin{cases} k_l & \text{if } a \le x \le b \\ 0 & \text{otherwise} \end{cases}$$

$$CL(k_c,\ \overset{a\ b}{x}\ ) = \begin{cases} k_c & \text{if } x \le a \text{ and } b \le x \\ 0 & \text{otherwise} \end{cases} \qquad (1)$$

while threshold operations are given by

$$\text{lower threshold, (th}_1\text{)}:\ {}^{z}_{y}|x = \begin{cases} z & \text{if } x \le y \\ 0 & \text{otherwise} \end{cases}$$

$$\text{upper threshold, (th}_\text{u}\text{)}:\ x|^{z}_{y} = \begin{cases} z & \text{if } x \ge y \\ 0 & \text{otherwise} \end{cases} \qquad (2)$$

It should be noted that threshold and literal operations are interrelated as $L(k_l,a,b) = {}^{k_l}_{b}\Big|\left(x|^{x}_{a}\right)$ and $x|^{b}_{a} = L(b,a,r-1)$, [17]. The cyclic (modulo-sum) operation, [16], is defined as

$$x\overset{y}{\rightarrow}(r) = x + y \pmod{\text{r}} = \begin{cases} x + y & \text{if } x + y \le r - 1 \\ x + y - r & \text{otherwise} \end{cases} \qquad (3)$$

A one-digit signed number x' withrespect to number x can be of the form

$$x' = s_x x \qquad (4)$$

where $s_x \in \{0, 1\}$ is the sign digit so is y' defined with respective digits. Multiplication of x' and y' is then written as

$$z' = s_z M_1 M_0 \qquad (5)$$

with sign digit s_z, lower and higher-significant digits M_0 and M_1,respectively. The sign digit s_z is given by

$$s_z = s_x\overset{s_y}{\rightarrow} = CL(1,\ s_x,\ s_y) \qquad (6)$$

It is obvious that $M_0 = S_{r-1}$and it can be obtained through iteration

$$S_j = S_{j-1}^{\overrightarrow{t_1|_j^{t_2}}}(r) \tag{7}$$

where $S_0 = 0$, $t_1 = \max(x, y)$, $t_2 = \min(x, y)$, [18], and $j = 1, 2, \ldots, r$-1. The higher-significant digit M_1 can be expressed as

$$M_1 = \sum_{j=1}^{r-1} C_j \tag{8}$$

where

$$C_j = \begin{cases} 1 & \text{if } S_{j-1} + t_1|_j^{t_2} \geq r \\ 0 & \text{otherwise} \end{cases} \tag{9}$$

It is seen that radix-8 multiplication with new algorithm requires 8 cyclic and 8 threshold operations.

Description of New High-radix Multiplication Algorithm

Number of cyclic operations can be reduced by introducing a relocating transformation, Rel(.), applied to t_1 as

$$\text{Rel}(t_1): \quad t = \begin{cases} t_1 & \text{if } t_1 \leq r/2] \\ r - t_1 & \text{otherwise} \end{cases} \tag{10}$$

where $r/2]$ refers to the smallest integer greater than or equal to $r/2$. By exploiting the symmetry, lower-significant digit can be written as

$$M_0 = {}^{M_{01}}_{r/2]} \mid t_1 + t_1 |_{r/2]+1}^{r-M_{01}} \tag{11}$$

where $M_0 = S_{r/2]}$ obtained from the iteration

$$S_j = S_{j-1}^{\overrightarrow{t|_j^{t_2}}}(r) \tag{12}$$

For $j = 1, 2, \ldots, r/2]$ with $S_0 = 0$. On the other hand, higher-significant digit M_1 is obtained as

$$M_1 = \sum_{j=1}^{r-1} C_j = \sum_{j=2}^{r/2]} C_j + \sum_{j=r/2]+1}^{r-1} C_j = M_{11} + M_{12} \tag{13}$$

Above, M_{11} is the sum of C_j terms given by

$$C_j = \begin{cases} 1 & \text{if } S_{j-1} + t|_j^{t_2} \geq r \\ 0 & \text{otherwise} \end{cases} \tag{14}$$

for $j = 2$ through $r/2]$ while M_{12} is the remaining sum. Thus, it can be written that

$$M_1 = {}^{M_{11}}_{r/2]}\big|t_1 + t_1\big|^{t_2 - M_{11} - 1}_{r/2]+1} \tag{15}$$

It is seen that in case of radix-8 multiplication, the number of cyclic operations is reduced to 4 from 8 while the number of threshold operations is reduced to 6 from 8. It is possible to reduce the number of cyclic operations further by relocating both operands simultaneously as

$$t_{x,y} = \begin{cases} x, y & \text{if } x, y \le r/2] \\ r - x, y & \text{otherwise} \end{cases} \tag{16}$$

for which the operation so performed is referred to one-quadrant since both operations are restricted to one quadrant of the multiplication table/chart of interest. The algorithm proceeds by calculating intermediate sums and carries as explained previously. By defining $M_{02} = S_{r/2]}$ with iteration

$$S_j = S_{j-1}^{\overrightarrow{t_x|_j^{t_y}}}(r) \tag{17}$$

for $j = 1, 2, \ldots, r/2]$ with $S_0 = 0$, the lower-significant digit M_0 can be obtained from successive operations as

$$M_0 = {}^{M_{01}}_{r/2]}\big|y + y\big|^{r - M_{01}}_{r/2]+1} \tag{18}$$

with

$$M_{01} = {}^{M_{02}}_{r/2]}\big|x + x\big|^{r - M_{02}}_{r/2]+1} \tag{19}$$

Similarly, the higher-significant digit M_1 can be obtained from successive operations as

$$M_1 = {}^{M_{11}}_{r/2]}\big|y + y\big|^{x - M_{11} - 1}_{r/2]+1} \tag{20}$$

with

$$M_{11} = {}^{M_{12}}_{r/2]}\big|x + x\big|^{t_y - M_{12} - 1}_{r/2]+1} \tag{21}$$

where

$$M_{12} = \sum_{j=2}^{r/2]} C_j \tag{22}$$

and

$$C_j = \begin{cases} 1 & \text{if } S_{j-1} + t_x\big|_j^{t_y} \ge r \\ 0 & \text{otherwise} \end{cases} \tag{23}$$

As verification of the proposed algorithm, some examples with differing radices are given below:

Radix $r = 5$:
1-) $x = 2, y = 3$ => $t_x = 2$, $t_y = 3$ => $S_0 = 0$, $S_1 = 3$, $S_2 = 1$, $S_3 = 1$
$M_{02} = 1$, $M_{01} = 1$ => $M_0 = 1$
$M_{12} = 1$, $M_{11} = 1$ => $M_1 = 1$
2-) $x = 3, y = 4$ => $t_x = 3$, $t_y = 1$ => $S_0 = 0$, $S_1 = 1$, $S_2 = 2$, $S_3 = 3$
$M_{02} = 3$, $M_{01} = 3$ => $M_0 = 2$
$M_{12} = 0$, $M_{11} = 0$ => $M_1 = 2$
3-) $x = 4, y = 4$ => $t_x = 1$, $t_y = 1$ => $S_0 = 0$, $S_1 = 1$, $S_2 = 1$, $S_3 = 1$
$M_{02} = 1$, $M_{01} = 4$ => $M_0 = 1$
$M_{12} = 0$, $M_{11} = 0$ => $M_1 = 3$

Radix $r = 8$:
1-) $x = 3, y = 4$ => $t_x = 3$, $t_y = 4$ => $S_0 = 0$, $S_1 = 4$, $S_2 = 0$, $S_3 = 4$, $S_4 = 4$
$M_{02} = 4$, $M_{01} = 4$ => $M_0 = 4$
$M_{12} = 1$, $M_{11} = 1$ => $M_1 = 1$
2-) $x = 4, y = 4$ => $t_x = 4$, $t_y = 4$ => $S_0 = 0$, $S_1 = 4$, $S_2 = 0$, $S_3 = 4$, $S_4 = 0$
$M_{02} = 0$, $M_{01} = 0$ => $M_0 = 0$
$M_{12} = 2$, $M_{11} = 2$ => $M_1 = 2$
3-) $x = 5, y = 7$ => $t_x = 3$, $t_y = 1$ => $S_0 = 0$, $S_1 = 1$, $S_2 = 2$, $S_3 = 3$, $S_4 = 3$
$M_{02} = 3$, $M_{01} = 5$ => $M_0 = 3$
$M_{12} = 0$, $M_{11} = 0$ => $M_1 = 4$

For a radix-8 multiplication, 3 cyclic and 6 threshold operations are needed. Generally, as seen from (17)-(23), the algorithm has a complexity, i.e. number of operations, of $O(nr/2] + 5)$ for multiplication of two n-digit radix-r numbers. Interested readers are referred to [13]-[20] for sample current-mode implementations of the relevant blocks and units utilized in this algorithm in detail.

Conclusions

A signed-multiplication algorithm for higher-radix numbers is proposed. The algorithm reduces the number of modulo-sums involved considerably. Maximum number of required addition is bound to half the radix. Each digit of product is obtained with regular operations regardless of digit position, which makes the algorithm useful for realization of highly reliable and compact multipliers with small overhead. Since whole operation is performed without coding between different number systems, further derivations are also possible.

Acknowledgement

This work was supported by The Scientific and Technical Research Council of Turkey (TUBITAK) under the project no: 107E059.

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