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RESEARCH ARTICLE

Numerical Study of Liquid Metal MHD Duct Flow Problem in Nuclear Fusion Reactor

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Abstract

A numerical method based on finite difference technique is employed to study liquid metal MHD duct flow problem that may arise in nuclear fusion power generation reactor. Numerical solutions for velocity and magnetic field distribution are presented with the aid of MATLAB software.

INTRODUCTION

At the beginning 21^{st} century, mankind is faced with a problem of energy crisis. Due to rapid industrial growth and rising population contributed to growing energy demand this is compounded by ever dwindling resources of conventional energy. For sustaining the standard of living depend on our ability to develop and implement new large scale energy generation technology. Nuclear fusion energy today provides us the most alternative energy source. Any research activity that may contribute global Endeavour for viable use of fusion energy may have high national and international status.

In the most of the ongoing experimental fusion reactors, detail knowledge of liquid metal MHD flow analysis in a duct under the influence of strong transverse magnetic field is become essential. In these experimental fusion reactors (and in some of reactors under planning) liquid lithium and liquid lead lithium blanket are used as coolant and tritium breeders.

In any experimental reactor, tritium-deuterium plasma is found to be most suitable for fusion reaction. Supply of deuterium can be made available from sea water, whereas tritium isotope is rare and are limited within certain locality.

However, tritium can be produced by escaping energetic neutrons in fusion reaction process, if these neutrons are allowed to react with lithium or lead lithium flow in the blanket of the reactor. The consideration of very high magnetic field acting transversely to duct flow in our model is relevant to fusion reactors blanket environment.

When a very strong magnetic field acts on flow in a duct the inside currents are large and as a consequent large flow opposing magnetic field develops. In ordinary case, where magnetic field acts transversely to the flow works for eliminating frictional forces and one may expect enhanced flow field in the channel. But for the case of very strong transverse magnetic field interaction between the moving conductor and strong magnetic field strong induced electric current appears in the fluid which in turn produces the Lorenz force $\vec{J} \times \vec{B}$, opposing the flow and creates large pressure loss. In such case Lorentz force is much larger than the frictional force and inertia forces which are confined to thin layers. In such thin layers with rapid changes of velocity field is generally observed. So the velocity profile of MHD flows in strong field quite different from those of ordinary hydrodynamic flow.

Hartmann and Lazarus (1937:1) were first to conduct experiments on liquid metal flow through pipes of different cross section under the action of transverse magnetic field. At the subsequent state, Shercliff (1962:513) investigated the flow of electricallyconducting fluid through non-conducting pipes of arbitrary cross sections under theaction of strong uniform transverse magnetic field theoretically as well asexperimentally. To study liquid metal MHD flows in a fusion reactor blanket Smolentsev (1999:231) considered two mathematical models, where the first one describes fully developed MHD flow and the second one describes non-uniform and non-steady flow. He developed numerical codes based on finite difference method which allows calculation for wide range of parameters associated with the MHD flow problems.

Recently Bhuyan and Goswami(2008:1955)made an analysis on the pressure drop for the flow of liquid metal through a rectangular duct under the action of transverse magnetic field.

In this numerical investigation we apply finite difference method to study the flow of liquid metal through a square duct with pair of Hartmann walls as slip walls under the action of very strong transverse inclined magnetic field. We have made an analysis on the velocity and induced magnetic field distribution associated with this liquid metal MHD duct flow from the contour plots are obtained using MATLAB software by utilizing a 5-point stencil finite central difference scheme.The effect of “slip” condition of the duct wall on velocity field distribution is also studied for different angle of inclination for imposed magnetic field.

Formulation of the problem:

We consider the steady motion of liquid metal through a square duct in presence of oblique transverse magnetic. It is assumed that flow in the duct has been driven by pressure gradient and a strong oblique uniform transverse magnetic field B_0 is acting on the flow whose direction is lying on the xy plane with an inclination ψ with the y axis.

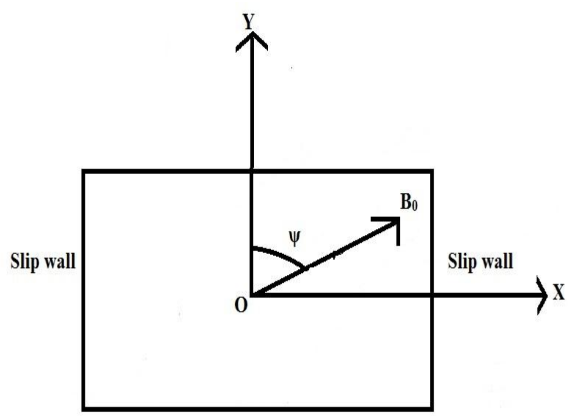

Fig 1: Geometry of the flow problem

Following assumptions are made in this study to solve governing equations of the flow problem

1) The flow is fully developed,
2) Hartmann walls are treated as slip walls and
3) Side walls are assumed to be of no-slip.

The fluid velocity, magnetic field and temperature field distribution for present flow problem are $\vec{V} = \left[0, 0, V_z\left(x, y\right)\right]$ and $\vec{B} = \left[B_{0x}, B_{0y}, B_z\left(x, y\right)\right] = \left[\sqrt{1-\gamma^2}B_0, \gamma B_0, B_z\left(x, y\right)\right]$ respectively, where $\gamma = \cos\psi$.

The equations describing the MHD duct flow problem are

$$\rho\left[\frac{\partial \vec{V}}{\partial t} + \left(\vec{V}.\nabla\right)\vec{V}\right] + \nabla p = \vec{J} \times \vec{B} + \mu\nabla^2\vec{V} \tag{1}$$

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times \left(\vec{V} \times \vec{B}\right) + \lambda\nabla^2\vec{B} \tag{2}$$

The above system of equations under given velocity and magnetic field distribution along with the assumption $\frac{\partial p}{\partial z} = -P$ (constant) finally becomes

$$\mu\nabla^2 V_z + \sqrt{1-\gamma^2}\,\frac{B_0}{\mu_e}\frac{\partial B_z}{\partial x} + \gamma\frac{B_0}{\mu_e}\frac{\partial B_z}{\partial y} = -P \tag{3}$$

$$\mu\nabla^2 B_z + \sqrt{1-\gamma^2}\,B_0\frac{\partial V_z}{\partial x} + \gamma B_0\frac{\partial V_z}{\partial y} = 0 \tag{4}$$

In this problem following non-dimensional quantities are used

$x^* = \frac{x}{a}, y^* = \frac{y}{a}, B^* = \frac{B_z}{B_0}, V^* = \frac{V_z}{V_0}, V_0 = \frac{Pa^2}{\mu}$ and $M = B_0 a\left(\frac{\sigma}{\mu}\right)^{1/2}$.

The non-dimensional equations are

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + M\sqrt{1-\gamma^2}\,\frac{\partial B}{\partial x} + \gamma M\frac{\partial B}{\partial y} = -1 \tag{5}$$

$$\frac{\partial^2 B}{\partial x^2} + \frac{\partial^2 B}{\partial y^2} + M\sqrt{1-\gamma^2}\,\frac{\partial V}{\partial x} + \gamma M\frac{\partial V}{\partial y} = 0 \tag{6}$$

The boundary conditions at the Hartmann walls (having slip) are:

$$\left.\begin{array}{l} V \pm \alpha\frac{\partial V}{\partial x} = 0 \\ B = 0 \end{array}\right\} \tag{7a}$$

and at the side walls (wall with no slip) are:

$$\left.\begin{array}{l} V = 0 \\ B = 0 \end{array}\right\} \tag{7b}$$

Numerical Method: Here, it is observed that Eqs.(5) –(6) are coupled non-linear equations which are to be solved using boundary condition given in Eqs.(7a)-(7b). In view of complexities in seeking closed form solutions, numerical solutions are considered. These equations are expressed in finite difference equation by utilizing a 5-point stencil centered finite difference scheme with mesh size $h = k = 1/N$,where N is a pre-assigned positive integer. The resulting finite difference representation for Eqs.(5) –(6) are as follows

$$\begin{aligned} V_{i,j} &= C_1\left(V_{i+1,j} + V_{i-1,j}\right) + C_2\left(V_{i,j+1} + V_{i,j-1}\right) \\ &\quad + C_3\left(B_{i+1,j} - B_{i-1,j}\right) + C_4\left(B_{i,j+1} - B_{i,j-1}\right) + C_5 \end{aligned} \tag{8}$$

$$\begin{aligned} V_{i,j} &= C_1\left(B_{i+1,j} + B_{i-1,j}\right) + C_2\left(B_{i,j+1} + B_{i,j-1}\right) \\ &\quad + C_3\left(V_{i+1,j} - V_{i-1,j}\right) + C_4\left(V_{i,j+1} - V_{i,j-1}\right) \end{aligned} \tag{9}$$

Where,

$$C_1 = \frac{k^2}{2\left(h^2+k^2\right)}, C_2 = \frac{h^2}{2\left(h^2+k^2\right)}, C_3 = \frac{hk^2 M\sqrt{1-\gamma^2}}{4\left(h^2+k^2\right)}, C_4 = \frac{kh^2 M\gamma}{4\left(h^2+k^2\right)}, C_5 = \frac{h^2k^2}{2\left(h^2+k^2\right)}.$$

The numerical boundary conditions for present duct flow are as follows:

a) for the case of side walls

$$\left.\begin{array}{l} V(i,1) = 0, V(i,N+1) = 0 \\ B(i,1) = 0, B(i,N+1) = 0 \end{array}\right\}$$

b) and for the case of Hartmann walls

$$\left.\begin{aligned} &V(1,j) = -\frac{2h}{\alpha}V(2,j) + V(3,j) \\ &V(N+1,j) = -\frac{2h}{\alpha}V(N,j) + V(N-1,j) \\ &B(1,j) = 0, B(N+1,j) = 0 \\ &B(i,1) = 0, B(i,N+1) = 0 \end{aligned}\right\}$$

The convergence of the computed values for each the variables V and B at different grid points are checked by using root-mean-square residuals R_s for each case. Convergence is considered to be achieved when $R_s < 10^{-7}$, where

$$R_s = \sqrt{\sum_{i=2}^{N}\sum_{j=2}^{N}\left(S_{i,j}^{n+1} - S_{i,j}^{n}\right)^2}$$

Results and Discussion:

In this study, a numerical investigation on the fully developed MHD flow of liquid metal through a square duct in presence of very strong inclined magnetic field is presented. We have considered the pair of Hartmann wall of the duct having slip due to insulation coating used to reduce pressure drop created by Lorentz force $\vec{J} \times \vec{B}$ in the flow region. We have emphasized on the effect of Hartmann number on the velocity and induced magnetic field contour for different angle of inclination of imposed magnetic field.

From the contour plots for velocity shown in Fig.2- Fig.7, we can conclude that with the increase of magnetic field strength velocity become uniform in the core region of the duct. It is also observed that the thick boundary layers are concentrated near the corners in the direction of applied oblique magnetic field. Again from the contour plots for induced magnetic field shown in Fig.8-Fig.13, we can conclude that with the increase of applied magnetic field strength the associated induced magnetic field become uniform and stronger in the core region of the duct flow. It is also observed that the boundary layer thicknesses are large enough near the corners of the duct.

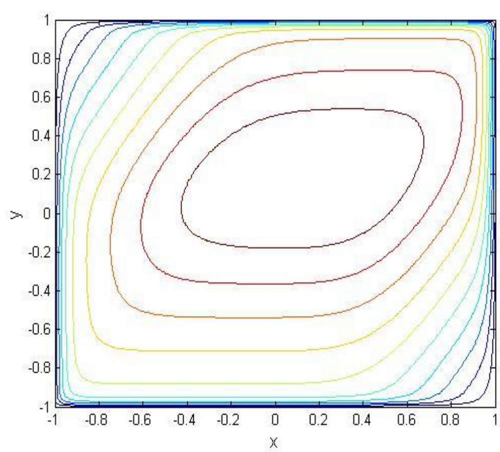

Fig 2: Velocity field contour for $M = 50$ and $\psi = \pi/6$

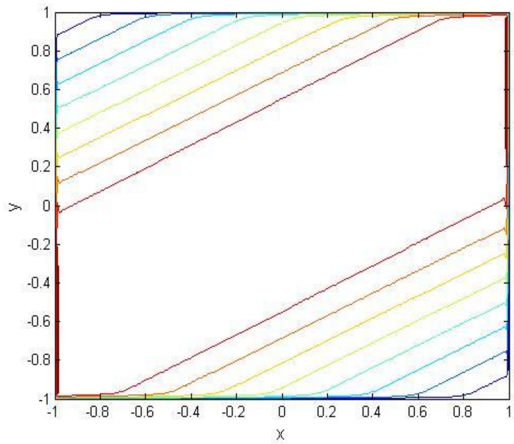

Fig 3: Velocity field contour for $M = 250$ and $\psi = \pi/6$

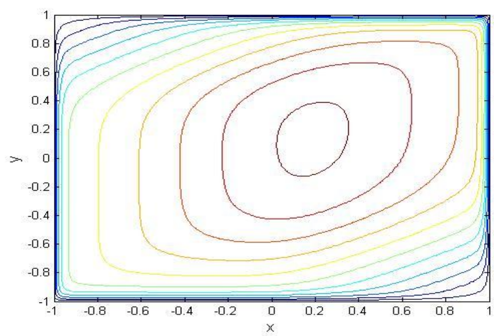

Fig 4: Velocity field contour for $M = 50$ and $\psi = \pi/3$

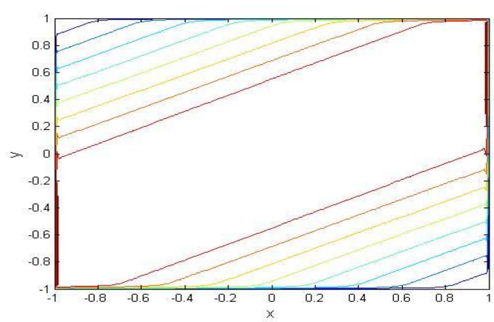

Fig 5: Velocity field contour for $M = 250$ and $\psi = \pi/3$

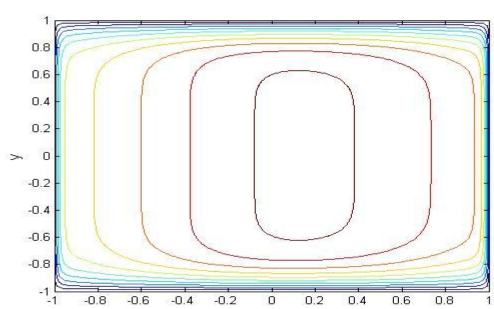

Fig 6: Velocity field contour for $M = 50$ and $\psi = \pi/2$

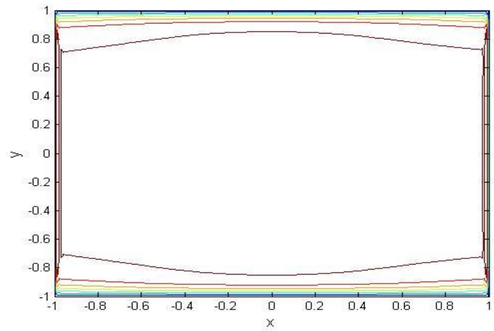

Fig 7: Velocity field contour for $M = 250$ and $\psi = \pi/2$

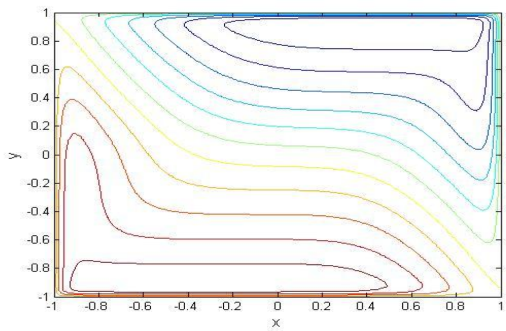

Fig 8: Induced magnetic field contour for $M = 50$ and $\psi = \pi/6$

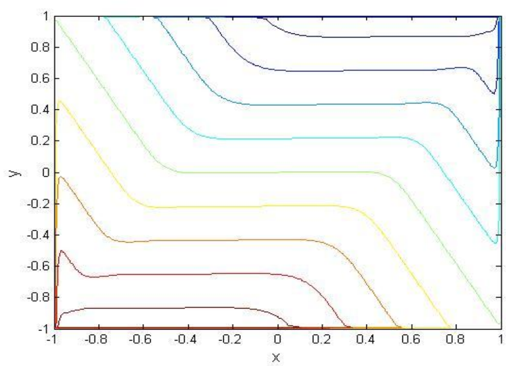

Fig 9: Induced magnetic field contour for $M = 250$ and $\psi = \pi/6$

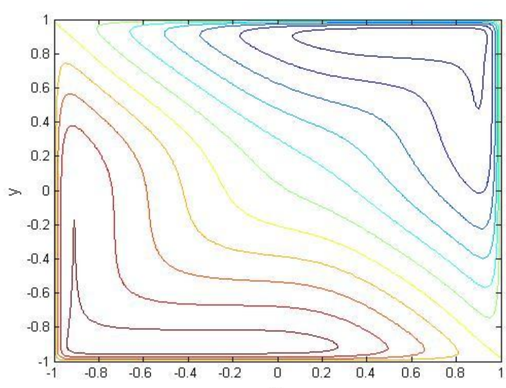

Fig 10: Induced magnetic field contour for $M = 50$ and $\psi = \pi/4$

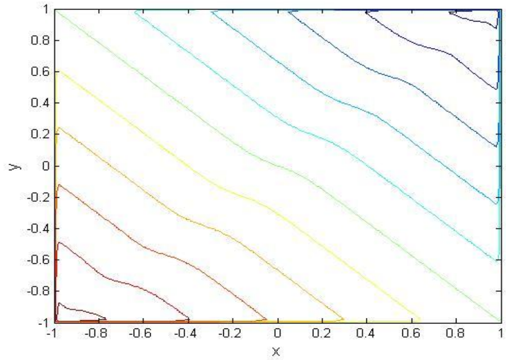

Fig 11: Induced magnetic field contour for $M = 250$ and $\psi = \pi/4$

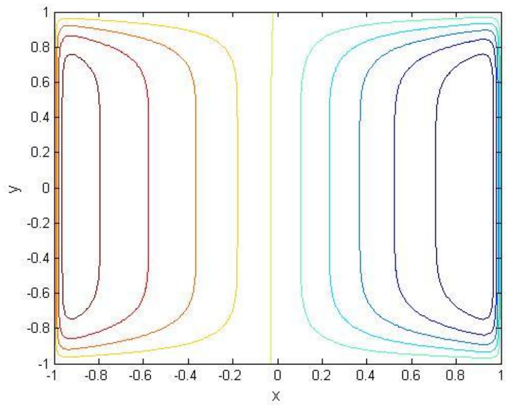

Fig 12: Induced magnetic field contour for $M = 50$ and $\psi = \pi/2$

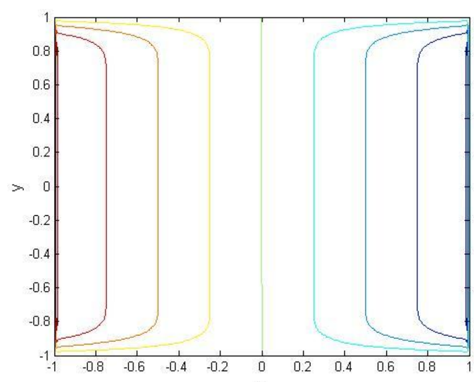

Fig 13: Induced magnetic field contour for $M = 250$ and $\psi = \pi/2$

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