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**RESEARCH ARTICLE**

# A WAVELET BASED APPROACH FOR MULTI-SCALE ANALYSIS AND PREDICTABILITY OF STOCK RETURNS.

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## *Abstract*

The aim of this paper is to demonstrate the effectiveness of alternative methods from the time-frequency domain in analyzing the behavior of financial time series. Multiresolution analysis of the discrete wavelet transform class is used to transform the BSE returns series into different sets of coefficients where each set contains coefficients that provide information corresponding to a particular time-scale resolution. The main coefficient series from the transformed time-series is used to make forecasts and the result is then compared with the result when forecasting is done directly using the original returns data. On the analyzed returns data we proved that forecasting using the wavelet transformed algorithm provides much accurate results as compared to forecasting applied directly to the original returns series The effectiveness of a wavelet based cross-correlation technique in analyzing the relation between two markets at different levels of time-frequency resolution is also demonstrated. This approach of multi-scale decomposition of a time-series using wavelet methodology allows us to detect changes in stock market behavior from a time-scale perspective where the data can be analyzed at different time horizons and frequencies simultaneously.



## Introduction:-

The decomposition of a time-series using wavelet filters helps us in breaking down the series into different levels, containing coefficients of the transformed series, with each level providing localization of the series in time and scale simultaneously. This decomposition is done via multiresolution analysis, which is an additive decomposition that helps in breaking down the time-series into components with varying levels of resolution with each level containing information in both time and scale (and frequency). Wavelet methods, therefore, are most suitable for the analysis of non-stationary financial and economic time-series due to its capability of breaking down the information into different layers of resolution and its time-scale localization properties. Moreover, wavelets are very handy in spotting the exact location in time of regime shifts, discontinuities, and isolated shocks to the dynamical system, Ramsey (1998). The capability of wavelet analysis to decompose a time-series on different time scales and at the same time preserve time localization is one of the main reasons for its induction into economic and financial research. One possesses a better understanding about the time-series and the dynamic market mechanisms behind the time-series by analyzing the time-series at different levels of resolution. This framework of analysis also allows us to isolate many interesting structures and other features of economic and financial time-series, which previously would not have been possible by the use of traditional time domain and Fourier based methods.

The increasing interest, in wavelet analysis, by economic researchers, and its applicability in areas like time-scale decomposition, forecasting, density estimation etc. have led to the emergence of various wavelet based techniques for the analysis of non-stationary financial time-series, Crowley (2005).

Wavelet based multiresolution analysis is ideal for the analysis of high frequency data generated by financial markets, providing valuable information for trading decisions, as the analyst can focus on a particular time scale where trading patterns are considered important. Therefore, wavelet analysis has tremendous potential in economics and finance, as relationships between different variables can be analyzed in time-frequency space, allowing one to analyze the relationships between variables at different frequencies and, simultaneously, the corresponding information about the evolution of a variable in time.

The application of wavelet theory was limited to the analysis of deterministic functions, as most of it was applied in the areas of engineering and the natural sciences. The application of wavelet analysis to study the behavior of stochastic processes, which characterize the underlying system in economics and finance, is relatively new. In the next section we review some of the important contribution of wavelet based methods in analyzing financial time-series.

## Literature review:-

The Nineties saw the introduction of wavelet based approaches in statistics. Nason and Silverman (1994) introduced discrete wavelet transforms for statistical applications. Percival and Walden (2000), provides a detailed introduction to wavelets methods for time-series analysis. The maximal overlap discrete wavelet transform (MODWT), Percival and Walden (2000), is particularly suitable in analyzing economic and financial data. This method is a modification of the discrete wavelet transform where the transform loses the property of orthogonality, but since it has the ability to analyze non-dyadic processes; it is very much suited for the analysis of financial time-series.

The application of wavelet methods, particularly in the field of economics and finance, is described by Gencay et al. (2001). High frequency foreign exchange rates were analyzed by Ramsey and Zhang (1995) using waveform dictionaries and a matching pursuit algorithm. Ramsey and Lampart (1998) found that the relationship between money and income varies according to scale. At higher scale levels, money supply Granger caused income and at lower scale, income granger caused money supply.

The multiresolution analysis of high frequency Nikkei stock market data, using the matching pursuit algorithm of Mallat and Zhang (1993), is carried out by Capobianco (2004). Hidden periodic components are unearthed using the algorithm. Maximal overlap discrete wavelet transform is applied by Crowley and Lee (2005) to analyze the frequency components of European business cycles. Date from countries with lesser degree of integration exhibited non-similar frequency components. The lead-lag relationship between the Dow Jones Industrial Average stock price series and the index of industrial production series of the US is analyzed by Gallegati (2008), using wavelet correlation and cross-correlation methods. The comovements between the stock markets of the US, Germany, UK and Japan were analyzed by Rua and Nunes (2009) using wavelet coherence analysis. Market interdependencies were found to change across frequencies and along the time horizon. Strongest comovements were observed between the markets of US and Europe, and the coherence between US-Germany and UK-Germany increased in time.

Conraria and Soares (2011) study business cycle synchronization across the European union-15 and Euro-12 countries using wavelet analysis. France and Germany are found to be highly synchronized with other European countries and French business cycle leads German business cycle as well as the business cycles from the rest of the European countries.
The next section gives a brief review of the methodology used in this analysis which will be followed by its use in analyzing equity prices, some empirical evidences and conclusions.

## Methodology:-

Wavelet analysis begins with the consideration of a function known as the mother wavelet, which is given by,

$$\psi_{a,b}(t) = \frac{1}{\sqrt{a}} \psi\left(\frac{t-b}{a}\right) \quad (1)$$

Where $a \neq 0$ and b are real constants. The parameter a is known as the scaling parameter which determines window widths, whereas the parameter b known as the translation parameter determine the position of the window. Unlike

Fourier analysis, a number of mother wavelets can be chosen depending upon the problem at hand. Just like in Fourier analysis, where a function is expressed as a combination of sines and cosines to transform the function into the spectral domain, wavelets can be used to project a function onto the time-frequency domain.

The scaling parameter a is typically taken to be a power of two so that $a = 2^j$ for some integer j. The compressed wavelet captures the finer scale resolution (high frequency components are captured and well localized in time) of a given signal while the dilated wavelet (broad time window widths) captures low-frequency components of a signal by having a broad range in time. Thus scaling and translation, which allow adjustment of time window widths and their locations, are the most fundamental operations which enable go for higher and higher refinements in terms of time and frequency resolutions. Thus, the scaling and translation operations facilitate a given signal or function to be represented as a basis function which in turn allow for higher and higher refinement in the time resolution of a signal.

Since the large scale structures of a given signal in time are captured with broad time-domain wavelets, in which case the window width is broad (spread out in the time axis), the time resolution of the signal is very poor and captures only the low frequency components. However, finer and finer time resolution of the signal along with its high frequency components can be obtained by successive rescaling of the dilation parameter in time. The information at low and high scales is all preserved so that a complete picture of the time-frequency domain can be constructed. Ultimately, the only limit in this process is the number of scaling levels to be considered.

In the case of spectral analysis, a signal is taken and projected into the space of sines and cosines. Similarly a function can be represented in terms of the wavelet basis. The wavelet basis can be accessed via the integral transform of the form

$$\int_t K(t, \omega) f(t) dt$$

(2)

where $K(t,\omega)$ is the kernel of the transform and f(t)is a time domain signal. The above equation represents any generic transform where a modification in the kernel gives the required transform. In the case of Fourier transform the kernel $K(t,\omega) = \exp(-i\omega t)$ represents the periodic oscillations. The key idea now is to define a transform which incorporates the mother wavelet as the kernel. Thus we define the continuous wavelet transform (CWT) as:

$$W_\psi[f](a,b) = (f, \psi_{a,b}) = \int_{-\infty}^{\infty} f(t)\overline{\psi}(t)\, dt$$

(3)

Where $W_\psi[f](a, b)$ is the CWT which is a function of the dilation parameter a and the translation parameter b, f and $\psi_{a,b}$ are as defined in equations (1) and (2). The CWT given in equation (3) should satisfy the following admissibility condition:

$$C_\psi = \int_{-\infty}^{\infty} \frac{|\widehat{\psi(\omega)}|}{|\omega|} d\omega < \infty$$

(4)

where $\widehat{\psi}(\omega)$, the Fourier transform of the wavelet is defined as:

$$\widehat{\psi} = \frac{1}{\sqrt{|a|}}\int_{-\infty}^{\infty} e^{-i\omega t}\psi\left(\frac{t-b}{a}\right)dt = \frac{1}{\sqrt{|a|}} e^{-ib\omega}\,\widehat{\psi}(a\omega)$$

(5)

The wavelet transform given in equation (3) is well defined subject to satisfying the admissibility condition given in equation (4). An important property of the wavelet transform is its ability to construct new wavelet bases

The working principle of wavelet transform is quite simple. A bunch of smaller signals are extracted from the main signal by translating (shifting time window location) the wavelet with parameter b over the entire time domain of the signal. Further, the scaling process is carried out where the same signal is processed at different frequency bands, or resolution, by scaling the wavelet window with the parameter a. This combination of translation and scaling allows for processing of signals at different times and frequencies which in turn allows reading the signal at different scales of time and frequency resolutions. The above process of analyzing a given signal or function at different scales of resolution is termed as multi resolution analysis.

## Multiresolution Analysis (MRA):-

Unlike Fourier analysis, where the frequency spectrum is generated from the time signal and resolution in time is completely lost, wavelet makes it possible to extract multi-scale information enabling us to analyze the function both in time and frequency, simultaneously. First, the signal is analyzed using a broader time window, which generates better resolution in frequency (also sometimes referred to as coarse scale resolution). This window is now translated (slid across the whole signal domain), and hence, the coarse scale resolution is extracted out of the signal. Next, the width of the time window is narrowed down, and by using translation operation, the next level of time-frequency resolution is extracted out of the signal. Every time dilation takes place, we are moving towards finer and finer scale resolution in time at the expense of poor resolution in frequency. This tradeoff between time and frequency is due to the Heisenberg uncertainty principle i.e. the generation of complete information about time and frequency, simultaneously, is not possible. Therefore, wavelet allows us to pull out scales of different resolution, enabled by the implementation of dilation and translation on the mother wavelet window. Fine window (smaller window widths) extracts high frequency content with good time localization and broad window extracts lower frequency content with reduced time localization.

### The Discrete Wavelet Transform (DWT):-

Let $x$ be a dyadic length $(N = 2^j)$ vector of observations. The length N vector of discrete wavelet coefficients $\mathbf{w}$ is obtained via

$$\mathbf{w} = \mathcal{W}\mathbf{x}$$

Where $\mathcal{W} = [\mathcal{W}_1, \mathcal{W}_2, ......\mathcal{W}_J, \mathbf{V}_J]^T$ $\mathcal{W}$ is an $N \times N$ orthonormal matrix defining the DWT. The vector of wavelet coefficients are organized into J+1 vectors,

$$w = [w_1, w_2, ......., w_J, v_J]^T$$

where $w_j$ is a length $N/2^j$ vector of wavelet coefficients associated with changes on a scale of length $\lambda_j = 2^{j-1}$ and $v_j$ is a length $N/2^J$ vector of scaling coefficients associated with averages on a scale of length $2\lambda_J$ . The matrix $\mathcal{W}$ is composed of the wavelet and scaling filter coefficients arranged on a row-by-row basis. The structure of the $N \times N$ matrix $\mathcal{W}$ looks like

$$\mathcal{W} = [\mathcal{W}_1, \mathcal{W}_2, .......\mathcal{W}_J, \mathbf{V}_J]^T$$

Wavelet coefficients are obtained by projecting the wavelet filter onto a vector of observations.
An additive decomposition of the series is formulated by using the DWT which gives us a multiresolution analysis (MRA) of the series. Let $d_j = \mathcal{W}^T \mathbf{w}_j$ for j=1…..J , define the jth level wavelet detail associated with changes in $\mathbf{x}$

at scale $\lambda_j$ . The series can be decomposed into different levels of details up to level J+1. The last element in the MRA decomposition is known as wavelet smooth while the remaining elements are known as wavelet rough; see Percival and Walden (2000) for a step by step computation of DWT using pyramid algorithm and MRA decomposition.

Following Grinsted et al. (2004), the cross wavelet transform (XWT) of two time series $x_n$ and $y_n$ is defined as $W^{XY} = W^X W^{Y*}$ , where * denotes complex conjugation. The absolute value of $W^{XY}$ , is then defined as the cross wavelet power and the phase angle between $x_n$ and $y_n$ in the time frequency space is given by the complex argument $\arg(W^{XY})$. The maximal overlap discrete wavelet transform (MODWT) is first applied to the time series. It is different from the standard wavelet transform in the sense that the discrete wavelet transform, a discrete analogue of the CWT, restricts the sample size T to a multiple of $2^J$ , where J is the level of DWT. MODWT can handle sample size of any length, therefore it is suitable for financial applications. The methodology developed by Javier Fernández-Macho (2012), is applied for calculating Wavelet correlation and cross correlations.

## Data:-

MRA decomposition is performed on a BSE returns series ranging from 01-07-1997 to 20-01-2014 comprising of 4096 observations which corresponds to a dyadic length of $N = 2^{12}$ . The length of the series was chosen so as to satisfy the dyadic length criteria for DWT computation.

## Empirical analysis:-

A multiscale decomposition of the dyadic length time-series comprising of BSE stock returns and volatility is carried out using the discrete wavelet transform. The level of decomposition is chosen to be J=4 giving us four details and one smooth series where both Haar and the least asymmetric (LA8) Daubechies filter are applied for the DWT decomposition of the series. Volatility clustering is observed at almost all four levels of decomposition with a significant clustering of wavelet crystals in the interval $[2700, 3000]$ (see figure 1 and figure 2) which correspond to the market crisis period between 2007 and early 2010. Figure 3 gives the energy distribution of the wavelet crystals comprising of four details and one scaling coefficent. High contribution to the total energy of the series is observed at levels one and two where significant clustering of volatility is present. This localisation of information at both time and scale helps heterogenous investors who operate at different time-scales.

The use of wavelet coefficents for forecasting purpose is demonstrated by comparing the forecasting accuracy of ARIMA models applied to wavelet coefficents with the model used directly to forecast the BSE return series. Scaling coefficents of the DWT decomposed series are extracted and used to analyze the forecasting accuracy . ARIMA(2,0,2) is the model which is selected for both the transformed series and the original returns series.

Table 1 Statistical criteria for ARIMA model

| Accuracy Criteria | Haar DWT and ARIMA | Daubechies DWT and ARIMA | Original BSE returns and ARIMA |
|---|---|---|---|
| Mean squared error (MSE) | 0.04060857 | -0.0504532 | 0.0271797 |
| Root mean squared error (RMSE) | 0.2419162 | 0.07319073 | 0.9916023 |
| Mean Absolute error (MAE) | 0.1907235 | 0.06492425 | 0.7278562 |

It is evident from Table 1 that forecasting accuracy is much better when wavelet transformed series is used in the ARIMA model as compared to the model used directly with the returns series. The root mean squared error (RMSE) is less in both Haar and Daubechies series where forecasting using the original series gives higher RMSE values.

This clearly demonstrates that the application of wavelet based smoothing filters to stock returns series leads to better forecasting accuracy.

Interdependence between markets can be studied using wavelet cross-correlation analysis. Wavelet cross correlation is performed between DAX (German stock market) and CAC (French stock market), with leads and lags up to 36 months. The variable that maximizes the correlation, as against the other variable, is shown in the upper-left portion of Figure 4. Wavelet cross correlation analysis at level 1 and level 2 reveal some significant correlation at lags 10, 11 and 12. As we increase the resolution level, there seems to be a slight increase in cross correlations with correlations oscillating between zero lag up to one year in the future. Statistically significant cross correlations are observed after increasing the level of decomposition, with strong cross correlations at level 5 and level 6, for almost all leads and lags. This suggests weak but positive correlations between DAX and CAC when the analyses are performed at smaller levels of resolution (intra–week and weekly period), but higher cross correlations when time scales are increased for analyzing the data at quarterly, biannual and annual periods. This gives us some evidence about good market integration between DAX and CAC.

Figure 1. Haar DWT decomposition of BSE returns series:-

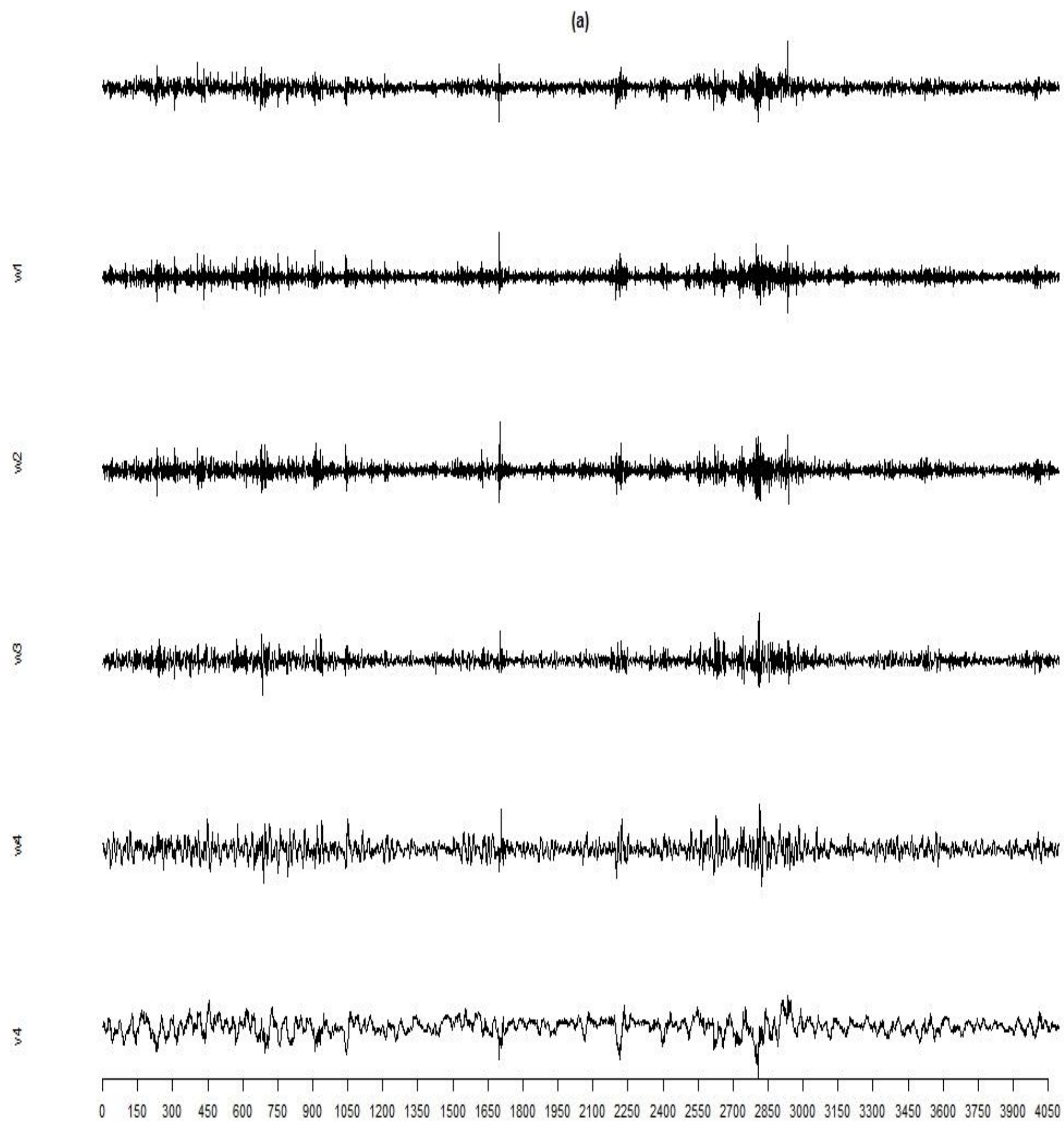


Figure 2. Daubechies DWT decomposition of BSE returns series:-

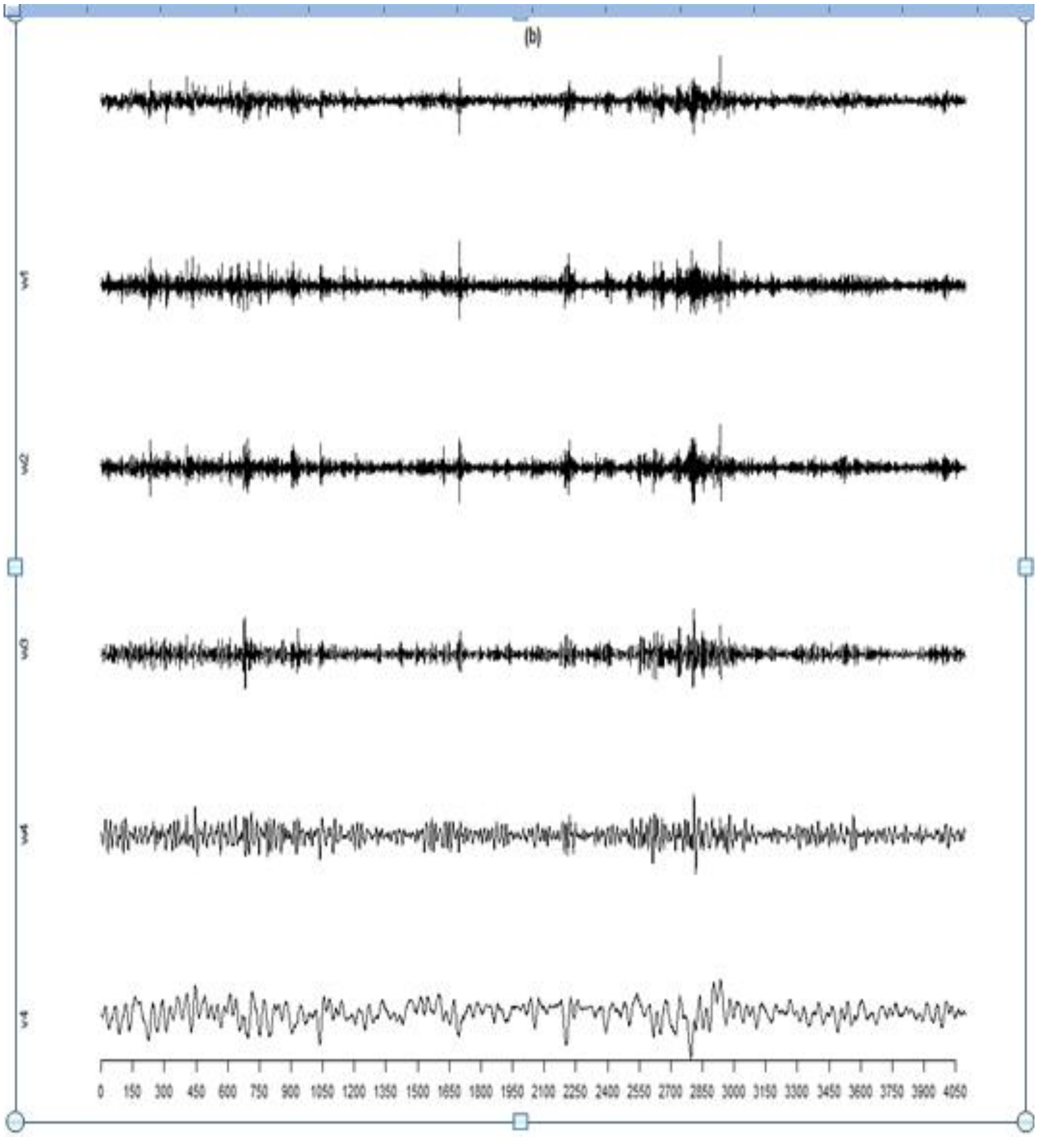


Figure 3. Energy Distribution of Wavelet coefficients at different levels:-

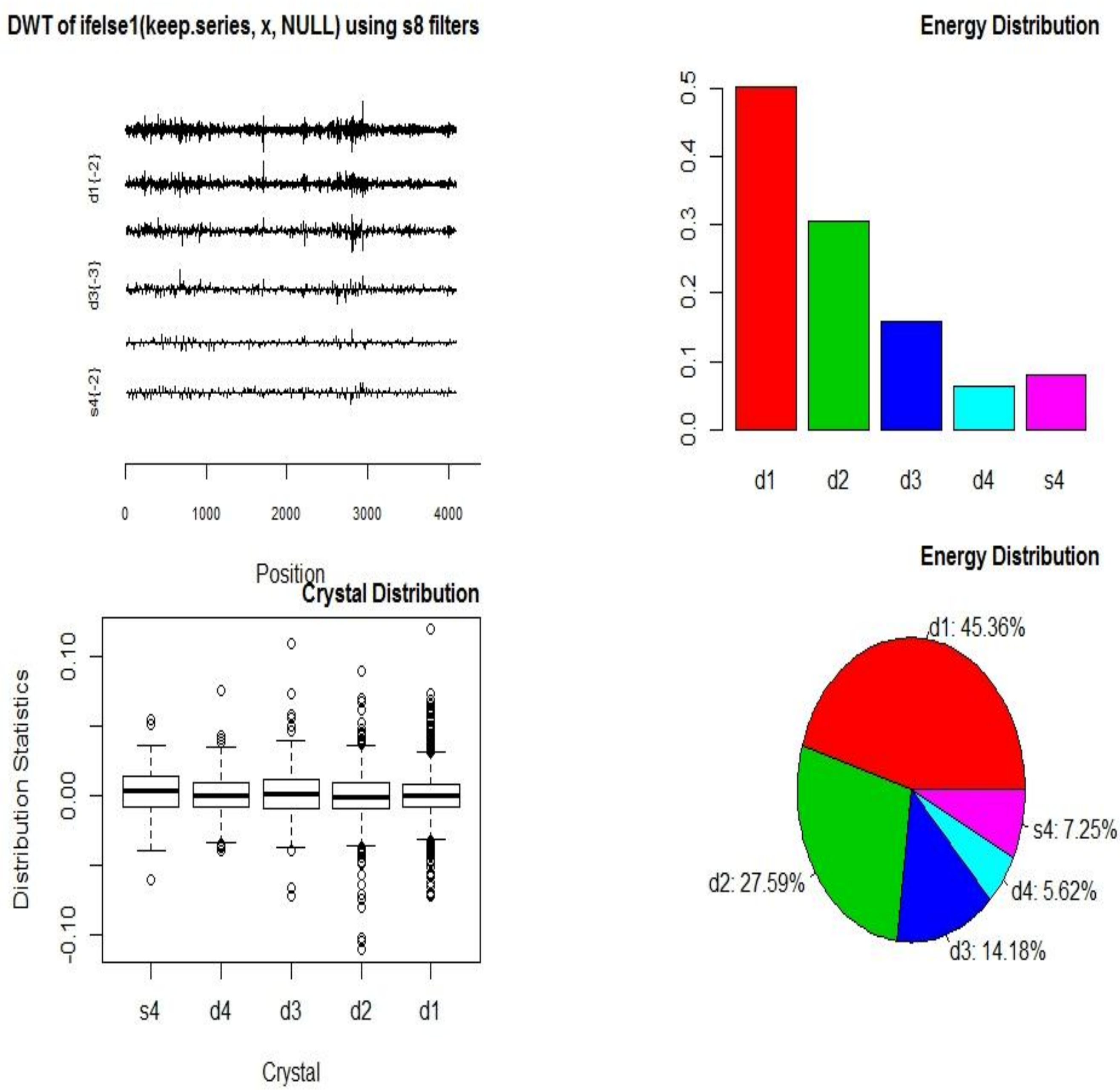
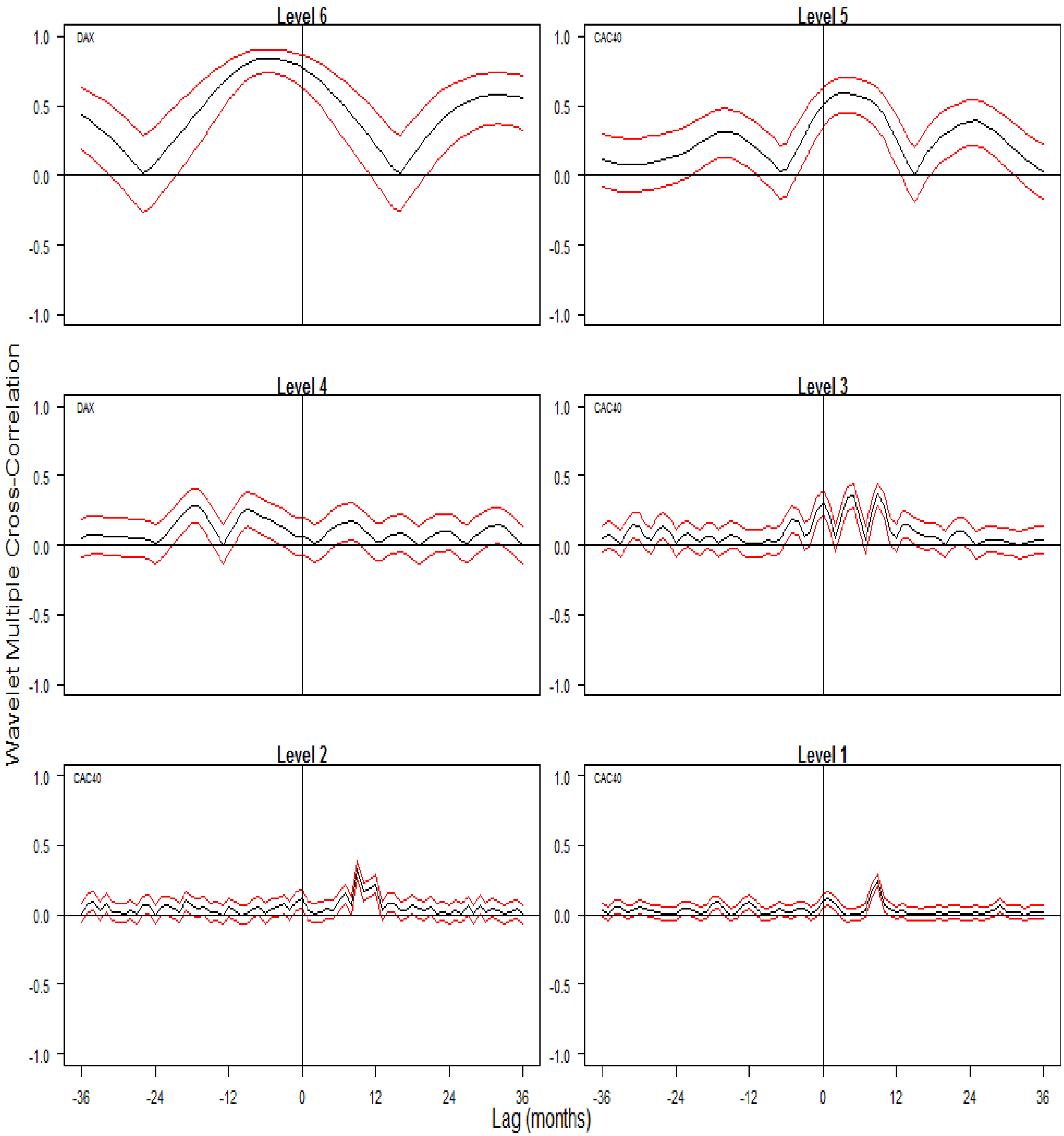


Figure 4. Wavelet Cross-Correlation between DAX and CAC:-




## Conclusion:-

Spectral methods project the time series data into a set of sines and cosines, where the output signal is a function of frequency only. As a result the time information is completely lost. This problem is resolved by using wavelets based decomposition where the output signal is a function of both time and scale, providing us with simultaneous information from both time and frequency domains. The dynamics of stock returns can be studied by decomposing the stock returns into several layers of time-scale resolution (i.e. Short time-scale analysis to long time-scale), which can provide useful insights for investors with different trading horizons in mind. Time-scale decomposition of the original series into different sets of wavelet coefficients associated with different scales makes it possible for the researcher to select the time-scale of interest by looking at the contribution of energy by coefficients at a particular scale to the total energy in the series. Moreover, forecasting using the transformed series generates more accurate results when compared to forecasting using the original series. We also are able to study market interdependence from a time-scale perspective by using wavelet cross-correlation methods. The analysis of stock returns at different multiscale resolution makes it easier for agents dealing with different trading horizons to make decisions based on the time-scale of interest.

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