ISSN NO. 2320-5407

Journal homepage: http://www.journalijar.com

INTERNATIONAL JOURNAL OF ADVANCED RESEARCH

RESEARCH ARTICLE

The Comparison of Bondage Number, Radius and Diameter of Circular-Arc Graphs

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Manuscript Info

Manuscript History:

Received: 13 November 2015
Final Accepted: 28 December 2015

Key words:
circular-arc family, circular-arc graph, dominating number, bondage number, radius, diameter.

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Abstract

A connected dominating set is used as a backbone for communications and vertices that are not in this set communicate by passing message through neighbors that are in the set. Among the various applications of the theory of domination and the distance, the most often discussed is a communication network. This network consists of communication links all distance between affixed set of sites. Circular-arc graphs are rich in combinatorial structures and have found applications in several disciplines such as Biology, Ecology, Psychology, Traffic control, Genetics, Computer sciences and particularly useful in cyclic scheduling and computer storage allocation problems etc. Suppose communication network does not work due to link failure. Then the problem is what is the fewest number of communication links such that at least one additional transmitter would be required in order that communication with all sites as possible. This leads to the introducing of the concept of the bondage number, radius and diameter. In this paper we present the comparison of bondage number, radius and diameter of circular-arc graphs..

Introduction:-

A graph G=(V,E) is called a circular-arc graph or an intersecting graph for a finite family A of a non empty set if there is a one to one correspondence between A and B . Such that two sets in A have non empty intersection if and only if there corresponding vertices in V are adjacent to each other. We call A an intersection model of G for an intersection model A we use G(A) to denote the intersection graph for G. If A is a family of arcs on a circle, then G is called circular-arc graph for A and A is called a circular-arc model of G[2]. Circular-arc graphs have many applications in different fields such as genetic research, traffic control, computer complier design etc.

Let A= $\{A_1, A_2, A_3,\ldots\ldots A_n\}$ be a family of n arcs on a circle C. Each end point of the arcs in assigned a positive integer called a coordinate. The end point of each arc are located on the circumference are C in the ascending order of the values of the coordinates in the clock wise direction. For convenience, each arcs are A_i, i=1,2,3…..n is represented as (h_i,t_i). Where h_i is the head point and t_i is the tail point respectively that starting and ending point_s of the arc when it is traversed in counter clock wise manner, starting with an arbitrary chosen point on C which is not an end of any arc in A. Without loss of generality, we will assume that the following condition

a) No two arcs share a common end point.
b) No single arc in A covers the entire circle C by itself otherwise the shortest path result becomes trivial and in this case the distance between any two arcs in either 1 or 2 unit.
c) $\bigcup_{i=1}^{n} Ci = C$, otherwise the result becomes one on trivial graph.
d) The end points of the arcs in A are already given and sorted, according to the order in which they are visited during clockwise traversal along C by starting at arc a_1 .
e) The arcs are sorted in increasing values of h_i's that is $h_i > h_j$ for I > j.
f) The family of arcs A is said to be canonical if h_i's and t_i's for i = 1,2,….n are all distinct integers between 1 and 2n and the point 1 is the head of the arc a_1.

And again alternatively, the circular arc graph can be defined as follows an undirected graph G (V, E) is a circular arc graph if and only if its vertices circularly indexed $v_1, v_2,\ldots\ldots v_n$ and $(v_i,v_j) \in E$ provided a_i and a_j intersect with each other, where v_i and v_j are the vertices in the graph G corresponding to the arcs a_i and a_j in A respectively.

A subset D of v is said to be a dominating set of G if every vertex not in D is adjacent to vertex in D. The domination number γ(G) of a graph G is the minimum cardinality of a dominating set in G[1]. The bondage number b(G) of a non empty graph G is the minimum cardinality[5,6] among all sets of edges E_1 for which $\gamma(G-E_1) > \gamma(G)$. In a simple graph, there is only one edge between two consecutive vertices of a walk, so one could abbreviate the walk as $W=\{v_{0,} e_1,v_1,e_2,\ldots..v_{n-1},e_n,v_n\}$. The length of a walk or directed walk is the number of edge steps in the walk sequence. A walk of length zero. i.e with one vertex and no edges is called a trivial walk. For a connected graph G the term distance we defined as d (ai,aj) ≥ 0, for all ai,aj $\in$V(G), d (ai,aj) = 0, if and only if ai=aj, d (ai,aj) = d (aj,ai), for all ai,aj $\in$V(G), d (ai,ak) ≤ d (ai,aj) + d (aj,ak), for all ai,aj,ak $\in$ V(G). Suppose an interval graph V(G) is not a connected and then G1 and G2 are two graphs of G such that G= G1 $\cup$ G2 and E(G1) $\cup$ E(G2). And G1∩G2= ϕ that is V (G1) ∩V (G2) = ϕ and E (G1) ∩E (G2) = ϕ. Then d (ai,aj) = ∞ for ai$\in$ V(G1).aj $\in$V(G2). Then G must be connected.The diameter and radius are two of the most basic graph parameters. The diameter of a graph is the largest distance between its vertices. The distance orientations of graphs by V.Chvatal (5). The diameter of a graph G is the maximum of eccentricity of all its vertices and is denoted by Diam (G) that is Diam (G) = max {e (v): v $\in$V (G)}, where the maximum distance from vertex u to any vertex of G is called eccentricity of the vertex v and is denoted by ecc (v) that is ecc (v) = max {d (u,v): u $\in$V (G)},where as the distance between two vertices u and v of a graph is the length of the shortest path between them and is denoted by d(u,v) or d (v,u).

Theorem1: Let $A = \{a_i, a_j,\ldots\ldots.a_n\}$ be circular-arc family and G be a circular-arc graph corresponding to circular-arc family A. Let $a_i, a_j \in A$ and suppose a_j is contained in a_i and there is no other arc that intersects a_j, other than a_i. Then the bondage number b(G) < the radius of G < the diameter of G.

Proof: Let $A = \{a_i, a_j \ldots\ldots.a_n\}$ be a circular-arc family. Let G be the circular-arc graph corresponding to the given arc family A. Let a_i, a_j any two arcs in A, which satisfy the hypothesis of the theorem. Then clearly $a_i \in D$, where D is a minimum dominating set of G, because there is no Other arc A, other than a_i, that dominates a_j.

Consider the edge $e = (a_i, a_j)$ in graph G. If we remove this edge from G, then a_j becomes an isolated vertex in G-e, as there is no other vertex in G, other than a_i, that is adjacent with a_j.

Now $D_1 = D \cup \{a_j\}$ becomes a dominating set of G-e and since D is a minimum dominating set of G it follows that D_1 is also the minimum dominating set of G-e.

Therefore γ (G-e) = $|D_1| = |D| +1 > |D| = \gamma$ (G)

Thus the bondage number b(G) = 1.

Again we will find the radius as well as the diameter of G. First we will find the distance of circular-arc graph G corresponding to circular-arc family $A = \{a_i, a_j,\ldots\ldots.a_n\}$.

For any pair of vertices $a_i, a_j \in A$, a path $a_i\, a_{i1}\, a_{i2}\ldots\ldots\ldots\ldots a_{in-1}\, a_j$ from a_i to a_j, where the vertices are not necessary distinct , is called an a_i to a_j path. The distance from a_i to a_j , that is the length of a shortest path from a_i to a_j , is denoted by d (a_i, a_j). If this does not lead to confusion and D = Diam(G) = max{d(a_i, a_j)}, where $a_i,a_j \in A$, which stands for the diameter of G. The distance from L to a_i, d(L, a_i) , is defined analogously. A Circular-arc graph G = (V,E) is said to be connected when for any pair of vertices ai, aj $\in$A , there always exists an a_i to a_j path. And again let G be a connected graph, the positive eccentricity of a vertex $a_i \in A$ is defined as e (a_i) = max { d(a_i, a_j)}: $a_j \in V$} and radius of G is rad(G) = min{e(a_i): $a_i \in V$}. Finally we have to compare said above all values we got b (G) < rad(G) < diam (G).

Therefore the theorem is hold.

PRACTICAL PROBLEM FOR THEOREM -1

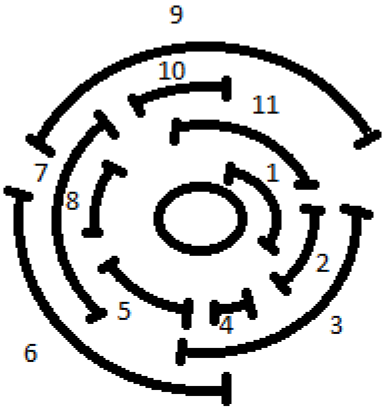

Fig. 1: Circular-arc family A

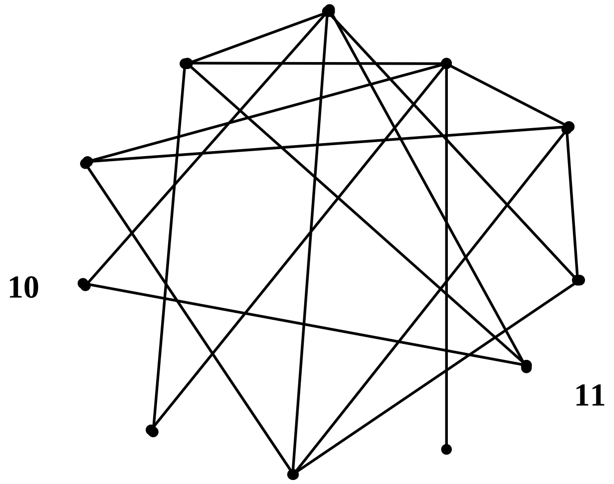

Fig. 2: Circular-arc Graph G

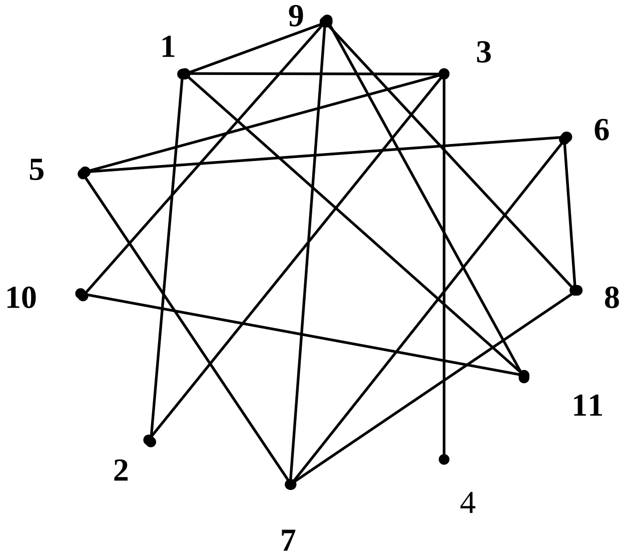

Fig. 3: Circular-arc Graph G-e

To find the distances from G

d(1,1)=0
d(1,2)=1
d(1,3)=1
d(1,4)=2
d(1,5)=2
d(1,6)=2
d(1,7)=2
d(1,8)=2
d(1,9)=1
d(1,10)=2
d(1,11)=1

d(2,1)=1
d(2,2)=0
d(2,3)=1
d(2,4)=2
d(2,5)=2
d(2,6)=2
d(2,7)=3
d(2,8)=3
d(2,9)=2
d(2,10)=3
d(2,11)=2

d(3,1)=1
d(3,2)=1
d(3,3)=0
d(3,4)=1
d(3,5)=1
d(3,6)=1
d(3,7)=2
d(3,8)=2
d(3,9)=2
d(3,10)=3
d(3,11)=2

d(4,1)=2
d(4,2)=2
d(4,3)=1
d(4,4)=0
d(4,5)=2
d(4,6)=2
d(4,7)=3
d(4,8)=3
d(4,9)=3
d(4,10)=4
d(4,11)=3

d(5,1)=2
d(5,2)=2
d(5,3)=1
d(5,4)=2
d(5,5)=0
d(5,6)=1
d(5,7)=1
d(5,8)=2
d(5,9)=2
d(5,10)=3
d(5,11)=3

d(6,1)=2
d(6,2)=2
d(6,3)=1
d(6,4)=2
d(6,5)=1
d(6,6)=0
d(6,7)=1
d(6,8)=1
d(6,9)=2
d(6,10)=3
d(6,11)=3

d(7,1)=2
d(7,2)=3
d(7,3)=2
d(7,4)=3
d(7,5)=1
d(7,6)=1
d(7,7)=0
d(7,8)=1
d(7,9)=1
d(7,10)=2
d(7,11)=2

d(8,1)=2
d(8,2)=3
d(8,3)=2
d(8,4)=3
d(8,5)=2
d(8,6)=1
d(8,7)=1
d(8,8)=0
d(8,9)=1
d(8,10)=2
d(8,11)=2

d(9,1)=1
d(9,2)=2
d(9,3)=2
d(9,4)=3
d(9,5)=2
d(9,6)=2
d(9,7)=1
d(9,8)=1
d(9,9)=0
d(9,10)=1
d(9,11)=1

d(10,1)=2
d(10,2)=3
d(10,3)=3
d(10,4)=3
d(10,5)=3
d(10,6)=3
d(10,7)=2
d(10,8)=2
d(10,9)=1
d(10,10)=0
d(10,11)=1

d(11,1)=1
d(11,2)=2
d(11,3)=2
d(11,4)=3
d(11,5)=3
d(11,6)=3
d(11,7)=2
d(11,8)=2
d(11,9)=1
d(11,10)=1
d(11,11)=0

To find an Eccentricity:-

e (i) = max {δ(i, j) : j $\in$ V}
e(1) = max {0, 1, 1, 2, 2, 2, 2, 2, 1, 2, 1}=2
e(2) = max {1, 0, 1, 2, 2, 2, 3, 3, 2, 3, 2}=3
e(3) = max {1, 1, 0, 1, 1, 1, 2, 2, 2, 3, 2}=3
e(4) = max {2, 2, 1, 0, 2, 2, 3, 3, 3, 4, 3}=4
e(5) = max {2, 2, 1, 2, 0, 1, 1, 2, 2, 3, 3}=3
e(6) = max {2, 2, 1, 2, 1, 0, 1, 1, 2, 3, 3}=3
e(7) = max {2, 3, 2, 3, 1, 1, 0, 1, 1, 2, 2}=3
e(8) = max {2, 3, 2, 3, 2, 1, 1, 0, 1, 2, 2}=3
e(9) = max {1, 2, 2, 3, 2, 2, 1, 1, 0, 1, 1}=3
e(10) = max {2, 3, 3, 4, 3, 3, 2, 2, 1, 0, 1}=4
e(11) = max {1, 2, 2, 3, 3, 3, 2, 2, 1, 1, 0}=3
e (v) = max {2, 3, 3, 4, 3, 3, 3, 3, 3, 4, 3}=4

To find the Diameter:-

Diam(G) = max {2, 3, 3, 4, 3, 3, 3, 3, 3, 4, 3}=4
Therefore Diam(G) = 4

To find the Radius:-

rad(G) = min{2, 3, 3, 4, 3, 3, 3, 3, 3, 4, 3}=2
rad(G) = 2.
Therefore b(G) < rad(G) < diam(G).

Theorem2:
Let the dominating set of G consists of two vertices only, say a_i and a_j .suppose a_i dominates the vertex set S_1 = {1,2,…….i} and a_j dominates the vertex set S_2 = { i+1,i+2,…..n }. If there is no vertex in S_1 and no vertex in S_2 other than a_j that dominates S_2 then b(G) < rad (G) < diam(G).

Proof:
Let A = {a_i, a_j,……..a_n} } be circular-arc family and let G be a circular-arc graph A . Let D = { a_i, a_j}. Suppose a_i and a_j satisfying the hypothisis of the theorem. Since a_i alone dominates S_1 , thre is no vertex in S_3 = {1, 2, …………i}- {a_i} that can dominate S_2. Let l_i be any vertex in S_3 and edge e = (a_i, l_i) .

Consider the graph G-e. In this graph, a_i dominates every vertex in S_1 except l_i . Now consider a vertex in S_1 which is adjacent with l_i , say m_i. Then clearly the set {a_i, m_i} dominates the set S_1 in G-e. If there is no vertex in S_1 that is adjacent with l_i , then clearly the graph G becomes disconnected. So, there is at least one vertex in S_1 that is adjacent with l_i.

Let us assume that there is a single vertex say x_i , $x_i \neq a_i$ such that x_i dominates the set S_1 in G-e. This implies that x_i also dominates set S_1 in G, a contradiction, because by hypothesis a_i is the only vertex that dominates the set S_1 in G. Hence a single vertex cannot dominate S_1 in G-e. Thus $D_1 = D \cup$ U {m_i} becomes a dominating set of G-e. Since D is minimum in G , D_1 is also minimum in G-e. So that γ (G-e) > γ (G) . Hence the bondage number b(G) = 1. A similar argument with vertex a_j is also gives b(G) = 1. Next we will find the radius of G. Our aim to show that b(G) < rad (G). We have already proved in theorem1. Again we have to show that rad (G) < diam (G). We know that any connected graph in the radius is less than the diameter of G. We have already shown in theorem1.

PRACTICAL PROBLEM FOR THEOREM -2

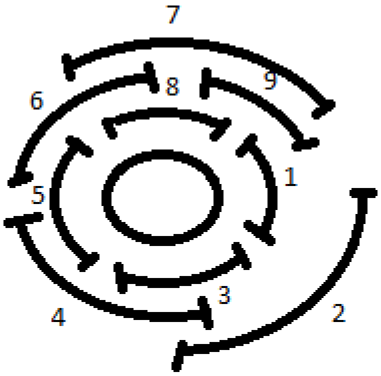

Fig. 4: Circular-arc family A

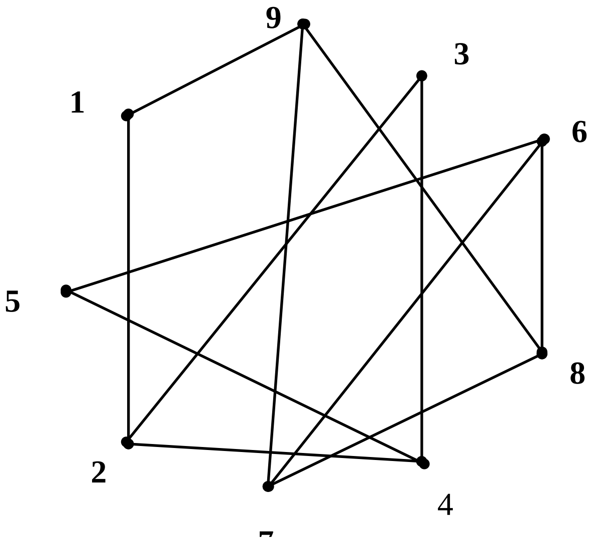

Fig. 5: Circular-arc graph G

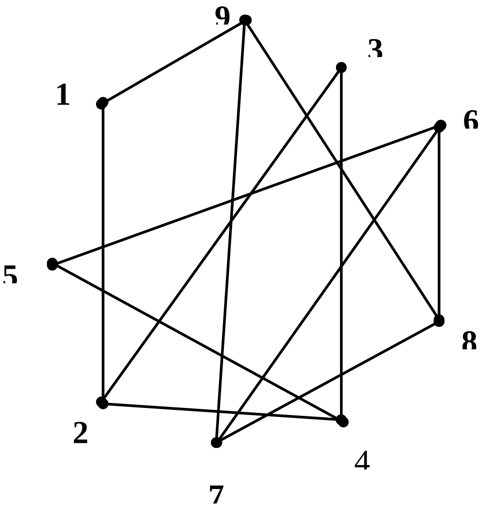

Fig. 6: Circular-arc graph G-e

To find the distances from G:-

d(1,1)=0
d (1,2)=1
d(1,3)=2
d(1,4)=2
d(1,5)=3
d(1,6)=2
d(1,7)=1
d(1,8)=2
d(1,9)=1

d(2,1)=1
d(2,2)=0
d(2,3)=1
d(2,4)=1
d(2,5)=2
d(2,6)=3
d(2,7)=2
d(2,8)=3
d(2,9)=2

d(3,1)=2
d(3,2)=1
d(3,3)=0
d(3,4)=1
d(3,5)=2
d(3,6)=3
d(3,7)=3
d(3,8)=4
d(3,9)=3

d(4,1)=2
d(4,2)=1
d(4,3)=1
d(4,4)=0
d(4,5)=1
d(4,6)=2
d(4,7)=3
d(4,8)=3
d(4,9)=3

d(5,1)=3
d(5,2)=2
d(5,3)=2
d(5,4)=1
d(5,5)=0
d(5,6)=1
d(5,7)=2
d(5,8)=2
d(5,9)=3

d(6,1)=2
d(6,2)=3
d(6,3)=3
d(6,4)=2
d(6,5)=1
d(6,6)=0
d(6,7)=1
d(6,8)=1
d(6,9)=2

d(7,1)=1
d(7,2)=2
d(7,3)=3
d(7,4)=3
d(7,5)=2
d(7,6)=1
d(7,7)=0
d(7,8)=1
d(7,9)=1

d(8,1)=2
d(8,2)=3
d(8,3)=4
d(8,4)=3
d(8,5)=2
d(8,6)=1
d(8,7)=1
d(8,8)=0
d(8,9)=1

d(9,1)=1
d(9,2)=2
d(9,3)=3
d(9,4)=3
d(9,5)=3
d(9,6)=2
d(9,7)=1
d(9,8)=1
d(9,9)=0

To find an Eccentricity:-

$e(i) = \max\{\delta(i, j) : j \in V\}$
e(1) = max {0, 1, 2, 2, 3, 2, 1, 2, 1}=3
e(2) = max {1, 0, 1, 1, 2, 3, 2, 3, 2}=3
e(3) = max {2, 1, 0, 1, 2, 3, 3, 4, 3}=4
e(4) = max { 2, 1, 1, 0, 1, 2, 3, 3, 3}=3
e(5) = max {3, 2, 2, 1, 0, 1, 2, 2, 3}=3
e(6) = max { 2, 3, 3, 2, 1, 0, 1, 1, 2}=3
e(7) = max {1, 2, 3, 3, 2, 1, 0, 1, 1}=3
e(8) = max {2, 3, 4, 3, 2, 1, 1, 0, 1}=4
e(9) = max {1, 2, 3, 3, 3, 2, 1, 1, 0}=3
e(v) = max {3, 3, 4, 3, 3, 3, 3, 4, 3} = 4

To find the Diameter:-

Diam(G) = max {3, 3, 4, 3, 3, 3, 3, 4, 3} = 4
Therefore Diam(G) = 4

To find the Radius:-

rad(G) = min {3, 3, 4, 3, 3, 3, 3, 4, 3} = 3
rad(G) = 3.
Therefore b(G) < rad(G) < diam(G).

Theorem3:

Let G be a circular-arc graph corresponding circular-arc family A. Let D = { a_i, a_j}. suppose a_i dominates $S_1 = \{1,2,\ldots\ldots i\}$ and a_j dominates $S_2 = \{ i+1, i+2,\ldots\ldots n \}$. Suppose there are two vertices say x_i , $y_i \in S_1$ or S_2 such that x_i , y_i also dominates S_1 or S_2 respectively. Then the bondage number b(G) ≤ the radius of G ≤ the diameter of G.

Proof:

Let A = {a_i, a_j, a_k ……..a_n} be the circular-arc family. Let G be circular-arc graph corresponding to circular-arc family A. We have to show that b(G) ≤ rad (G) ≤ diam(G) from the given circular-arc graph. First we will prove that b(G) ≤ rad (G). Let D = { a_i, a_j} and a_i , a_j satisfying the hypothesis of the theorem. Suppose x_i , $y_i \in S_1$ that x_i

, y_i also dominates S_1 . Let z_i be an arbitrary vertex in S_1 , $z_i \neq I$, a_i , x_i , y_i. Now delete the edges a_iz_i , x_iz_i , y_iz_i that are incident with z_i from G. If $d(z_i) = 3$, then z_i becomes an isolated vertex in $G_1 = G-\{ a_iz_i , x_iz_i , y_iz_i \}$. Thus $D_1 = D \cup \{z_i\}$ becomes a dominating set of G_1 and since D is minimum it follows that D_1 is also minimum in G_1. Therefore $\gamma (G_1) > \gamma (G)$, where $\gamma (G\text{-}e) = \gamma (G_1)$ and hence b(G) =3.

Suppose $d(z_i) >3$. Then there is at least one vertex say j in S_1 such that j is adjacent to z_i and$j\neq a_i, x_i , y_i$. Let $G_1 = G-\{ a_iz_i , x_iz_i , y_iz_i \}$. In G_1, z_i is not dominated by a_i, x_i , y_i but is dominated by j. Further every vertex in S_1 other z_i is dominated by a_i or x_i or y_i in G_1.
Therefore every vertex in S_1 is dominated by $\{a_i,j\}$ or $\{x_i,j\}$ or $\{y_i,j\}$ in G_1. Thus $D_1 = D \cup \{j\}$ becomes a dominating set of G_1 and since D is minimum in G it follows that D_1 is minimum in G_1. Hence $\gamma (G_1) > \gamma (G)$. So that the bondage number b(G) = 3.Similar is the case if $w_1, w_2 \in S_2$. Again we have to find the radius of G. Already we have seen the procedure of radius in theorem1. From the statement we can get the radius greater than or equal to the bondage number b(G).

Finally we have to find the radius is less than or equal to the diameter of G. This procedure already proved in theorem1. From the theorem it must be the radius less than the diameter of G. Its symbolic form is $rad(G) \leq diam(G)$. As follows above said that $b(G) \leq rad (G) \leq diam(G)$.
Therefore the theorem is proved from G

Practical problem for theorem -3:-

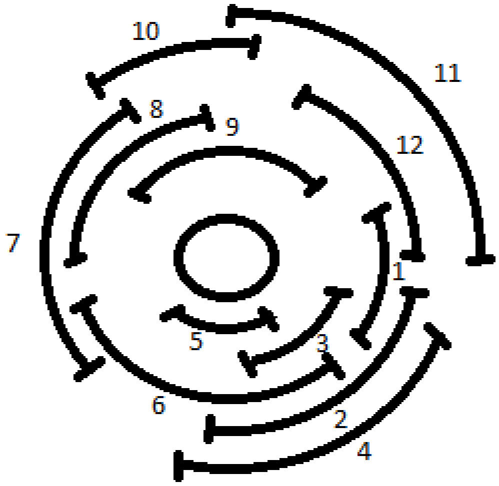

Fig. 7: Circular-arc family A

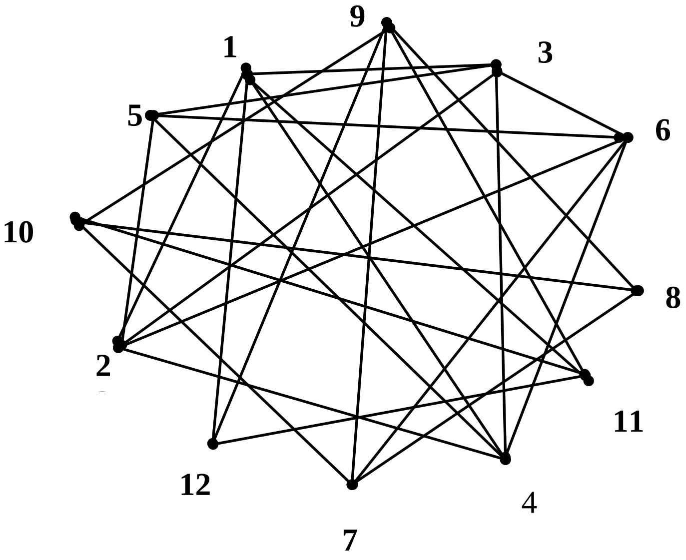

Fig. 8: Circular-arc graph G

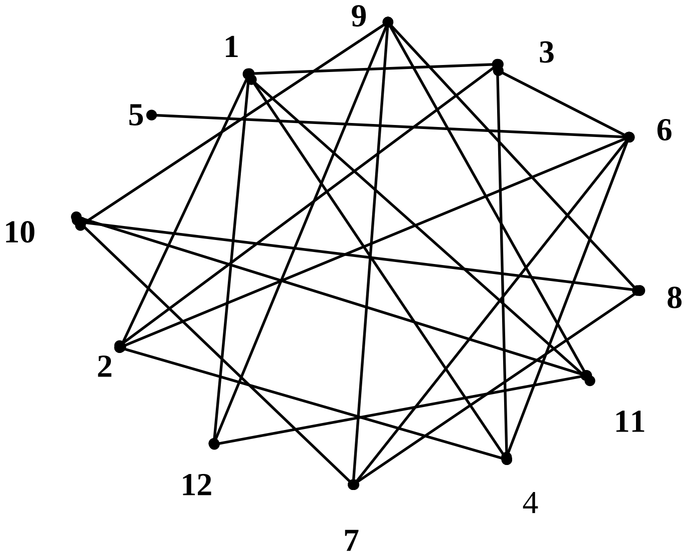

Fig. 9: Circular-arc graph G-e

To find the distances from G

d(1,1)=0
d(1,2)=1
d(1,3)=1
d(1,4)=1
d(1,5)=2
d(1,6)=2
d(1,7)=3
d(1,8)=3
d(1,9)=2
d(1,10)=2
d(1,11)=1
d(1,12)=1

d(2,1)=1
d(2,2)=0
d(2,3)=1
d(2,4)=1
d(2,5)=1
d(2,6)=1
d(2,7)=2
d(2,8)=3
d(2,9)=3
d(2,10)=3
d(2,11)=2
d(2,12)=2

d(3,1)=1
d(3,2)=1
d(3,3)=0
d(3,4)=1
d(3,5)=1
d(3,6)=1
d(3,7)=2
d(3,8)=3
d(3,9)=3
d(3,10)=3
d(3,11)=2
d(3,12)=2

d(4,1)=1
d(4,2)=1
d(4,3)=1
d(4,4)=0
d(4,5)=1
d(4,6)=1
d(4,7)=2
d(4,8)=3
d(4,9)=3
d(4,10)=3
d(4,11)=2
d(4,12)=2

d(5,1)=2
d(5,2)=1
d(5,3)=1
d(5,4)=1
d(5,5)=0
d(5,6)=1
d(5,7)=2
d(5,8)=3
d(5,9)=3
d(5,10)=3
d(5,11)=3
d(5,12)=3

d(6,1)=2
d(6,2)=1
d(6,3)=1
d(6,4)=1
d(6,5)=1
d(6,6)=0
d(6,7)=1
d(6,8)=2
d(6,9)=2
d(6,10)=2
d(6,11)=3
d(6,12)=3

d(7,1)=3
d(7,2)=2
d(7,3)=2
d(7,4)=2
d(7,5)=2
d(7,6)=1
d(7,7)=0
d(7,8)=1
d(7,9)=1
d(7,10)=1
d(7,11)=2
d(7,12)=2

d(8,1)=3
d(8,2)=3
d(8,3)=3
d(8,4)=3
d(8,5)=3
d(8,6)=2
d(8,7)=1
d(8,8)=0
d(8,9)=1
d(8,10)=1
d(8,11)=2
d(8,12)=2

d(9,1)=2
d(9,2)=3
d(9,3)=3
d(9,4)=3
d(9,5)=2
d(9,6)=2
d(9,7)=1
d(9,8)=1
d(9,9)=0
d(9,10)=1
d(9,11)=2
d(9,12)=2

d(10,1)=2
d(10,2)=3
d(10,3)=3
d(10,4)=3
d(10,5)=3
d(10,6)=2
d(10,7)=1
d(10,8)=1
d(10,9)=1
d(10,10)=0
d(10,11)=1
d(10,12)=2

d(11,1)=1
d(11,2)=2
d(11,3)=2
d(11,4)=2
d(11,5)=3
d(11,6)=3
d(11,7)=2
d(11,8)=2
d(11,9)=1
d(11,10)=1
d(11,11)=0
d(11,12)=1

d(12,1)=1
d(12,2)=2
d(12,3)=2
d(12,4)=2
d(12,5)=3
d(21,6)=3
d(12,7)=2
d(12,8)=2
d(12,9)=1
d(12,10)=2
d(12,11)=1
d(12,12)=0

To find an Eccentricity

e (i) = max {δ(i, j) : j ∈ V}
e(1) = max {0, 1, 1, 1, 2, 2, 3, 3, 2, 2, 1, 1}=3
e(2) = max {1, 0, 1, 1, 1, 1,2, 3, 3, 3, 2, 2}=3
e(3) = max {1, 1, 0, 1, 1, 1,2, 3, 3, 3, 2, 2}=3
e(4) = max {1, 1, 1, 0, 1, 1, 2, 3, 3, 3, 2, 2}=3
e(5) = max {2, 1, 1, 1, 0, 1, 2, 3, 3, 3, 3, 3}=3
e(6) = max {2, 1, 1, 1, 1, 0, 1, 2, 2, 2, 3, 3}=3
e(7) = max {3, 2, 2, 2, 2, 1, 0, 1, 1, 1, 2, 2}=3
e(8) = max {3, 3, 3, 3, 3, 2, 1, 0, 1, 1, 2, 2}=3
e(9) = max {2, 3, 3, 3, 3, 2, 1, 1, 0, 1, 1, 1}=3
e(10) = max {2, 3, 3, 3, 3, 2, 1, 1, 1, 0, 1, 2}=3
e(11) = max {1, 2, 2, 2, 3, 3, 2, 2, 1, 1, 0, 1}=3
e(12) = max {1, 2, 2, 2, 3, 3, 2, 2, 1, 2, 1, 0}=3
e(v) = max {3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3}=3

To find the Diameter

Diam(G) = max {3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3}=3
Therefore Diam(G) = 3

To find the Radius

rad(G) = min {3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3} = 3
rad(G) = 3.
Therefore b(G) ≤ rad(G) ≤ diam(G)

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