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RESEARCH ARTICLE

On Semi Maximal Soft Pre-α-Open Sets and Semi Minimal Soft Pre-α-Closed Sets in Soft Topological Spaces

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Abstract

In this paper, we introduce new classes of soft open sets and soft closed sets in soft topological spaces called maximal soft pre-α-open sets, minimal soft pre-α-open sets, maximal soft pre-α-closed sets, minimal soft pre-α-closed sets, semi maximal soft pre-α-open sets and semi minimal soft pre-α-closed sets. Also some fundamental properties of these concepts has been studied.

Introduction:-

Nakaoka and Oda in [9,10,11] introduced the concepts of minimal open sets, maximal open sets, minimal closed sets and maximal closed sets in topological spaces resp. Also, the concept of soft set theory was first introduced by Molodtsov [8] as a general mathematical tool for dealing with uncertain objects. He has shown several applications of this theory in solving many practical problems in economics, engineering, social science, medical science, etc. Shabir and Naz [12] introduced the notion of soft topological spaces which are defined over an initial universe with a fixed set of parameters. Arockiarani and Arokia Lancy [3] and Mahmood [6] introduced and investigated soft pre-open sets and soft pre-α-open sets in soft topological spaces. The purpose of the present paper is to introduce new classes of soft open sets and soft closed sets in soft topological spaces called maximal soft pre-α-open sets, minimal soft pre-α-open sets, maximal soft pre-α-closed sets, minimal soft pre-α-closed sets, semi maximal soft pre-α-opens and semi minimal soft pre-α-closed and investigate some of their fundamental properties.

1. Preliminaries:-

First we recall the following definitions and propositions.

Definition(1.1)[8]: Let X be an initial universe and E be a set of parameters. Let $P(X)$ denote the power set of X and A be a non-empty subset of E. A pair (F,A) is called a soft set over X, where F is a mapping given by $F : A \to P(X)$. In other words, a soft set over X is a parameterized family of subsets of the universe X. For $e \in A$, $F(e)$ may be considered as the set of e-approximate elements of the soft set (F,A) .

Definition(1.2)[7]: A soft set (F,A) over X is said to be a null soft set denoted by $\tilde{\phi}$ if for each $e \in A$, $F(e) = \phi$.

Definition(1.3)[7]: A soft set (F,A) over X is said to be an absolute soft set denoted by $\tilde{X}$ if for each $e \in A$, $F(e) = X$.

Definition(1.4)[7]: For two soft sets (F,A) and (G,B) over a common universe X, we say that (F,A) is a soft subset of (G,B) denoted by $(F,A) \tilde{\subseteq} (G,B)$ if $A \subseteq B$ and $F(e) \subseteq G(e)$ for each $e \in A$.

Definition(1.5)[7]: The union of two soft sets (F,A) and (G,B) over a common universe X is the soft set (H,C), where $C = A \bigcup B$, and $\forall e \in C$,

$$H(e) = \begin{cases} F(e) & \text{if } e \in A - B \\ G(e) & \text{if } e \in B - A \\ F(e) \bigcup G(e) & \text{if } e \in A \bigcap B \end{cases},$$

We write $(H,C) = (F,A) \tilde{\bigcup} (G,B)$.

Definition(1.6)[7]: The intersection of two soft sets (F,A) and (G,B) over a common universe X is the soft set (H,C), where $C = A \bigcap B$, and $\forall e \in C$,
$H(e) = F(e) \bigcap G(e)$. We write $(H,C) = (F,A) \tilde{\bigcap} (G,B)$.

Definition(1.7)[2]: For a soft set (F,A) over X, the relative complement of (F,A) is denoted by $(F,A)^c$ and is defined by $(F,A)^c = (F^c, A)$, where $F^c : A \to p(X)$ is a mapping given by
$F^c(e) = X - F(e)$ for each $e \in A$.

Definition(1.8)[5]: Let (F,A) a soft set over X. We say that $\tilde{x} = (e, \{x\})$ is a non-empty soft element of (F,A) if $e \in A$ and $x \in F(e)$. The pair (e, ϕ) is called an empty soft element of (F,A). Nonempty soft elements of (F,A) and empty soft elements of (F,A) are called the soft elements of (F,A) . The fact that $\tilde{x}$ is a soft element of (F,A) will be denoted by $\tilde{x} \tilde{\in} (F,A)$.

Definition(1.9)[12]:. Let $\tilde{\tau}$ be the collection of soft sets over X. Then $\tilde{\tau}$ is called a soft topology on X if $\tilde{\tau}$ satisfies the following axioms:

i) $\tilde{\phi}$, $\tilde{X}$ belong to $\tilde{\tau}$.

ii) The union of any number of soft sets in $\tilde{\tau}$ belongs to $\tilde{\tau}$.

iii) The intersection of any two soft sets in $\tilde{\tau}$ belongs to $\tilde{\tau}$.

The triplet $(X, \tilde{\tau}, E)$ is called a soft topological space over X. The members of $\tilde{\tau}$ are called the soft open sets in X**.**

Definition(1.10)[4]: Let (F,E) be a soft subset of a soft topological space $(X, \tilde{\tau}, E)$. Then:

i) The intersection of all soft closed sets in $(X, \tilde{\tau}, E)$ which contains (F,E) is called the soft closure of (F,E) and is denoted by cl(F,E).

ii) The union of all soft open sets in $(X, \tilde{\tau}, E)$ which are contained in (F,E) is called the soft interior of (F, E) and is denoted by int(F,E).

Definition(1.11)[3]: A soft subset (F,E) of a soft topological space $(X, \tilde{\tau}, E)$ is said to be a soft pre-open set if $(F,E) \tilde{\subseteq} \text{int}(\text{cl}(F,E))$. The complement of a soft pre-open set is said to be a soft pre-closed set**.**

Definition(1.12)[1]: Let (F,E) be a soft subset of a soft topological space $(X, \tilde{\tau}, E)$. Then:

i) The intersection of all soft pre-closed sets in $(X, \tilde{\tau}, E)$ which contains (F,E) is called the soft pre-closure of (F,E) and is denoted by pcl(F,E).

ii) The union of all soft pre-open sets in $(X, \tilde{\tau}, E)$ which are contained in (F,E) is called the soft pre-interior of (F, E) and is denoted by pint(A,E) .

Clearly int(F,E) $\tilde{\subseteq}$ pint(F,E) $\tilde{\subseteq}$ (F,E) and (F,E) $\tilde{\subseteq}$ pcl(F,E) $\tilde{\subseteq}$ cl(F,E).

Definition(1.13)[6]: A soft subset (F,E) of a soft topological space $(X, \tilde{\tau}, E)$ is said to be a soft pre-α-open set if $(F, E) \tilde{\subseteq} \text{int}(\text{pcl}(\text{int}(F, E)))$. The complement of a soft pre-α-open set is said to be soft pre-α-closed. The family of all soft pre-α-open subsets of $(X, \tilde{\tau}, E)$ is denoted by $\tilde{\tau}^{\text{pre}-\alpha}$.

Clearly, every soft open set is soft pre-α-open, but the converse is not true. Consider the following example.

Example(1.14): Let $X = \{a, b, c\}$, $E = \{e_1, e_2\}$ and $\tilde{\tau} = \{\tilde{X}, \tilde{\phi}, (F, E)\}$ be a soft topology over X, where $(F, E) = \{(e_1, \{a\}), (e_2, \{X\})\}$. Thus $\tilde{\tau}^{\text{pre}-\alpha} = \{\tilde{X}, \tilde{\phi}, (F, E), (F_1, E), (F_2, E)\}$, where $(F_1, E) = \{(e_1, \{a, b\}), (e_2, \{X\})\}$ and $(F_2, E) = \{(e_1, \{a, c\}), (e_2, \{X\})\}$. Then (F_2, E) is a soft pre-α-open set in $(X, \tilde{\tau}, E)$, but is not soft open.

Proposition(1.15)[6]: A soft subset (F,E) of a soft topological space $(X, \tilde{\tau}, E)$ is soft pre-α-open if and only if there exists a soft open set (O,E) in $(X, \tilde{\tau}, E)$ such that $(O, E) \tilde{\subseteq} (F, E) \tilde{\subseteq} \text{int}(\text{pcl}(O, E))$.

Definition(1.16)[6]: The intersection of all soft pre-α-closed sets in $(X, \tilde{\tau}, E)$ which contains (F,E) is called the soft pre-α-closure of (F,E) and is denoted by p-α-cl(F,E).

Definition(1.17): Let $(X, \tilde{\tau}, E)$ be a soft topological space. Then:

i) A proper nonnull soft open subset (O,E) of $(X, \tilde{\tau}, E)$ is said to be minimal soft open if any soft open set which is contained in (O,E) is $\tilde{\phi}$ or (O,E).

ii) A proper nonnull soft open subset (O,E) of $(X, \tilde{\tau}, E)$ is said to be maximal soft open if any soft open set which contains (O,E) is (O,E) or $\tilde{X}$.

iii) A proper nonnull soft closed subset (F,E) of $(X, \tilde{\tau}, E)$ is said to be minimal soft closed if any soft closed set which is contained in (F,E) is $\tilde{\phi}$ or (F,E) .

iv) A proper nonnull soft closed subset (F,E) of $(X, \tilde{\tau}, E)$ is said to be maximal soft closed if any soft closed set which contains (F,E) is (F,E) or $\tilde{X}$.

2. Minimal and Maximal Soft Pre-α-Open Sets:-

Definition(2.1): Let $(X, \tilde{\tau}, E)$ be a soft topological space. Then:

i) A proper nonnull soft pre-α-open subset (O,E) of $(X, \tilde{\tau}, E)$ is said to be minimal soft pre-α-open if any soft pre-α-open set which is contained in (O,E) is $\tilde{\phi}$ or (O,E) .

ii) A proper nonnull soft pre-α-open subset (O,E) of $(X, \tilde{\tau}, E)$ is said to be maximal soft pre-α-open if any soft pre-α-open set which contains (O,E) is $\tilde{X}$ or (O,E).

iii) A proper nonnull soft pre-α-closed subset (F,E) of $(X,\tilde{\tau},E)$ is said to be minimal soft pre-α-closed if any soft pre-α-closed set which is contained in (F,E) is $\tilde{\phi}$ or (F,E) .

iv) A proper nonnull soft pre-α-closed subset (F,E) of $(X,\tilde{\tau},E)$ is said to be maximal soft pre-α-closed if any soft pre-α-closed set which contains (F,E) is $\tilde{X}$ or (F,E) .

Remarks(2.2):

i) The family of all minimal soft pre-α-open (resp. minimal soft pre-α-closed) sets of a soft topological space $(X,\tilde{\tau},E)$ is denoted by M_i pre-α-O(X) (resp. M_i pre-α-C(X)).

ii) The family of all maximal soft pre-α-open (resp. maximal soft pre-α-closed) sets of a soft topological space $(X,\tilde{\tau},E)$ is denoted by M_a pre-α-O(X) (resp. M_a pre-α-C(X)).

Remark(2.3): The concept of minimal soft pre-α-open sets, maximal soft pre-α-open sets, minimal soft pre-α-closed sets and maximal soft pre-α-closed sets are independent of each other as in the following example.

Example(2.4): Let $X=\{a,b,c\}$, $E=\{e_1,e_2\}$ and $\tilde{\tau}=\{\tilde{X},\tilde{\phi},(F,E)\}$ be a soft topology over X, where $(F,E)=\{(e_1,F(e_1)),(e_2,F(e_2))\}=\{(e_1,\{a\}),(e_2,\{X\})\}$. So $\tilde{\tau}^{\text{pre}-\alpha}=\{\tilde{\phi},\tilde{X},(F,E),(F_1,E),(F_2,E)\}$, where $(F_1,E)=\{(e_1,\{a,b\}),(e_2,\{X\})\}$ and $(F_2,E)=\{(e_1,\{a,c\}),(e_2,\{X\})\}$ and pre-α-closed(X) $=\{\tilde{\phi},\tilde{X},(G,E),(G_1,E),(G_2,E)\}$, where $(G,E)=\{(e_1,\{b,c\}),(e_2,\{\phi\})\}$, $(G_1,E)=\{(e_1,\{c\}),(e_2,\{\phi\})\}$ and $(G_2,E)=\{(e_1,\{b\}),(e_2,\{\phi\})\}$. Hence M_i pre-α-O(X) $=\{(F,E)\}$, M_i pre-α-C(X)) $=\{(G_1,E),(G_2,E)\}$, M_a pre-α-O(X) $=\{(F_1,E),(F_2,E)\}$, M_a pre-α-C(X) $=\{(G,E)\}$.

	Minimal soft pre-α-open	Maximal soft pre-α-open	Minimal soft pre-α-closed	Maximal soft pre-α-closed
(F,E)	Yes	No	No	No
(F_1,E)	No	Yes	No	No
(G_1,E)	No	No	Yes	No
(G,E)	No	No	No	Yes

Remarks(2.5):i) The concept of maximal soft open (resp. soft open) sets and maximal soft pre-α-open sets are in general independent. In example (2.4) (F_1,E) is a maximal soft pre-α-open set which is not maximal soft open (resp. soft open) and (F,E) is a maximal soft open (resp. soft open) set which is not maximal soft pre-α-open.

ii) The concept of minimal soft closed (resp. soft closed) sets and minimal soft pre-α-closed sets are in general independent. In example (2.4) (G_1,E) is a minimal soft pre-α-closed set which is not minimal soft closed (resp. soft closed), and (G,E) is a minimal soft closed (resp. soft closed) set which is not minimal soft pre-α-closed.

iii) The Intersection of two maximal soft pre-α-open sets need not to be maximal soft pre-α-open. In example (2.4) (F_1,E) and (F_2,E) are maximal soft pre-α-open sets, but $(F_1,E)\tilde{\bigcap}(F_2,E)=(F,E)$ which is not maximal soft pre-α-open.

iv) The union of two minimal soft pre-α-closed sets need not to be minimal soft pre-α-closed. In example (2.4) (G_1, E) and (G_2, E) are minimal soft pre-α-closed sets, but $(G_1, E) \tilde{\bigcup} (G_2, E)$ $= (G, E)$ which is not minimal soft pre-α-closed.

Proposition(2.6):i) Every minimal soft pre-α-open subset of a soft topological space $(X, \tilde{\tau}, E)$ is soft open.
ii) Every minimal soft pre-α-open subset of a soft topological space $(X, \tilde{\tau}, E)$ is minimal soft open.

Proof:i) Let (A,E) be a minimal soft pre-α-open set in $(X, \tilde{\tau}, E)$, then (A,E) is soft pre-α-open. Hence by proposition (1.15) there is a soft open set (U,E) in $(X, \tilde{\tau}, E)$ such that $(U, E) \tilde{\subseteq}$ $(A, E) \tilde{\subseteq} \mathrm{int}(p\mathrm{cl}(U, E))$. But (U,E) is soft open so it is soft pre-α-open. Since (A,E) is minimal so either $(U, E) = \tilde{\phi}$ contradiction or $(U, E) = (A, E)$ thus (A, E) is soft open.

ii) Let (A,E) be a minimal soft pre-α-open set in $(X, \tilde{\tau}, E)$. To prove that (A,E) is minimal soft open. If not, then there is a soft open set (U,E) in $(X, \tilde{\tau}, E)$ such that $\tilde{\phi} \neq (U, E) \tilde{\subset} (A, E)$.
Since every soft open set is soft pre-α-open, hence (U,E) is a soft pre-α-open set such that $(U, E) \tilde{\subset} (A, E)$, this is a contradiction, since (A, E) is a minimal soft pre-α-open set. Thus (A, E) is a minimal soft open set.

Remark(2.7): The converse of proposition (2.6) may not be true in general as shown in the following example.

Example(2.8): Let $X = \{a, b, c\}$, $E = \{e_1, e_2\}$ and $\tilde{\tau} = \{\tilde{X}, \tilde{\phi}, (F_1, E), (F_2, E)\}$ be a soft topology over X, where $(F_1, E) = \{(e_1, \{a\}), (e_2, \{X\})\}$ and $(F_2, E) = \{(e_1, \{a, c\}), (e_2, \{X\})\}$. Then (F_2, E) is a soft open set in $(X, \tilde{\tau}, E)$, but is not minimal soft pre-α-open.

Proposition(2.9): Let (A, E) be a soft subset of a soft topological space $(X, \tilde{\tau}, E)$. Then the following duality principle holds:
i) (A, E) is a maximal soft pre-α-closed set if and only if $(A, E)^c$ is a minimal soft pre-α-open set
ii) (A, E) is a minimal soft pre-α-closed set if and only if $(A, E)^c$ is a maximal soft pre-α-open set
Proof: i) $\Rightarrow$ Let (A, E) be a maximal soft pre-α-closed set and suppose that $(A, E)^c$ is not a minimal soft pre-α-open set. Then there exists a nonnull soft pre-α-open set (B, E) such that $(B, E) \tilde{\subset} (A, E)^c$. That is $(A, E) \tilde{\subset} (B, E)^c$ and $(B, E)^c$ is a soft pre-α-closed set. This is a contradiction to the fact that (A, E) is a maximal soft pre-α-closed set. Hence $(A, E)^c$ is a minimal soft pre-α-open set.

Conversely, let $(A, E)^c$ be a minimal soft pre-α-open set and suppose that (A, E) is not a maximal soft pre-α-closed set. Then there exists a soft pre-α-closed set $(B, E) \neq (A, E)$ such that $(A, E) \tilde{\subset} (B, E) \neq \tilde{X}$. That is $\tilde{\phi} \neq (B, E)^c \tilde{\subset} (A, E)^c$ and $(B, E)^c$ is a soft pre-α-open set. This is a contradiction to the fact that $(A, E)^c$ is a minimal soft pre-α-open set. Hence (A, E) is a maximal soft pre-α-closed set.
ii) Similar to (i).

Theorem(2.10): The following statements are true for any soft topological space $(X, \tilde{\tau}, E)$.

i) If (U,E) is a maximal soft pre-α-open set and (V,E) is a soft pre-α-open set. Then either $(U,E)\tilde{\bigcup}(V,E)=\tilde{X}$ or $(V,E)\tilde{\subseteq}(U,E)$.

ii) If (U,E) and (V,E) are maximal soft pre-α-open sets. Then either $(U,E)\tilde{\bigcup}(V,E)=\tilde{X}$ or $(U,E)=(V,E)$.

iii) If (F,E) is a minimal soft pre-α-closed set and (K,E) is a soft pre-α-closed set. Then either $(F,E)\tilde{\bigcap}(K,E)=\tilde{\phi}$ or $(F,E)\tilde{\subseteq}(K,E)$.

iv) If (F,E) and (K,E) are minimal soft pre-α-closed sets. Then either $(F,E)\tilde{\bigcap}(K,E)=\tilde{\phi}$ or $(F,E)=(K,E)$.

Proof: i) If $(U,E)\tilde{\bigcup}(V,E)=\tilde{X}$, then the proof is complete. If $(U,E)\tilde{\bigcup}(V,E)\neq\tilde{X}$, then we have to prove that $(V,E)\tilde{\subseteq}(U,E)$. Now $(U,E)\tilde{\bigcup}(V,E)\neq\tilde{X}$ means $(U,E)\tilde{\subseteq}(U,E)\tilde{\bigcup}(V,E)$. Since (U,E) is maximal soft pre-α-open, then by definition ((2.1),(ii)) we have $(U,E)\tilde{\bigcup}(V,E)=\tilde{X}$ or $(U,E)\tilde{\bigcup}(V,E)=(U,E)$, but $(U,E)\tilde{\bigcup}(V,E)\neq\tilde{X}$, then $(U,E)\tilde{\bigcup}(V,E)=(U,E)$ which implies $(V,E)\tilde{\subseteq}(U,E)$.

ii) If $(U,E)\tilde{\bigcup}(V,E)=\tilde{X}$, then the proof is complete. If $(U,E)\tilde{\bigcup}(V,E)\neq\tilde{X}$, then we have to prove that $(U,E)=(V,E)$. Now $(U,E)\tilde{\bigcup}(V,E)\neq\tilde{X}$ means $(U,E)\tilde{\subseteq}(U,E)\tilde{\bigcup}(V,E)$ and $(V,E)\tilde{\subseteq}(U,E)\tilde{\bigcup}(V,E)$. Since $(U,E)\tilde{\subseteq}(U,E)\tilde{\bigcup}(V,E)$ and (U,E) is maximal soft pre-α-open, then by definition ((2.1),(ii)) we have $(U,E)\tilde{\bigcup}(V,E)=\tilde{X}$ or $(U,E)\tilde{\bigcup}(V,E)=(U,E)$, but $(U,E)\tilde{\bigcup}(V,E)\neq\tilde{X}$ therefore $(U,E)\tilde{\bigcup}(V,E)=(U,E)$ which implies $(V,E)\tilde{\subseteq}(U,E)$.
Similarly if $(V,E)\tilde{\subseteq}(U,E)\tilde{\bigcup}(V,E)$ we obtain $(U,E)\tilde{\subseteq}(V,E)$. Therefore $(U,E)=(V,E)$.

iii) Since (F,E) is a minimal soft pre-α-closed set, then by proposition ((2.9),(ii)) $(F,E)^c$ is a maximal soft pre-α-open set. Also, since (K,E) is a soft pre-α-closed set, then $(K,E)^c$ is a soft pre-α-open set. Hence by (i) $(F,E)^c\tilde{\bigcup}(K,E)^c=\tilde{X}$ or $(K,E)^c\tilde{\subseteq}(F,E)^c$. Therefore $(F,E)\tilde{\bigcap}(K,E)=\tilde{\phi}$ or $(F,E)\tilde{\subseteq}(K,E)$.

iv) Since (F,E) and (K,E) are minimal soft pre-α-closed sets, then by proposition ((2.9),(ii)) $(F,E)^c$ and $(K,E)^c$ are maximal soft pre-α-open sets. Hence by (ii) $(F,E)^c\tilde{\bigcup}(K,E)^c=\tilde{X}$ or $(K,E)^c=(F,E)^c$. Therefore $(F,E)\tilde{\bigcap}(K,E)=\phi$ or $(F,E)=(K,E)$.

Theorem(2.11): Let (A,E) be a soft subset of a soft topological space $(X,\tilde{\tau},E)$. Then

i) If (A,E) is a maximal soft pre-α-open set and $\tilde{x} \tilde{\in} (A,E)^c$, then $(A,E)^c \tilde{\subset} (B,E)$ for any soft pre-α-open set (B,E) containing $\tilde{x}$.

ii) If (A,E) is a maximal soft pre-α-open set. Then, either of the following (1) and (2) holds:

1) For each $\tilde{x} \tilde{\in} (A,E)^c$ and each soft pre-α-open set (B,E) containing $\tilde{x}$, $(B,E) = \tilde{X}$.

2) There exists a soft pre-α-open set (B,E) such that $(A,E)^c \tilde{\subset} (B,E)$ and $(B,E) \tilde{\subset} \tilde{X}$.

iii) If (A,E) is a maximal soft pre-α-open set. Then, either of the following (1) and (2) holds:

1) For each $\tilde{x} \tilde{\in} (A,E)^c$ and each soft pre-α-open set (B,E) containing $\tilde{x}$, we have $(A,E)^c \tilde{\subset} (B,E)$

2) There exists a soft pre-α-open set (B,E) such that $(A,E)^c = (B,E) \neq \tilde{X}$.

Proof: i) Since $\tilde{x} \tilde{\in} (A,E)^c$, we have $(B,E) \tilde{\not\subset} (A,E)$ for any soft pre-α-open set (B,E) containing $\tilde{x}$. Hence by theorem ((2.10),(i)) we get $(A,E) \tilde{\bigcup} (B,E) = \tilde{X}$, therefore $(A,E)^c \tilde{\bigcap} (B,E)^c = \tilde{\phi}$, thus $(A,E)^c \tilde{\subset} (B,E)$.

ii) If (1) does not hold, then there exists $\tilde{x} \tilde{\in} (A,E)^c$ and a soft pre-α-open set (B,E) containing $\tilde{x}$ such that $(B,E) \tilde{\subset} \tilde{X}$. By (i) we have $(A,E)^c \tilde{\subset} (B,E)$.

iii) If (2) does not hold, then, by (i), we have $(A,E)^c \tilde{\subset} (B,E)$ for each $\tilde{x} \tilde{\in} (A,E)^c$ and each soft pre-α-open set (B,E) containing $\tilde{x}$.

Theorem(2.12): Let (A,E), (B,E) and (C,E) be maximal soft pre-α-open subsets of a soft topological space $(X, \tilde{\tau}, E)$ such that $(A,E) \neq (B,E)$. If $(A,E) \tilde{\bigcap} (B,E) \tilde{\subseteq} (C,E)$, then either $(A,E) = (C,E)$ or $(B,E) = (C,E)$.

Proof: Suppose that $(A,E) \tilde{\bigcap} (B,E) \tilde{\subseteq} (C,E)$. If $(A,E) = (C,E)$, then the proof is complete.

If $(A,E) \neq (C,E)$, then we have to prove $(B,E) = (C,E)$. $(B,E) \tilde{\bigcap} (C,E) = (B,E) \tilde{\bigcap} [(C,E) \tilde{\bigcap} \tilde{X}]$

$$= (B,E) \tilde{\bigcap} [(C,E) \tilde{\bigcap} ((A,E) \tilde{\bigcup} (B,E))]$$
$$= (B,E) \tilde{\bigcap} [((C,E) \tilde{\bigcap} (A,E)) \tilde{\bigcup} ((C,E) \tilde{\bigcap} (B,E))]$$
$$= [(B,E) \tilde{\bigcap} (C,E) \tilde{\bigcap} (A,E)] \tilde{\bigcup} [(B,E) \tilde{\bigcap} (C,E) \tilde{\bigcap} (B,E)]$$
$$= [(A,E) \tilde{\bigcap} (B,E)] \tilde{\bigcup} [(C,E) \tilde{\bigcap} (B,E)] \quad (\text{since } (A,E) \tilde{\bigcap} (B,E) \tilde{\subseteq} (C,E))$$
$$= [(A,E) \tilde{\bigcup} (C,E)] \tilde{\bigcap} (B,E)$$
$$= \tilde{X} \tilde{\bigcap} (B,E) = (B,E) \quad (\text{since } (A,E) \tilde{\bigcup} (C,E) = \tilde{X})$$

This implies $(B,E) \tilde{\subseteq} (C,E)$, but (B,E) is a maximal soft pre-α-open set therefore $(B,E) = (C,E)$.

Theorem(2.13): Let (A,E), (B,E) and (C,E) be maximal soft pre-α-open subsets of a soft topological space $(X,\tilde{\tau},E)$ which are different from each other. Then $(A,E)\tilde{\cap}(B,E)\tilde{\not\subset}$.
$(A,E)\tilde{\cap}(C,E)$.

Proof: Let $(A,E)\tilde{\cap}(B,E)\tilde{\subseteq}(A,E)\tilde{\cap}(C,E)$. Then
$[(A,E)\tilde{\cap}(B,E)]\tilde{\cup}[(C,E)\tilde{\cap}(B,E)]\tilde{\subseteq}[(A,E)\tilde{\cap}(C,E)]\tilde{\cup}[(C,E)\tilde{\cap}(B,E)]$
Hence $[(A,E)\tilde{\cup}(C,E)]\tilde{\cap}(B,E)\tilde{\subseteq}(C,E)\tilde{\cap}[(A,E)\tilde{\cup}(B,E)]$. But by theorem ((2.10),(ii)) we get $(A,E)\tilde{\cup}(C,E)=(A,E)\tilde{\cup}(B,E)=\tilde{X}$. Therefore $\tilde{X}\tilde{\cap}(B,E)\tilde{\subseteq}(C,E)\tilde{\cap}\tilde{X}$ which implies $(B,E)\tilde{\subseteq}(C,E)$. From the definition of maximal soft pre-α-open set it follows that $(B,E)=$
(C,E). Contradiction to the fact that (A,E), (B,E) and (C,E) are different from each other. Hence $(A,E)\tilde{\cap}(B,E)\tilde{\not\subset}(A,E)\tilde{\cap}(C,E)$.

Theorem(2.14): Let $(X,\tilde{\tau},E)$ be a soft topological space, (M,E),(N,E) be maximal soft pre-α-open sets in $(X,\tilde{\tau},E)$ and (U,E)$\tilde{\subseteq}\tilde{X}$ such that $(N,E)\tilde{\subseteq}(U,E)\tilde{\subseteq}$p-α-$cl(N,E)$ if $(M,E)\tilde{\cap}$
$(N,E)=\tilde{\phi}$ then $(U,E)\tilde{\cap}(M,E)=\tilde{\phi}$

Proof: Since $(M,E)\tilde{\cap}(N,E)=\tilde{\phi}$, it follows that $(N,E)\tilde{\subseteq}(M,E)^c$ therefore p-α-cl$(N,E)\tilde{\subseteq}$
p-α-cl($(M,E)^c$). Since $(M,E)^c$ is a minimal soft pre-α-closed set and every minimal soft pre-α-closed set is soft pre-α-closed, then p-α-cl($(M,E)^c$)$=(M,E)^c$. But $(N,E)\tilde{\subseteq}(U,E)\tilde{\subseteq}$
p-α-$cl(N,E)$. Therefore $(U,E)\tilde{\subseteq}$p-α-$cl(N,E)\tilde{\subseteq}(M,E)^c$. Thus $(U,E)\tilde{\subseteq}(M,E)^c$ which means $(U,E)\tilde{\cap}(M,E)=\tilde{\phi}$.

Theorem(2.15): If (F,E) is a minimal soft pre-α-closed set in $(X,\tilde{\tau},E)$ and $\tilde{x}\tilde{\in}(F,E)$, then $(F,E)\tilde{\subset}(G,E)$ for any soft pre-α-closed set (G,E) containing $\tilde{x}$.

Proof: Let (F,E) be a minimal soft pre-α-closed set such that $\tilde{x}\tilde{\in}(F,E)$ and (G,E) be a soft pre-α-closed set containing $\tilde{x}$. If $(F,E)\tilde{\not\subset}(G,E)$, then $(F,E)\tilde{\cap}(G,E)\tilde{\subset}(F,E)$ and $(F,E)\tilde{\cap}$
$(G,E)\neq\tilde{\phi}$. Since (F,E) is minimal soft pre-α-closed, then by definition ((2.1),(iii)) $(F,E)\tilde{\cap}$
$(G,E)=(F,E)$ which contradicts the relation $(F,E)\tilde{\cap}(G,E)\tilde{\subset}(F,E)$. Therefore $(F,E)\tilde{\subset}$
(G,E).

Theorem(2.16): Let (F,E) and $\{(F_\alpha, E)\}_{\alpha\in\Lambda}$ be minimal soft pre-α-closed sets. Then:

i) If $(F,E) \tilde{\subset} \bigcup_{\alpha\in\Lambda}(F_\alpha,E)$, then there exists $\alpha_0 \in \Lambda$ such that $(F,E) = (F_{\alpha_0}, E)$.

ii) If $(F,E) \neq (F_\alpha, E)$ for each $\alpha \in \Lambda$, then $(\bigcup_{\alpha\in\Lambda}(F_\alpha,E)) \tilde{\bigcap} (F,E) = \tilde{\phi}$.

Proof:i) Since $(F,E) \tilde{\subset} \bigcup_{\alpha\in\Lambda}(F_\alpha,E)$, we get $(F,E) = (F,E) \tilde{\bigcap} \bigcup_{\alpha\in\Lambda}(F_\alpha,E) = \bigcup_{\alpha\in\Lambda}((F,E) \tilde{\bigcap} (F_\alpha, E))$. If $(F,E) \neq (F_\alpha, E)$ for each $\alpha \in \Lambda$, then by theorem ((2.10,(iv)) $(F,E) \tilde{\bigcap} (F_\alpha, E) = \tilde{\phi}$ for each $\alpha \in \Lambda$, hence we have $(F,E) = \bigcup_{\alpha\in\Lambda}((F_\alpha,E) \tilde{\bigcap} (F,E)) = \tilde{\phi}$. This contradicts our assumption that (F,E) is a minimal soft pre-α-closed set. Thus there exists $\alpha_0 \in \Lambda$ such that $(F,E) = (F_{\alpha_0}, E)$.

ii) If $(F,E) \neq (F_\alpha, E)$ for each $\alpha \in \Lambda$, then by theorem ((2.10,(iv)) $(F,E) \tilde{\bigcap} (F_\alpha, E) = \tilde{\phi}$ for each $\alpha \in \Lambda$, therefore $(\bigcup_{\alpha\in\Lambda}(F_\alpha,E)) \tilde{\bigcap} (F,E) = \bigcup_{\alpha\in\Lambda}((F_\alpha,E) \tilde{\bigcap} (F,E)) = \tilde{\phi}$.

3. Semi Maximal Soft Pre-α-Open Sets:-

Definition(3.1): A soft subset (A,E) of a soft topological space $(X, \tilde{\tau}, E)$ is said to be semi maximal soft pre-α-open if there exists a maximal soft pre-α-open set (M,E) in $(X, \tilde{\tau}, E)$ such that $(M,E) \tilde{\subseteq} (A,E) \tilde{\subseteq} cl(M,E)$. The family of all semi maximal soft pre-α-open sets in $(X, \tilde{\tau}, E)$ is denoted by SM_a pre-α-O(X) .

Remark(3.2): Every maximal soft pre-α-open set is semi maximal soft pre-α-open, but the converse is not true as shown by the following example.

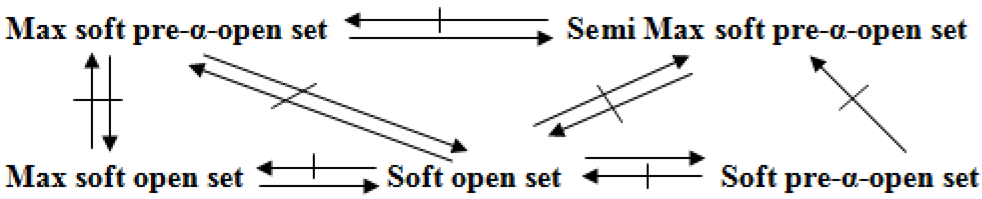
Example(3.3): Let $X = \{a,b,c\}$, $E = \{e_1, e_2\}$ and $\tilde{\tau} = \{\tilde{X}, \tilde{\phi}, (F,E)\}$ be a soft topology over X, where $(F,E) = \{(e_1,\{a\}),(e_2,\{X\})\}$. Then $\tilde{X}$ is a semi maximal soft pre-α-open set in $(X, \tilde{\tau}, E)$, since $\exists$ a maximal soft pre-α-open set $(M,E) = \{(e_1,\{a,b\}),(e_2,\{X\})\}$ in $(X, \tilde{\tau}, E)$ such that $(M,E) \tilde{\subseteq} \tilde{X} \tilde{\subseteq} cl(M,E)$, but $\tilde{X}$ is not maximal soft pre-α-open.

Remark(3.4): Semi maximal soft pre-α-open sets and soft open sets are in general independent. Consider the following examples:

Examples(3.5): Let $X = \{a,b,c\}$, $E = \{e_1, e_2\}$ and $\tilde{\tau} = \{\tilde{X}, \tilde{\phi}, (F_1,E), (F_2,E)\}$ be a soft topology over X, where $(F_1,E) = \{(e_1,\{a\}),(e_2,\{X\})\}$ and $(F_2,E) = \{(e_1,\{a,c\}), (e_2,\{X\})\}$. Since $(A,E) = \{(e_1,\{a,b\}),(e_2,\{X\})\}$ is a maximal soft pre-α-open set in $(X, \tilde{\tau}, E)$, thus (A,E) is a semi maximal soft pre-α-open set in $(X, \tilde{\tau}, E)$, but (A,E) is not soft open in $(X, \tilde{\tau}, E)$. Also, (F_1,E) is a soft open set in $(X, \tilde{\tau}, E)$, but is not semi maximal soft pre-α-open.

Remark(3.6): In example (3.3) $(F,E) = \{(e_1,\{a\}),(e_2,\{X\})\}$ is soft pre-α-open in $(X, \tilde{\tau}, E)$, but is not semi maximal soft pre-α-open.

The following diagram shows the relationships between semi maximal soft pre-α-open sets and some other soft open sets:



Theorem(3.7): If (M,E) is a semi maximal soft pre-α-open set in a soft topological space $(X,\tilde{\tau},E)$ and $(M,E)\tilde{\subseteq}(A,E)\tilde{\subseteq}cl(M,E)$ then (A,E) is also semi maximal soft pre-α-open in $(X,\tilde{\tau},E)$.

Proof: Since (M,E) is a semi maximal soft pre-α-open set in $(X,\tilde{\tau},E)$, then by definition (3.1) there exists a maximal soft pre-α-open set (U,E) in $(X,\tilde{\tau},E)$ such that $(U,E)\tilde{\subseteq}(M,E)\tilde{\subseteq}cl(U,E)$. Since $(M,E)\tilde{\subseteq}cl(U,E)$ it follows that $cl(M,E)\tilde{\subseteq}cl(cl(U,E))=cl(U,E)$. But from hypothesis $(A,E)\tilde{\subseteq}cl(M,E)$ it follows that $(U,E)\tilde{\subseteq}(A,E)\tilde{\subseteq}cl(U,E)$. Hence by definition (3.1) (A,E) is a semi maximal soft pre-α-open set in $(X,\tilde{\tau},E)$.

Remark(3.8): The intersection of two semi maximal soft pre-α-open sets need not to be semi-maximal soft pre-α-open. It can be shown by the following example:

Example(3.9): Let $X=\{a,b,c\}$, $E=\{e_1,e_2\}$ and $\tilde{\tau}=\{\tilde{X},\tilde{\phi},(F_1,E),(F_2,E)\}$ be a soft topology over X. Also, SM_a pre-α-O(X) $=\{\tilde{X},(F_2,E),(F_3,E)\}$, where $(F_1,E)=\{(e_1,\{a\}),(e_2,\{X\})\}$, $(F_2,E)=\{(e_1,\{a,b\}),(e_2,\{X\})\}$ and $(F_3,E)=\{(e_1,\{a,c\}),(e_2,\{X\})\}$. Then (F_2,E) and (F_3,E) are semi maximal soft pre-α-open sets, but $(F_2,E)\tilde{\cap}(F_3,E)=(F_1,E)$ which is not semi maximal soft pre-α-open.

4. Semi Minimal Soft Pre-α-Closed Sets

Definition(4.1): A soft subset (A,E) of a soft topological space $(X,\tilde{\tau},E)$ is said to be semi minimal soft pre-α-closed set if $(A,E)^c$ is semi maximal soft pre-α-open. The family of all semi minimal soft pre-α-closed sets in $(X,\tilde{\tau},E)$ is denoted by SM_i pre-α-C(X).

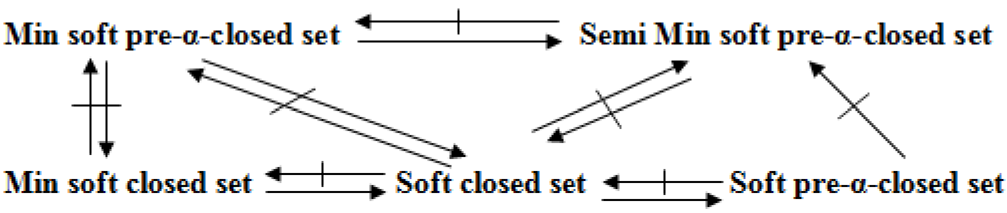
Remark(4.2): Every minimal soft pre-α-closed set is semi minimal soft pre-α-closed, but the converse is not true in general. In example (3.3) $\tilde{\phi}$ is semi minimal soft pre-α-closed set in $(X,\tilde{\tau},E)$, since $(\tilde{\phi})^c=\tilde{X}$ is semi maximal soft pre-α-open set, but $\tilde{\phi}$ is not minimal soft pre- α-closed set in $(X,\tilde{\tau},E)$.

Remark(4.3): Semi minimal soft pre-α-closed sets and soft closed sets are in general independent. In example (3.5) $(B,E)=\{(e_1,\{c\}),(e_2,\{\phi\})\}$ is a semi minimal soft pre-α-closed set in $(X,\tilde{\tau},E)$, since $(B,E)^c=(A,E)$ is semi maximal soft pre-α-open set in $(X,\tilde{\tau},E)$, but

(B,E) is not soft closed in $(X,\tilde{\tau},E)$. Also, $(G,E)=\{(e_1,\{b,c\}),(e_2,\{\phi\})\}$ is soft closed in $(X,\tilde{\tau},E)$, but is not semi minimal soft pre-α-closed.

Remark(4.4): In example (3.3) $(G,E)=\{(e_1,\{b,c\}),(e_2,\{\phi\})\}$ is soft pre-α-closed in $(X,\tilde{\tau},E)$, but is not semi minimal soft pre-α-closed.

The following diagram shows the relationships between semi minimal soft pre-α-closed sets and some other soft closed sets:



Theorem(4.5): A soft subset (A,E) of a soft topological space $(X,\tilde{\tau},E)$ is semi minimal soft pre-α-closed if and only if there exists a minimal soft pre-α-closed set (N,E) in $(X,\tilde{\tau},E)$ such that $\operatorname{int}(N,E)\tilde{\subseteq}(A,E)\tilde{\subseteq}(N,E)$.

Proof: Suppose that (A,E) is a semi minimal soft pre-α-closed set in $(X,\tilde{\tau},E)$, then by definition (4.1) $(A,E)^c$ is semi maximal soft pre-α-open in $(X,\tilde{\tau},E)$. Therefore by definition (3.1) there exists a maximal soft pre-α-open set (M,E) in $(X,\tilde{\tau},E)$ such that $(M,E)\tilde{\subseteq}(A,E)^c$ $\tilde{\subseteq}\operatorname{cl}(M,E)$ which implies that $(\operatorname{cl}(M,E))^c\tilde{\subseteq}(A,E)\tilde{\subseteq}(M,E)^c$. Since $(\operatorname{cl}(M,E))^c=$ $\operatorname{int}((M,E)^c)$, therefore $\operatorname{int}((M,E)^c)\tilde{\subseteq}(A,E)\tilde{\subseteq}(M,E)^c$. Put $(M,E)^c=(N,E)$, hence (N,E) is a minimal soft pre-α-closed set in $(X,\tilde{\tau},E)$ such that $\operatorname{int}(N,E)\tilde{\subseteq}(A,E)\tilde{\subseteq}(N,E)$.

Conversely, suppose that there exists a minimal soft pre-α-closed set (N,E) in $(X,\tilde{\tau},E)$ such that $\operatorname{int}(N,E)\tilde{\subseteq}(A,E)\tilde{\subseteq}(N,E)$. Hence $(N,E)^c\tilde{\subseteq}(A,E)^c\tilde{\subseteq}(\operatorname{int}(N,E))^c$. Since $\operatorname{cl}((N,E)^c)=(\operatorname{int}(N,E))^c$, therefore there exists a maximal soft pre-α-open set $(N,E)^c$ in $(X,\tilde{\tau},E)$ such that $(N,E)^c\tilde{\subseteq}(A,E)^c\tilde{\subseteq}\operatorname{cl}((N,E)^c)$. Thus by definition (3.1) $(A,E)^c$ is a semi maximal soft pre-α-open set in $(X,\tilde{\tau},E)$. Hence by definition (4.1) (A,E) is a semi minimal soft pre-α-closed set in $(X,\tilde{\tau},E)$.

Theorem(4.6): If (N,E) is a semi minimal soft pre-α-closed set in a soft topological space $(X,\tilde{\tau},E)$ and $\operatorname{int}(N,E)\tilde{\subseteq}(A,E)\tilde{\subseteq}(N,E)$ then (A,E) is also semi minimal soft pre-α-closed in $(X,\tilde{\tau},E)$.

Proof: Since (N,E) is a semi minimal soft pre-α-closed set in $(X,\tilde{\tau},E)$, then by theorem (4.5) there exists a minimal soft pre-α-closed set (F,E) in $(X,\tilde{\tau},E)$ such that $\operatorname{int}(F,E)\tilde{\subseteq}(N,E)\tilde{\subseteq}$ (F,E). Now, $\operatorname{int}(F,E)\tilde{\subseteq}(N,E)$ which implies $\operatorname{int}(F,E)=\operatorname{int}(\operatorname{int}(F,E)\tilde{\subseteq}\operatorname{int}(N,E)$. Since $\operatorname{int}(N,E)\tilde{\subseteq}(A,E)$, then $\operatorname{int}(F,E)\tilde{\subseteq}(A,E)$. Hence $\operatorname{int}(F,E)\tilde{\subseteq}\operatorname{int}(N,E)\tilde{\subseteq}(A,E)\tilde{\subseteq}(N,E)$ $\tilde{\subseteq}(F,E)$. It follows that $\operatorname{int}(F,E)\tilde{\subseteq}(A,E)\tilde{\subseteq}(F,E)$. Thus there exists a minimal soft pre-α-closed set (F,E) in $(X,\tilde{\tau},E)$ such that $\operatorname{int}(F,E)\tilde{\subseteq}(A,E)\tilde{\subseteq}(F,E)$. Therefore (A,E) is a semi minimal soft pre-α-closed set in $(X,\tilde{\tau},E)$.

Remark(4.7): The union of two semi minimal soft pre-α-closed sets need not to be semi-minimal soft pre-α-closed. In example (3.8) SM_i pre-α-C(X) $=\{\tilde{\phi},(G_2,E),(G_3,E)\}$, where $(G_2,E)=\{(e_1,\{c\}),(e_2,\{\phi\})\}$ and $(G_3,E)=\{(e_1,\{b\}),(e_2,\{\phi\})\}$. Then (G_2,E) and (G_3,E) are semi minimal soft pre-α-closed sets, but $(G_2,E)\tilde{\bigcup}(G_3,E)=\{(e_1,\{c,b\}),(e_2,\{\phi\})\}$ which is not semi minimal soft pre-α-closed.

References:-

[1]. Akdag, M. and Ozkan, A.; " On soft preopen sets and soft pre separation axioms", Gazi University Journal of Science, 27(4), 2014, 1077-1083.

[2]. Ali, M.I., Feng, F., Liu, X., Min, W. K. and Shabir, M.; " On some new operations in soft set theory", Comput. Math. Appl., 57, 2009,1547-1553.

[3]. Arockiarani, I.and Arokia Lancy, A.; "Generalized soft gβ-closed sets and soft gsβ-closed sets in soft topological spaces", Int. J. Math. Arch. 4(2), 2013, 17-23.

[4]. Cağman, N., Karataş, S.and Enginoglu, S.; " Soft topology", Computers and Mathematics with Applications 62, 2011, 351-358.

[5]. Dariusz Wardowski ; " On a soft mapping and its fixed points", Fixed Point Theory and Applications Springer open Journal,182, 2013, 1-11.

[6]. Mahmood, S. I.; " On soft Pre-α-open sets and soft Pre-α-continuous functions in soft topological spaces", Journal College of Education Al-Mustansiriyah University, 2015 (to appear).

[7]. Maji, P. K., Biswas, R. and Roy, A. R.; "Soft set theory", Computers and Mathematics with Applications, 45, 2003, 555-562.

[8]. Molodtsov, D.; "Soft set theory-First results", Comput. Math. Appl. 37 (4–5), 1999, 19-31.

[9]. Nakaoka, F. and Oda, N.; "Some applications of minimal open sets", Int. Journal of Math. Math. Sci. 27(8), 2001, 471-476.

[10]. Nakaoka, F. and Oda, N.; "Some properties of maximal open sets", Int. Journal of Math. Math. Sci. 21(21), 2003, 1331-1340.

[11]. Nakaoka, F. and Oda, N.; "On minimal closed sets", Proceeding of topological spaces and its applications,5, 2003, 19-21.

[12]. Shabir, M. and Naz, M.; "On soft topological spaces", Computers and Mathematics with Applications, 61(7), 2011,1786-1799.