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## RESEARCH ARTICLE

## NUMERICAL ANALYSIS OF MHD FLOW OF VISCOUS FLUID BETWEEN PARALLEL POROUS BOUNDING WALLS.

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### Abstract

This study deals the numerical analysis of MHD flow of viscous fluid between parallel porous walls in steady state flow of fluid in the presence of transverse magnetic field when the fluid is being withdrawn through porous bounding walls of a channel at the same rate. A solution for the case of small (suction) Reynolds Number  $Re$ , Hartman Number  $M$ , width of the channel is discussed. The expression for the velocity components and the pressure are obtained. The governing differential equations are solved by using perturbation method and the graphs of axial and radial velocity profiles have been drawn.

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### Nomenclature:-

$h$	Height of the channel;
$x$	The axial distance from the channel entrance;
$y$	The coordinate axis perpendicular to the channel walls measured from the Non-porous wall;
$u$	Velocity component in the $x$ -direction;
$u_0$	Average velocity over the channel at channel inlet;
$Re_w$	Wall Reynolds number, $\frac{v_w h}{\nu}$ ;
$Re_{ent}$	Reynolds Number for flow entering the channel, $u_0 h/\nu$ ;
$v$	Velocity component in the $y$ - direction;
$v_w$	Velocity of the fluid through the membrane;
$V$	$v/v_w$ ;
$B$	Magnetic Field
$M$	Hartman Number

### Greek symbols:-

$\rho$	Solution density;
$\lambda$	Dimensionless variable in the $y$ – direction, $y/h$ ;
$\mu$	Viscosity;
$\nu$	Kinematic viscosity;
$\alpha$	Surface characteristic of the membrane;
$\Delta p$	Dimensionless pressure drop, $2[p(0, \lambda) - p(x, \lambda)]/(\rho u_0^2)$ ;
$\psi$	Stream Function;

**Introduction:-**

The Numerical study of flow of an electrically conducting incompressible viscous fluid through a channel of permeable wall not only possesses a theoretical appeal but also model many biological and engineering problems such as magneto hydrodynamics (MHD) generators, plasma studies, nuclear reactors, geothermal energy extraction, the boundary layer control in the field of aerodynamics, blood flow problem etc. Following has studied the MHD flow of fluid such as Abo-El-Dhat (1993), has studied Hartmann flow with uniform suction and injection at the bounding plates. Kuznetsov (1997) has studied Analytical investigation of the fluid flow in the interface region between a porous medium and a clear fluid in channels partially filled with a porous medium. Alpher (1961) has studied Heat Transfer in Magneto hydro Flow Between plates. A. Raptis et. al (1987), has studied Hydro magnetic free-convective flow through porous media. Abdou et. al (2007), has studied Effect of mixed thermal boundary conditions and magnetic field on free convective flow about a cone in micropolar fluids. B Prasad et. al (2011), has studied Flow of Hydro magnetic Fluid through porous media between permeable Beds under Exponentially Decaying Pressure Gradient. B.P. Garg and K.D. Singh (2014), has studied Hydro-Magnetic mixed convection flow through porous medium in a Hot vertical channel with span wise Co sinusoidal temperature and Heat Radiation. Beavers G.S. and D.D. Joseph (1967), has studied Boundary conditions at a naturally permeable wall. Beavers et. al (1970), has studied Experiments on coupled parallel Flows in a Channel and a Bounding porous medium. Bujurke et. al (2010), has studied Analysis of Laminar flow in a channel with one porous bounding wall. Cox S. M. (1991), has studied Analysis of Steady Flow in a Channel with One Porous Wall or with Accelerating Walls. D.A. Nield, (1994), has studied modeling high speed flow of an incompressible fluid in a saturated porous medium.

J. A. Ochoa-Tapia and S. Whitaker (1995a, 1995b), has studied Momentum transfer at the boundary between a porous medium and a homogeneous fluid I and II. Lage (1992), has studied Effect of the convective inertia term on Bonnard convection in a porous medium. Lage (1998), has studied The fundamental theory of flow through permeable media from Darcy to turbulence. J.R. Sellers (1955), has studied Laminar flow in channels with porous walls at high suction Reynolds Numbers. K. Vafai and S. J. Kimi (1990), has studied Fluid mechanics of an interface region between a porous medium and a fluid layer-an exact solution. Kishan Naikati and Meenakshi Vadithya (2014), has studied thermal Radiation Effects on Magneto Hydrodynamic Flow and Heat Transfer in a Channel with Porous walls of Different Permeability. P.K. Mahanta (2012) has studied Numerical study on Heat transfer of Non-Newtonian Fluid flow over Stretching surface with variable viscosity in uniform magnetic Field. M. Jafaryar et. al (2014), has studied Analytical investigation of laminar flow through expanding or contracting gaps with porous walls. Mohamed Y. Abou Zeid (2009), has studied Numerical treatment of heat and mass transfer of MHD flow of carreau fluid with Diffusion and chemical reaction through a Non-Darcy porous medium. Hossain et. al (1996), has studied Radiation effect on mixed convection along a vertical plate with uniform surface temperature. M. Hosseini et. al (2013), has studied Non-Newtonian fluid flow in an axisymmetric channel with porous wall. N. T. El-Dabe et. al (2013), has studied Peristaltic transport of a magneto non-Newtonian fluid through a porous medium in a horizontal finite channel. Bujurke et. al (2010), has studied Analysis of Laminar flow in a channel with one porous bounding wall. Nigam et. al (1961), has studied Heat Transfer by Laminar Flow Between Parallel plates. Makinde and E. Osalusi (2006), has studied MHD Steady flow in a channel with slip at the permeable boundaries.

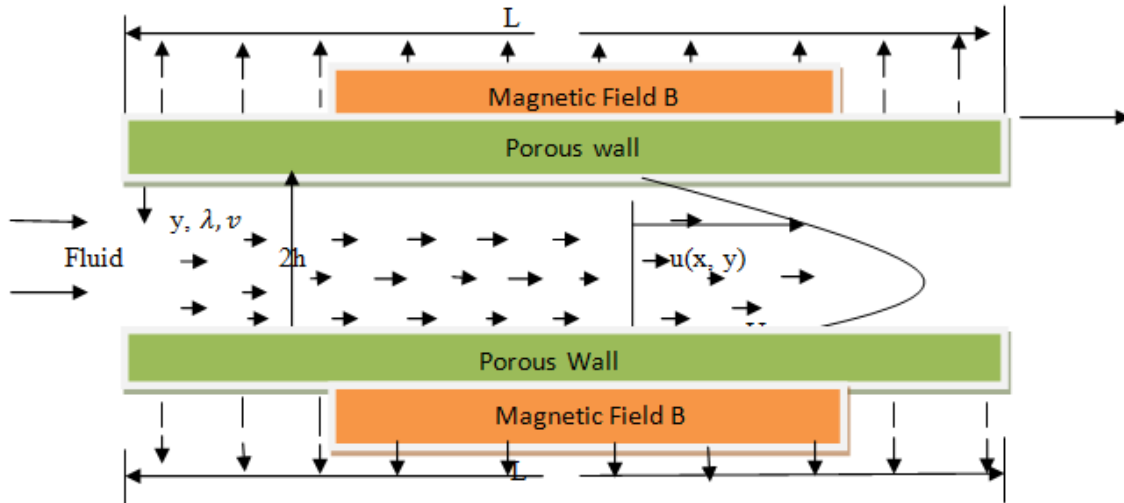
Robinson W. A (1976), has studied The Existence of Multiple Solutions for the Laminar Flow in a Uniformly Porous Channel with Suction at Both Walls. Robinson W. A (1976), has studied The Existence of Multiple Solutions for the Laminar Flow in a Uniformly Porous Channel with Suction at Both Walls. R. Muthuraj and S. Srinivas (2009), has studied Influence of Magnetic field and wall slip conditions on steady flow between parallel flat wall and a long wavy wall with sores effect. Sato H. (1961), has studied The Hall Effect in the viscous flow of ionized gas between parallel plates under transverse magnetic field. Saffman P. G. (1971), has studied On the Boundary condition at the Surface of a Porous medium. S. Ganesh and S. Krishnambal (2006), has studied Magneto hydrodynamics Flow of Viscous fluid between two parallel plates. Srinamulu A. et. al (2003), has studied Magneto hydrodynamic Flow with suction or Injection at the Forward Stagnation Point of an infinite plane wall. Terrill, R. M (1964), has studied Laminar Flow in a Uniformly Porous Channel. Terrill, R. M (1965), has studied Laminar Flow through a channel with uniformly Porous Walls of Different Permeability. Terrill R. M (1968), has studied Laminar Flow with Large Injection Through parallel and Uniformly Porous Walls of Different Permeability. Y. Takatsu and T. Masuka (1998), has studied turbulent phenomena in flow through porous media.

The objective of this paper is to study the combined effect of magnetic field and Reynolds numbers, width of the channel on the steady flow of an electrically conducting fluid in a channel of uniform width. Using Berman (1953) similarity approach, the governing Navier Stokes equations is reduced to a non-linear boundary value fourth order

parameter dependent ordinary differential equations and semi-numerical technique. In the following sections, the problem is formulated, analyzed and discussed.

**Mathematical formulation:-**

The study of MHD flow of viscous fluid between two parallel porous bounding walls is considered in the presence of a transverse magnetic field of strength  $H_0$  applied perpendicular to the walls. The origin is considered at the centre of the channel and let  $x$  and  $y$  be the coordinate axes parallel and perpendicular to the channel walls. The length of the channel is assumed to be  $L$  and  $2h$  is the distance between the two parallel porous walls,  $u$  and  $v$  be the velocity components in the  $x$  and  $y$  directions, respectively.



**Fig. 1:** The coordinate system used in the solution of the two dimensional steady State flow of viscous fluid.

The equation of continuity is

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

The momentum equations are

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) - \frac{\sigma B^2}{\rho} u \tag{2}$$

$$u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial y} + \nu \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) - \frac{\sigma B^2}{\rho} v \tag{3}$$

Where  $\sigma$  is electrical conductivity and  $B = \mu_e H_0$  and  $\mu_e$  being the magnetic permeability.

Let  $\lambda = y/h$ , putting it in the above equation

We get 
$$\frac{\partial u}{\partial x} + \frac{1}{h} \frac{\partial v}{\partial \lambda} = 0 \tag{4}$$

$$u \frac{\partial u}{\partial x} + \frac{v}{h} \frac{\partial u}{\partial \lambda} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \left( \frac{\partial^2 u}{\partial x^2} + \frac{1}{h^2} \frac{\partial^2 u}{\partial \lambda^2} \right) - \frac{\sigma B^2}{\rho} u \tag{5}$$

$$u \frac{\partial v}{\partial x} + \frac{v}{h} \frac{\partial v}{\partial \lambda} = -\frac{1}{\rho h} \frac{\partial p}{\partial \lambda} + \nu \left( \frac{\partial^2 v}{\partial x^2} + \frac{1}{h^2} \frac{\partial^2 v}{\partial \lambda^2} \right) - \frac{\sigma B^2}{\rho} v \tag{6}$$

Where,  $\nu$  is the kinematic viscosity,  $p$  the Pressure,  $\rho$  the density of the fluid

The boundary conditions are

$$u(x, h) = 0, \quad u(x, -h) = 0 \tag{7}$$

$$v(x, h) = V \text{ and } v(x, -h) = -V \tag{8}$$

Where,  $V$  is the velocity of suction at the walls of the channel.

The boundary conditions are converted into

$$u(x, 1) = 0, \quad u(x, -1) = 0, \tag{9}$$

$$v(x, 1) = V \text{ and } v(x, -1) = -V \quad (10)$$

$$\text{Introducing the stream function } \psi(x, y), \text{ we know that } u = \frac{\partial \psi}{\partial y} \text{ \& } v = -\frac{\partial \psi}{\partial x} \quad (11)$$

$$\text{Introducing the dimensionless variable in equation (7) we get } u = \frac{1}{h} \frac{\partial \psi}{\partial \lambda} \text{ \& } v = -\frac{\partial \psi}{\partial x} \quad (12)$$

The equation of continuity can be satisfied by a stream function of the form

$$\psi(x, \lambda) = (h u_0 - v_w x) f(\lambda) \quad (13)$$

From Navier Stokes equation (5), (6) and eliminating p we get

$$\frac{\partial}{\partial \lambda} \left[ \frac{v_w}{h} (f'^2 - f f'') - \frac{\sigma B^2 u}{\rho} f' + \frac{v}{h^2} f''' \right] = 0 \quad (14)$$

Which is true for all x, Let Re be the Reynolds Number of the suction and M be the Hartmann Number.

Then we get the expression

$$Re = hV/v \text{ \& } M = Bh \left( \frac{\sigma}{\nu \rho} \right)^{1/2} \quad (15)$$

Integrating the above equation (14) w.r.t '  $\lambda$  ' we get the above expression in the form

$$Re \left[ (f'^2 - f f'') - \frac{H_0^2 \mu_e^2 \sigma h}{\rho \nu} f' \right] + f''' = A \quad (16)$$

Or

$$Re \left[ (f'^2 - f f'') - \xi f' \right] + f''' = A \quad (17)$$

Where,  $\xi = H_0^2 \mu_e^2 \sigma h / \rho \nu$  and A is an arbitrary Constant

The above differential equation is Non-linear differential equation and solving by using perturbation method. The boundary condition of f( $\lambda$ ) are

$$\left. \begin{aligned} f(1) = 1, f(-1) = -1 \\ \text{and } f'(1) = f'(-1) = 0 \end{aligned} \right\} \quad (18)$$

We solve third order Non-linear differential equation (17) Subject to the boundary conditions (18).

### Method of solution:-

We solve the non-linear ordinary differential equation (17), subject to condition (18). However for special case when Re small,  $\xi$  small and large, the approximate analytical results can be obtained by use of a regular perturbation approach. In this situation f may be expanded in the form

$$\text{Let } f(\lambda) = f_0(\lambda) + Re f_1(\lambda) + Re^2 f_2(\lambda) + Re^3 f_3(\lambda) + \dots = \sum_{n=0}^{\infty} Re^n f_n(\lambda) \quad (19)$$

And

$$A = A_0 + A_1 Re + A_2 Re^2 + A_3 Re^3 + \dots = \sum_{n=0}^{\infty} A_n Re^n \quad (20)$$

Where the boundary conditions are

$$\left. \begin{aligned} f_0(-1) = -1, f_0'(-1) = 0 \\ f_0(1) = 1, f_0'(1) = 0 \end{aligned} \right\} \quad (21)$$

And

$$f_n(0) = f_n'(0) = 0, f_n(1) = f_n'(1) = 0 \quad (22)$$

when  $n > 0$

Putting the value of f( $\lambda$ ) & A in above equation (17), we get

$$\begin{aligned} & Re \left[ (f_0'(\lambda) + Re^1 f_1'(\lambda) + Re^2 f_2'(\lambda) + Re^3 f_3'(\lambda) + \dots)^2 - (f_0(\lambda) + Re^1 f_1(\lambda) + Re^2 f_2(\lambda) + Re^3 f_3(\lambda) + \dots) \cdot (f_0''(\lambda) + Re^1 f_1''(\lambda) + Re^2 f_2''(\lambda) + Re^3 f_3''(\lambda) + \dots) - \xi (f_0^{(\lambda)} + Re^1 f_1^{(\lambda)} + Re^2 f_2^{(\lambda)} + Re^3 f_3^{(\lambda)} + \dots) \right] + \\ & f_0'''(\lambda) + Re^1 f_1'''(\lambda) + Re^2 f_2'''(\lambda) + Re^3 f_3'''(\lambda) + \dots = A_0 + A_1 Re + A_2 Re^2 + A_3 Re^3 + \dots \end{aligned} \quad (23)$$

Taking the coefficient of  $Re^0, Re^1, Re^2, Re^3, \dots$

We get  $f_0''' = A_0$  (24)

$f_1''' = A_1 - f_0'^2 + f_0 f_0'' + \xi f_0'$  (25)

$f_2''' = A_2 - 2f_0' f_1' + f_0 f_1'' + f_1 f_0'' + \xi f_1'$  (26)

$f_1''' = A_3 - f_1'^2 - 2f_0' f_1' + f_1 f_1'' + f_2 f_0'' + f_0 f_2'' + \xi f_2'$  (27)

Integrating above equation (24),

We get  $f_0 = \frac{A_0}{6} \lambda^3 + \frac{C_1}{2} \lambda^2 + C_2 \lambda + C_3$  (28)

Using the boundary condition (21) in equation (28)

We get the solution

$f_0 = \frac{3\lambda}{2} - \frac{\lambda^3}{2} = 1.5 \lambda - 0.5 \lambda^3$  (29)

Using similar process by putting the value of  $f_0, f_1, f_2$  in the order of equation (25, (26), (27) by using above boundary condition (22) we get,

$f_1(\lambda) = -0.014286 \lambda^2 + 0.01786 \lambda^3 - 0.00357143 \lambda^7 + \xi(-0.1 \lambda^2 - 0.2 \lambda^3 - 0.025 \lambda^5)$  (30)

$f_2(\lambda) = 0.003525 \lambda^2 - 0.0061 \lambda^3 + 0.001788 \lambda^4 + 0.0008342 \lambda^6 + 0.0002551 \lambda^7 - 0.00313 \lambda^9 + 0.0000114 \lambda^{11} + \xi(-0.01655 \lambda^2 + 0.0117 \lambda^3 + 0.01131 \lambda^4 + 0.000893 \lambda^5 - 0.001667 \lambda^6 - 0.004643 \lambda^7 - 0.001116 \lambda^9 + 0.0000796 \lambda^{11} + \xi^2 - 0.03075 \lambda^2 + 0.04967 \lambda^3 - 0.0083 \lambda^4 - 0.01 \lambda^5 - 0.000595 \lambda^7)$  (31)

$f_3(\lambda) = -0.00383 \lambda^2 + 0.003394 \lambda^3 + 0.004015 \lambda^4 - 0.003601 \lambda^5 - 0.0005558 \lambda^6 + 0.00094 \lambda^7 - 0.0000958 \lambda^8 + 0.00018824 \lambda^9 - 0.000095 \lambda^{10} - 0.000428 \lambda^{11} + 0.000072232 \lambda^{13} - 0.00000028 \lambda^{15} + \xi(0.02187 \lambda^2 - 0.06321 \lambda^3 - 0.005223 \lambda^4 + 0.03136 \lambda^5 - 0.002723 \lambda^6 - 0.00689 \lambda^7 - 0.0005283 \lambda^8 - 0.0013404 \lambda^9 + 0.00005 \lambda^{10} - 0.0000331 \lambda^{11} - 0.001772 \lambda^{13} - 0.00000169 \lambda^{15} + \xi^2 - 0.0034105 \lambda^2 + 0.00455 \lambda^3 - 0.005223 \lambda^4 + 0.007703 \lambda^5 - 0.00051 \lambda^6 - 0.0034 \lambda^7 + 0.00021 \lambda^8 + 0.0000794 \lambda^9 + 0.00000112 \lambda^{11} + 0.0000005103 \lambda^{13} + \xi^3 0.00057305 \lambda^2 + 0.0000292 \lambda^3 - 0.0025625 \lambda^4 + 0.002483 \lambda^5 - 0.0002767 \lambda^6 - 0.0002381 \lambda^7 - 0.000008264 \lambda^9)$  (32)

Putting these values in the equation (19) we get the solution of non-linear differential equation (17)

$f(\lambda) = 1.5 \lambda - 0.5 \lambda^3 + Re[-0.014286 \lambda^2 + 0.01786 \lambda^3 - 0.00357143 \lambda^7 + \xi(-0.1 \lambda^2 - 0.2 \lambda^3 - 0.025 \lambda^5)] + Re^2[0.003525 \lambda^2 - 0.0061 \lambda^3 + 0.001788 \lambda^4 + 0.0008342 \lambda^6 + 0.0002551 \lambda^7 - 0.00313 \lambda^9 + 0.0000114 \lambda^{11} + \xi - 0.01655 \lambda^2 + 0.0117 \lambda^3 + 0.01131 \lambda^4 + 0.000893 \lambda^5 - 0.001667 \lambda^6 - 0.004643 \lambda^7 - 0.001116 \lambda^9 + 0.0000796 \lambda^{11} + \xi^2 - 0.03075 \lambda^2 + 0.04967 \lambda^3 - 0.0083 \lambda^4 - 0.01 \lambda^5 - 0.000595 \lambda^7 + Re^3 0.00383 \lambda^2 + 0.003394 \lambda^3 + 0.004015 \lambda^4 - 0.003601 \lambda^5 - 0.0005558 \lambda^6 + 0.00094 \lambda^7 - 0.0000958 \lambda^8 + 0.00018824 \lambda^9 - 0.000095 \lambda^{10} - 0.000428 \lambda^{11} + 0.000072232 \lambda^{13} - 0.0000028 \lambda^{15} + \xi 0.02187 \lambda^2 - 0.06321 \lambda^3 - 0.005223 \lambda^4 + 0.03136 \lambda^5 - 0.002723 \lambda^6 - 0.00689 \lambda^7 - 0.0005283 \lambda^8 - 0.0013404 \lambda^9 + 0.00005 \lambda^{10} - 0.0000331 \lambda^{11} - 0.001772 \lambda^{13} - 0.00000169 \lambda^{15} + \xi^2 - 0.0034105 \lambda^2 + 0.00455 \lambda^3 - 0.005223 \lambda^4 + 0.007703 \lambda^5 - 0.00051 \lambda^6 - 0.0034 \lambda^7 + 0.00021 \lambda^8 + 0.0000794 \lambda^9 + 0.00000112 \lambda^{11} + 0.0000005103 \lambda^{13} + \xi^3 0.00057305 \lambda^2 + 0.0000292 \lambda^3 - 0.0025625 \lambda^4 + 0.002483 \lambda^5 - 0.0002767 \lambda^6 - 0.0002381 \lambda^7 - 0.000008264 \lambda^9]$  (33)

Or

$f(\lambda) = 1.5 \lambda - 0.5 \lambda^3 + Re[-0.014286 \lambda^2 + 0.01786 \lambda^3 - 0.00357143 \lambda^7] + Re \cdot \xi(-0.1 \lambda^2 - 0.2 \lambda^3 - 0.025 \lambda^5) + Re^2[-0.003525 \lambda^2 - 0.0061 \lambda^3 + 0.001788 \lambda^4 + 0.0008342 \lambda^6 + 0.0002551 \lambda^7 - 0.00313 \lambda^9 + 0.0000114 \lambda^{11}] + Re^2 \cdot \xi(-0.01655 \lambda^2 + 0.0117 \lambda^3 + 0.01131 \lambda^4 + 0.000893 \lambda^5 - 0.001667 \lambda^6 - 0.004643 \lambda^7 - 0.001116 \lambda^9 + 0.0000796 \lambda^{11}) + Re^2 \cdot \xi^2(-0.03075 \lambda^2 + 0.04967 \lambda^3 - 0.0083 \lambda^4 - 0.01 \lambda^5 - 0.000595 \lambda^7) + Re^3[0.00383 \lambda^2 + 0.003394 \lambda^3 + 0.004015 \lambda^4 - 0.003601 \lambda^5 - 0.0005558 \lambda^6 + 0.00094 \lambda^7 - 0.0000958 \lambda^8 + 0.00018824 \lambda^9 - 0.000095 \lambda^{10} - 0.000428 \lambda^{11} + 0.000072232 \lambda^{13} - 0.0000028 \lambda^{15}] + Re^3 \cdot \xi(0.02187 \lambda^2 - 0.06321 \lambda^3 - 0.005223 \lambda^4 + 0.03136 \lambda^5 - 0.002723 \lambda^6 - 0.00689 \lambda^7 - 0.0005283 \lambda^8 - 0.0013404 \lambda^9 + 0.00005 \lambda^{10} - 0.0000331 \lambda^{11} - 0.001772 \lambda^{13} - 0.00000169 \lambda^{15}) + Re^3 \cdot \xi^2(-0.0034105 \lambda^2 + 0.00455 \lambda^3 - 0.005223 \lambda^4 + 0.007703 \lambda^5 - 0.00051 \lambda^6 - 0.0034 \lambda^7 + 0.00021 \lambda^8 + 0.0000794 \lambda^9 + 0.00000112 \lambda^{11} + 0.0000005103 \lambda^{13}) +$

$$Re^3 \cdot \xi^3 (0.00057305 \lambda^2 + 0.0000292 \lambda^3 - 0.0025625 \lambda^4 + 0.002483 \lambda^5 - 0.0002767 \lambda^6 - 0.0002381 \lambda^7 - 0.000008264 \lambda^9) + \dots \tag{34}$$

We know that  $\xi Re = M^2$ , putting these values in the equation (34) we get

$$f(\lambda) = 1.5 \lambda - 0.5 \lambda^3 + Re[-0.014286 \lambda^2 + 0.01786 \lambda^3 - 0.00357143 \lambda^7] + M^2(-0.1 \lambda^2 - 0.2 \lambda^3 - 0.025 \lambda^5) + Re^2[-0.003525 \lambda^2 - 0.0061 \lambda^3 + 0.001788 \lambda^4 + 0.0008342 \lambda^6 + 0.0002551 \lambda^7 - 0.00313 \lambda^9 + 0.0000114 \lambda^{11}] + Re M^2(-0.01655 \lambda^2 + 0.0117 \lambda^3 + 0.01131 \lambda^4 + 0.000893 \lambda^5 - 0.001667 \lambda^6 - 0.004643 \lambda^7 - 0.001116 \lambda^9 + 0.0000796 \lambda^{11} + M^4 - 0.03075 \lambda^2 + 0.04967 \lambda^3 - 0.0083 \lambda^4 - 0.01 \lambda^5 - 0.000595 \lambda^7 + Re^3 0.00383 \lambda^2 + 0.003394 \lambda^3 + 0.004015 \lambda^4 - 0.003601 \lambda^5 - 0.0005558 \lambda^6 + 0.00094 \lambda^7 - 0.0000958 \lambda^8 + 0.00018824 \lambda^9 - 0.000095 \lambda^{10} - 0.000428 \lambda^{11} + 0.000072232 \lambda^{13} - 0.0000028 \lambda^{15} + Re^2 M^2 0.02187 \lambda^2 - 0.06321 \lambda^3 - 0.005223 \lambda^4 + 0.03136 \lambda^5 - 0.002723 \lambda^6 - 0.00689 \lambda^7 - 0.0005283 \lambda^8 - 0.0013404 \lambda^9 + 0.00005 \lambda^{10} - 0.0000331 \lambda^{11} - 0.001772 \lambda^{13} - 0.00000169 \lambda^{15} + Re M^4 - 0.0034105 \lambda^2 + 0.00455 \lambda^3 - 0.005223 \lambda^4 + 0.007703 \lambda^5 - 0.00051 \lambda^6 - 0.0034 \lambda^7 + 0.00021 \lambda^8 + 0.0000794 \lambda^9 + 0.00000112 \lambda^{11} + 0.000005103 \lambda^{13} + M^6 0.00057305 \lambda^2 + 0.0000292 \lambda^3 - 0.0025625 \lambda^4 + 0.002483 \lambda^5 - 0.0002767 \lambda^6 - 0.0002381 \lambda^7 - 0.000008264 \lambda^9 + \dots \tag{35}$$

Equation (35) is approximate solution of non-linear differential equation (18).

**Result and discussion:-**

We have solving the non-linear differential equation (17) by perturbation method using the equation (19, 20) subject to (21, 22) we get the solution of differential equation (17) is (35), by analytic procedure. However the process of numerical calculation by using matlab software, the velocity of viscous fluid have been drawn for different values of Magnetic Number M (Hartman Number), Reynolds Number Re, width of the channel in the figure 2 to figure 10. We know that when we increase the magnetic field B then the Hartman Number M increases or we can say that magnetic field is increases.

Figure 2 a, b, shows the graph between dimensionless variable  $\lambda$  and velocity of fluid  $f(\lambda)$ , As M increases the velocity of fluid decreases in the central region and increase near the walls. In Figure 3 a, b, shows the graph between the dimensionless variable  $\lambda$  and velocity of fluid  $f(\lambda)$ , as increase of Reynolds number Re the velocity of fluid decreases. when we plot the graph between Hartman number M and velocity of fluid  $f(\lambda)$ , we see that the increase of width of the channel the velocity of fluid is decreases slowly whereas when we increase the width of the channel is greater than unity the velocity of fluid decreases fast, which is shown in the figure 4 a, b and 5 a and b. Figure 6 a and b is the graph between the Reynolds number Re and velocity of the fluid  $f(\lambda)$  at constant width of the channel, velocity of fluid decreases with increase of magnetic parameter M (which is known as Hartman Number) where as in the figure 7 a and b is the graph between the Reynolds number Re and Velocity of the fluid at constant Hartman number which shows that the velocity of the fluid decreases with the increase of the width of the channel.

In the Figure 8 a and b is the graph between the dimensionless variable  $\lambda$  and velocity of fluid  $f(\lambda)$  at different small and large constant Reynolds number Re, we see that the increase of the magnetic field the velocity of fluid decreases, where as in the figure 9 a and b which is given the graph between dimensionless variable  $\lambda$  and velocity of fluid  $f(\lambda)$  at different magnetic field and Reynolds number, it is observed that  $Re > 0$  and for different values of m in the region  $-1 \leq \lambda < 0$ ,  $f(\lambda)$  decreases with increase of Reynolds number Re. while in region  $0 < \lambda \leq 1$ ,  $f(\lambda)$  increases with increase of Re.

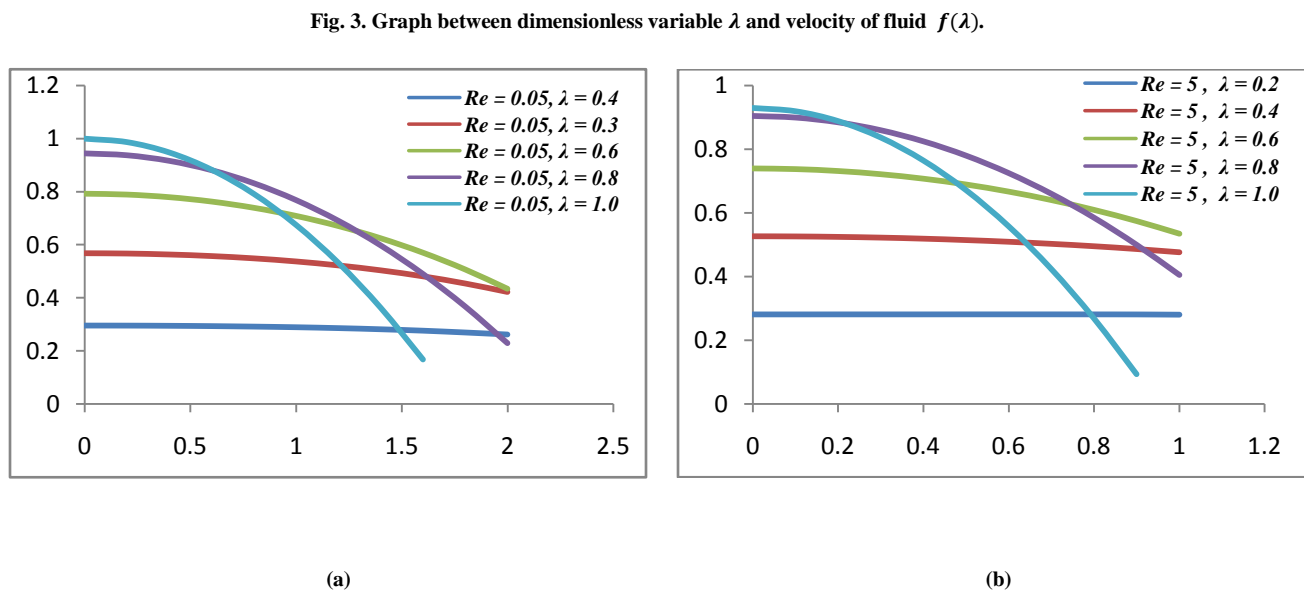
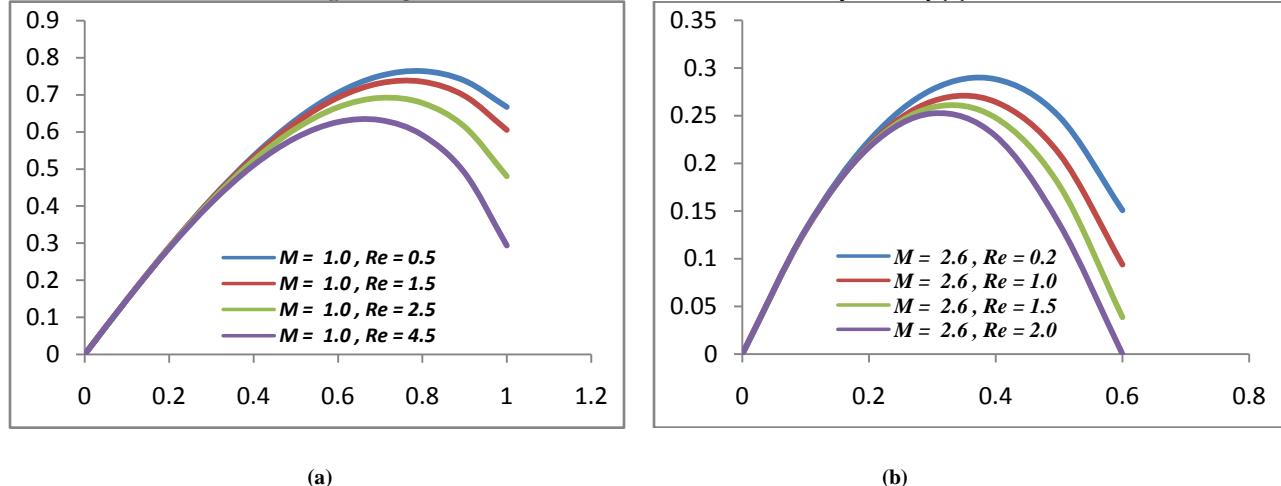
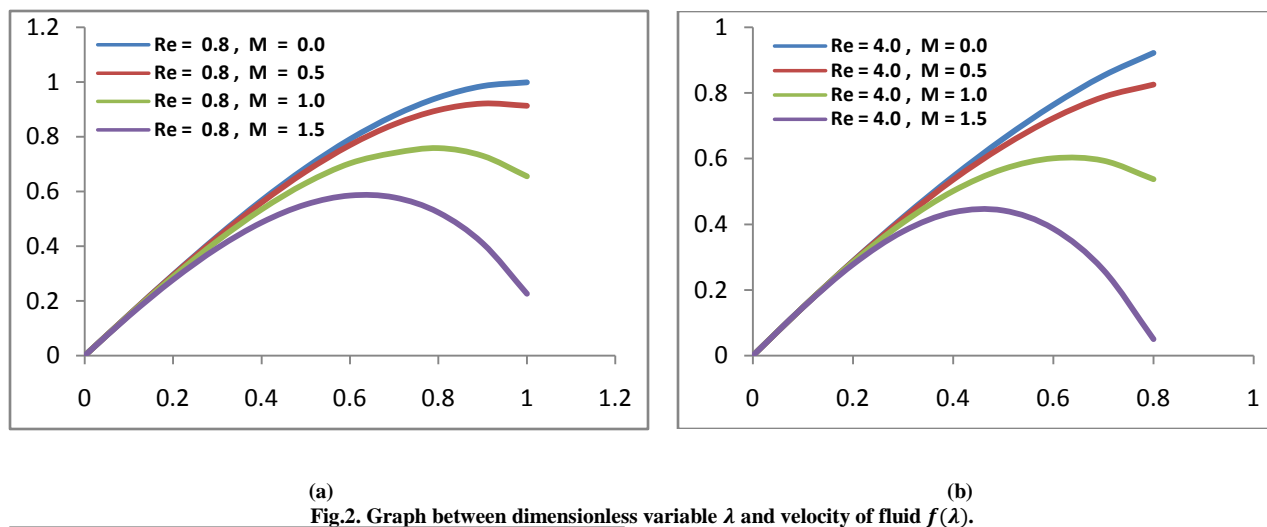


Fig. 4. Graph between magnetic field  $M$  and velocity of fluid  $f(\lambda)$ .

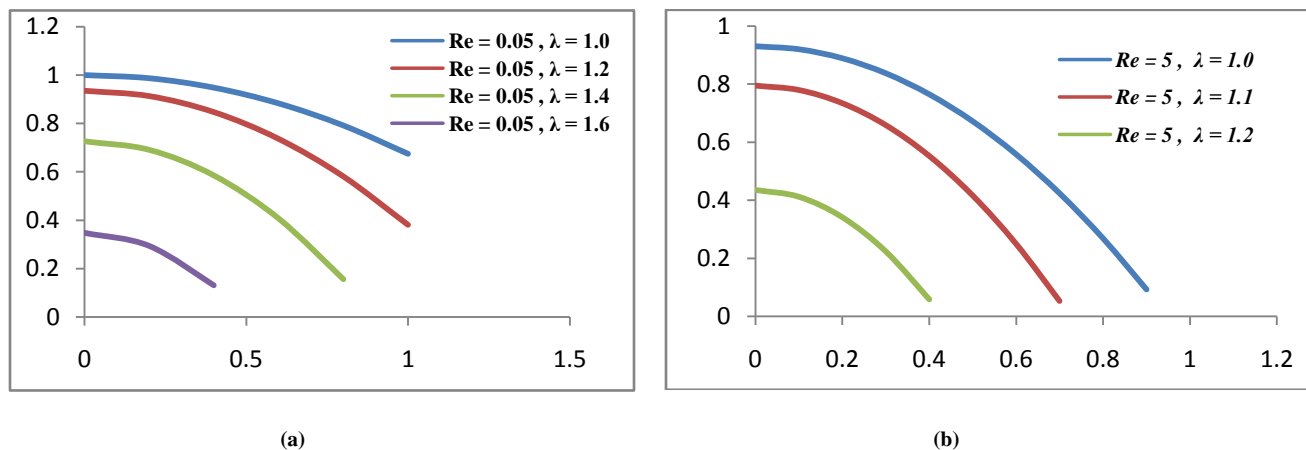
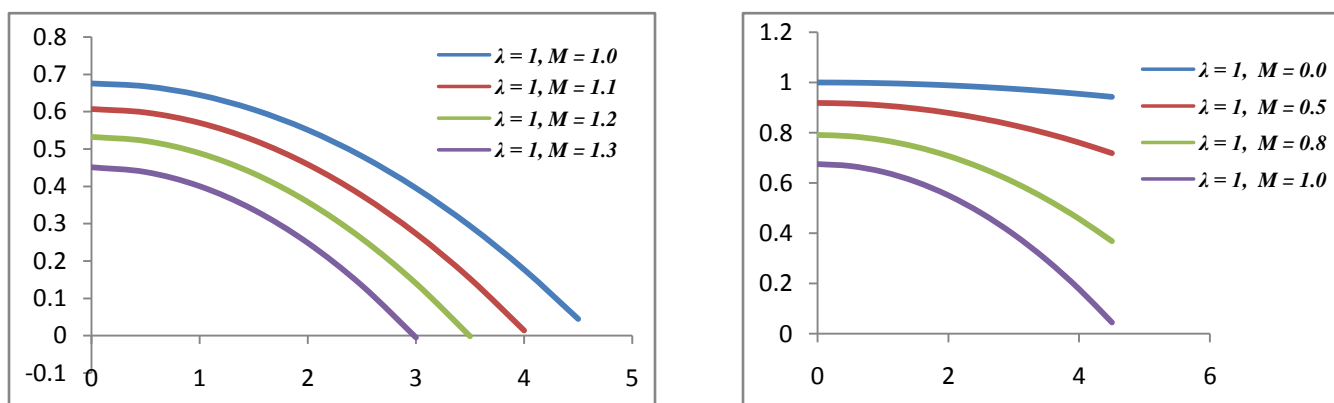


Fig. 5. Graph between magnetic field M and velocity of fluid  $f(\lambda)$ .



(a) (b)  
Fig.6. Graph between dimensionless variable  $\lambda$  and velocity of fluid  $f(\lambda)$ .

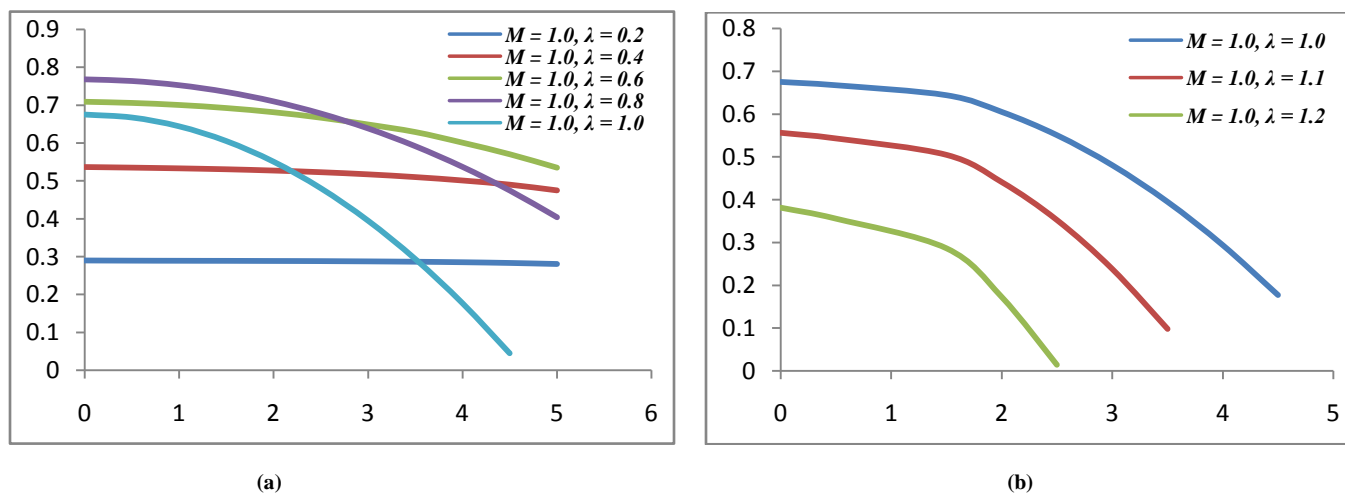


Fig. 7. Graph between Reynolds number Re and velocity of fluid  $f(\lambda)$ .

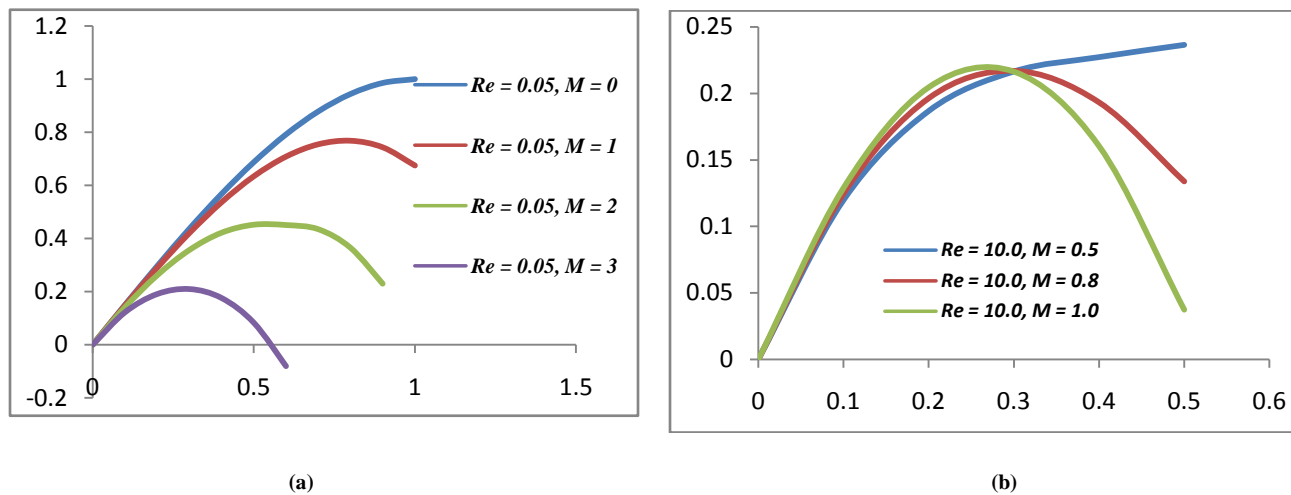


Fig. 8. Graph between dimensionless variable  $\lambda$  and velocity of fluid  $f(\lambda)$ .

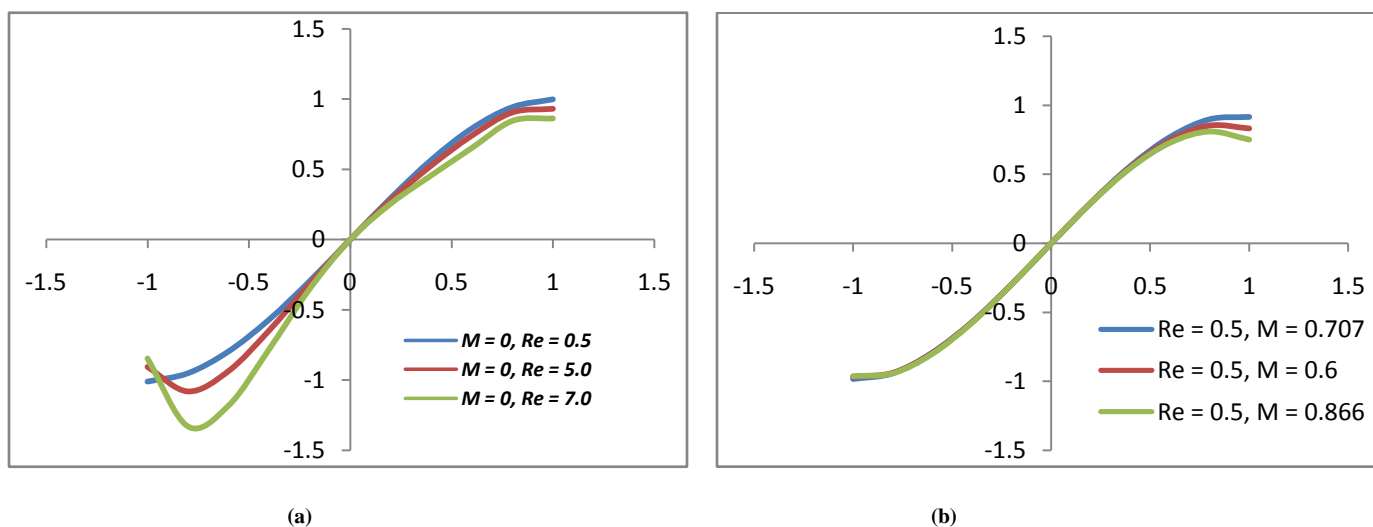
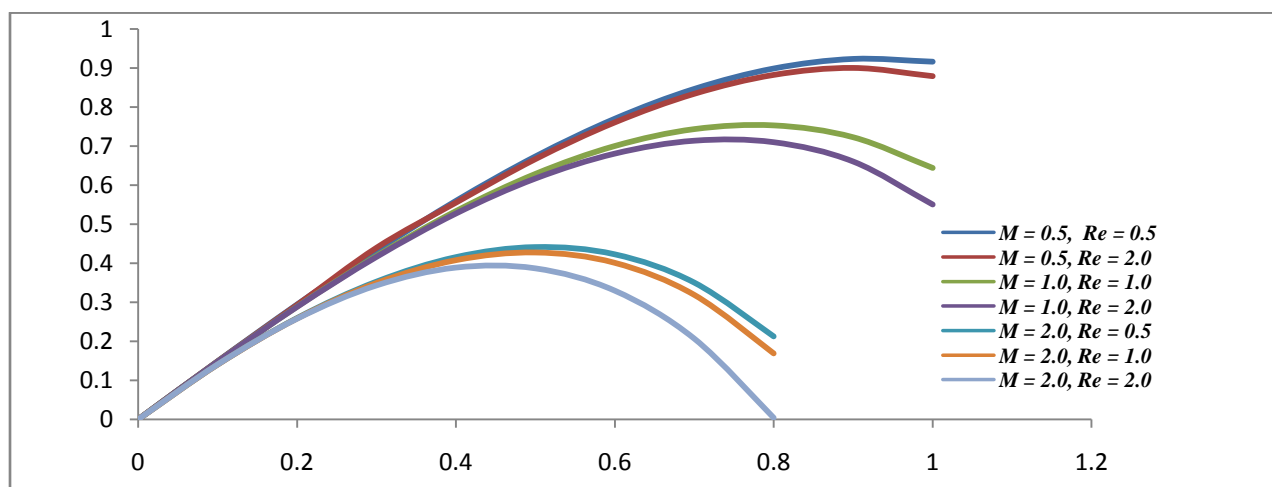


Fig. 9. Graph between dimensionless variable  $\lambda$  and velocity of fluid  $f(\lambda)$ .



**Fig. 10. Graph between dimensionless variable  $\lambda$  and velocity of fluid  $f(\lambda)$ .**

The above figure 10 is the graph between the dimensionless variable  $\lambda$  and velocity of fluid  $f(\lambda)$  at different magnetic field  $M$  and different Reynolds number  $Re$  which show the increase of Reynolds number  $Re$  and increase of Magnetic field  $M$ , the velocity of fluid decreases.

### Conclusion:-

In this paper we have analyzed the MHD flow of viscous fluid between parallel porous bounding walls in the presence of transverse magnetic field; in this we study the combined effect of magnetic field, Reynolds number and width of the channel on the steady flow of viscous fluid. We see that the enhancement of the magnetic field, Reynolds number and width of the channel the velocity of fluid decreases. This result obtained for this problem reduces to the result of (Berman 1953) when the Hartman number is zero.

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