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RESEARCH ARTICLE

ANALYSIS ON ISOMORPHISM OF FUZZY LINE GRAPHS

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Abstract

This paper develops the properties of fuzzy line graph L(G) which is obtained from the fuzzy graph G=(S,σ,μ). The aim of this paper is to find the isomorphism, co – weak isomorphism, homomorphism and weak isomorphism of fuzzy line graph from their corresponding fuzzy graphs.

1. Source of the study:-

Fuzzy set theory is the broad research areas in Mathematics which is largely applied in Computer Science, Artificial Intelligence, Decision Analysis, Information Science, System Science, Control Engineering, Expert System, Pattern Recognition, Management Science, Operations Research and Robotics etc. It also can be applied in Sociology, Political Science and Anthropology, as well as in any field of inquiry dealing with complex patterns of caution.

L.A. Zadeh[1] introduced the concept of uncertainty in 1965, which was described by the fuzzy relation. Nowadays the theory of fuzzy sets turned out to be a significant concept of investigation in different areas including Logic, Topology, Algebra and Analysis etc. Rosenfeid.A[2] introduced the fuzzy graphs and obtained several theoretical concepts. Bhutani. K. R.[3] introduced the notion of isomorphism between fuzzy graphs. The theory of fuzzy line graphs were analyzed by Mordeson, J.N. and P.S.Nair[4]. Some properties of fuzzy line graphs were analyzed by[5].

Section 2 is committed to some necessary definitions which is mandatory throughout this paper. Section 3 analyzes the property of fuzzy line graphs and last Section concludes the investigation.

2. Mandatory Definitions:-

Fuzzy set[1] :-

Fuzzy sets are the sets with limitations that are not accurate. The membership function of a fuzzy set A is denoted by μ_A and it is defined by $\mathbf{0 \leq \mu_A(x) \leq 1}$.

Fuzzy graph[2] :-

A fuzzy graph $G = (S,\sigma,\mu)$ is a non empty set S jointly with a couple of functions. Where $\sigma : S\rightarrow[0,1]$ is a fuzzy vertex set of G, $\mu:S\times S\rightarrow[0,1]$ is a fuzzy relation on the fuzzy edge set of G such that

$$\mu(x,y) \leq \min(\sigma(x), \sigma(y)) \quad \forall\ x,y\in S.$$

Strong Fuzzy Graph:-

A fuzzy graph G : (σ,μ) is said to be strong if $\mu(x,y) = \sigma(x) \wedge \sigma(y)\ \forall\ x,y \in \mu^*$. Where $\mu^* = supp(\mu) = \{(x, y) \in S \times S / \mu(x, y)>0\}$.

Homomorphism of Fuzzy Graph[3] :-

A homomorphism of fuzzy graph h: G→G' is a map h: S→S' which satisfies,

$\sigma(x) \leq \sigma'[\ h(x)\ ]\ \ \forall\ x \in S$

$\mu(x,y) \leq \mu'[\ h(x), h(y)\ ]\ \ \ \forall\ x, y \in S$

Weak- Isomorphism of Fuzzy Graph[3&5] :-

A weak isomorphism h: G→G' is a map h: S→S' which is a bijective that satisfies,

$\sigma(x) = \sigma'[\ h(x)\ ]\ \ \forall\ x \in S$

$\mu(x,y) \leq \mu'[\ h(x), h(y)\ ]\ \ \ \forall\ x, y \in S.$

That is the weak isomorphism preserves the weight of the nodes but not necessarily the weight of the edges.

Co-Weak Isomorphism of Fuzzy Graph[3&5] :-

A co-weak isomorphisam h: G→G' is a map h: S→S' which is a bijective that satisfies,

$\sigma(x) \leq \sigma'[\ h(x)\ ]\ \ \forall\ x \in S$

$\mu(x,y) = \mu'[\ h(x), h(y)\ ]\ \ \ \forall\ x, y \in S$

That is the co - weak isomorphism preserves the weight of the edges but not necessarily the weight of the nodes.

Isomorphism of Fuzzy Graph[3&5] :-

An isomorphism h: G→G' is a map h: S→S' which is bijective that satisfies,

$\sigma(x) = \sigma'[\ h(x)\ ]\ \ \forall\ x \in S$

$\mu(x,y) = \mu'[\ h(x), h(y)\ ]\ \ \ \forall\ x, y \in S.$

Isomorphism between fuzzy graphs denoted as G≅G'. That is the isomorphism preserves the weight of the edges and the weight of the nodes.

Fuzzy Intersection Graph[4] :-

Consider *G: (σ,μ)* be a fuzzy graph with the underlying graph (V,E). The fuzzy intersection graph G is defined by I(S) : (i,γ) with the underlying graph (S,T). The node set S is defined as,

$S = \{S_i / S_i = v_i, x_{i1}, \ldots, x_{iqi}, x_{ij} \in E, j=1,2,\ldots,q_i, i=1,2,\ldots,n\}$

$T = \{(S_i,S_j) / S_i,S_j \in S, S_i \cap S_j \neq \emptyset, i \neq j\}$

$i(S_i) = \sigma(V_i)\ \forall\ S_i \in S$ ***and***

$\gamma(S_i,S_j) = \mu(V_i, V_j)\ \forall\ S_i \in T$

Fuzzy Line Graph[4&5]:-

Fuzzy line graph L(G) is the intersection graph of the set of edge of G. hence the vertices of G are the edges of L(G). Let G*: (σ,μ)* be a fuzzy graph with the underlying graph (V,E). Then the fuzzy line graph of G is *L(G): (ω,λ)* with the underlying graph (Z,W). Where Z and W defined as

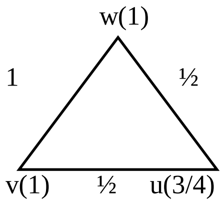
$Z = \{S_x = \{x\} U \{u_x, v_x\} / x \in E, u_x, v_x \in V, x=(u_x,v_x)\}$

$W = \{\ (S_x,S_y) / S_x \cap S_y \neq \emptyset, x \neq y, x,y \in E\}$

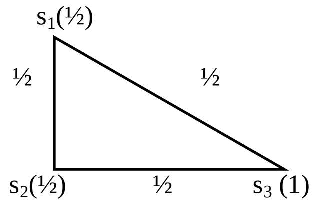
$\omega(S_x) = \mu(x)\ \forall\ S_x \in Z$

$\lambda(S_x,S_y) = \mu(x) \wedge \mu(y), \forall\ (S_x,S_y) \in W$

Example:-



G :(σ,μ)



L(G) :(ω, λ)

$Z = \{S_1, S_2, S_3\}$;
$S_1 = \{(w,u), \{u\}, \{w\}\}$;
$S_2 = \{(u,v), \{u\}, \{v\}\}$;
$S_3 = \{(w,v), \{w\}, \{v\}\}$ and
$W = \{ (S_x, S_y) / S_x \cap S_y \neq \emptyset, x \neq y, x,y \in E\}$

3. Investigation of Fuzzy Line Graphs:-

Isomorphism of Fuzzy Line Graph:-

As in any area of mathematics, it is important to know when two objects are the "same" or "different". If two graphs differ from one another only by way they are drawn or by the way their vertices (or edges) are labeled, we say that they are isomorphic.

- The isomorphism of fuzzy line graph exists only when the given fuzzy graph is either an isomorphism or a co – weak isomorphism. Since the isomorphism graphs preserves the weight of the edges and weight of the nodes.
- The isomorphism of fuzzy line graphs can't exist when the given fuzzy graph is weak isomorphic. Since weak isomorphism of graphs doesn't preserves the weight of the edges.

Theorem 3.1
If G_1: (σ_1, μ_1) and G_2: (σ_2, μ_2) are the co-weak isomorphic then $L(G_1)$: (ω_1, λ_1) and $L(G_2)$: (ω_2, λ_2) are isomorphic fuzzy line graphs corresponding to G_1 and G_2.

Proof:- Given G_1: (σ_1, μ_1) and G_2: (σ_2, μ_2) are co-weak isomorphic fuzzy graphs then there exist a bijective map $h : S_1 \rightarrow S_2$ such that, $h(x) = \dot{x} \ \forall \ x \in S_1$

satisfies, $\sigma_1(x) \leq \sigma_2[h(x)]$
$\mu_1(x, y) = \mu_2[h(x), h(y)]$
from the definition of FLG,

$$\omega_1(S_x) = \mu_1(x) = \mu_2[h(x)] = \omega_2(S_x)$$

$\Rightarrow \boldsymbol{\omega_1(S_x) = \omega_2(S_x)}$

$$\lambda_1(S_x, S_y) = \mu_1(x) \wedge \mu_1(y) = \mu_2[h(x)] \wedge \mu_2[h(y)] = \lambda_2(S_x, S_y)$$

$\Rightarrow \boldsymbol{\lambda_1(S_x, S_y) = \lambda_2(S_x, S_y)}$

Hence if G_1 and G_2 are the co-weak isomorphic then their corresponding line graphs $L(G_1)$ and $L(G_2)$ are isomorphic.

Example:-

(A). co – weak isomorphism of Fuzzy graphs

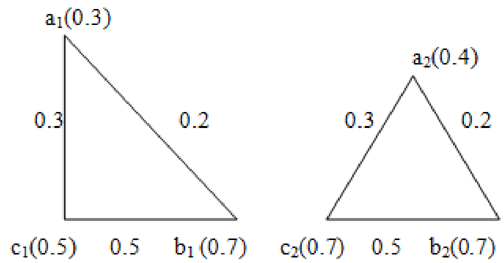


Fig (a) $G_1 :(\sigma_1,\mu_1)$ Fig (b) $G_2 :(\sigma_2,\mu_2)$

Isomorphism of fuzzy line graphs
from (A) when ($\mu(x,y) = \sigma(x) \wedge \sigma(y)\ \forall\ x,y \in$ supp μ)

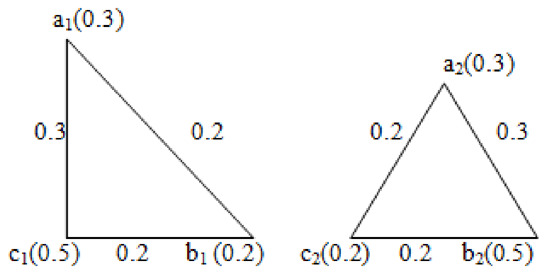


Fig (a) $L(G_1) :(\omega, \lambda)$ Fig (b) $L(G_2) :(\omega, \lambda)$

Isomorphism of fuzzy line graphs
from (A) when $\mu(x,y) \leq \min(\sigma(x), \sigma(y))$

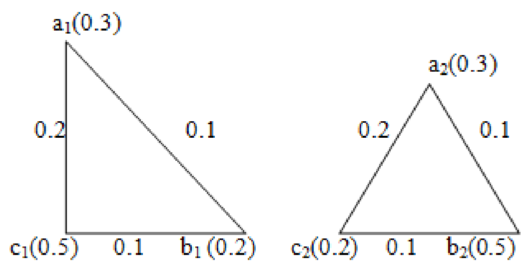


Fig (a) $L(G_1) :(\omega, \lambda)$ Fig (b) $L(G_2) :(\omega, \lambda)$

Weak isomorphism of Fuzzy Line Graph:-

Weak isomorphism of fuzzy line graphs can be derived from the isomorphism or co – weak isomorphism of fuzzy graphs by the condition $\mu(x,y) \leq \min(\sigma(x), \sigma(y))$. Since isomorphism and co– weak isomorphism preserves the weight of the edges. That is, the weak isomorphism of fuzzy line graphs preserves the weight of the nodes.

Example:-

Weak isomorphism of fuzzy line graphs
from (A) when $\mu(x,y) \leq \min(\sigma(x), \sigma(y))$

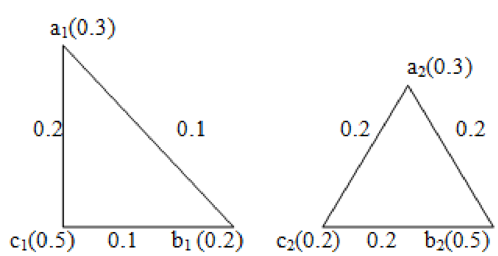


Fig (a) $L(G_1) :(\omega, \lambda)$ Fig (b) $L(G_2) :(\omega, \lambda)$

Homomorphism of Fuzzy Line Graph:-

When the fuzzy graph is weak isomorphism then the homomorphism of a fuzzy line graph (ω_1,λ_1) onto (ω_2,λ_2) is defined as

$\boldsymbol{\omega_1(S_x) \leq \omega_2(S(h(x))), \forall\, S_x \in Z}$

$\boldsymbol{\lambda_1(S_x,S_y) \leq \lambda_2(S(h(x)), S(h(y))), \forall\, (S_x,S_y) \in W}$

Theorem 3.2

If G_1: (σ_1, μ_1) and G_2: (σ_2, μ_2) are the weak-isomorphic fuzzy graphs then $L(G_1) : (\omega_1, \lambda_1)$ and $L(G_2) : (\omega_2, \lambda_2)$ are the homomorphism of fuzzy line graph corresponding to G_1, G_2.

Proof:- Consider $L(G_1) : (\omega_1, \lambda_1)$ and $L(G_2) : (\omega_2, \lambda_2)$ are the fuzzy line graphs of $G_1 : (\sigma_1, \mu_1)$ and $G_2 : (\sigma_2, \mu_2)$. If two fuzzy graphs G_1 and G_2 satisfies the weak-isomorphism, then there exist a bijective map $h : S_1 \to S_2$ such that,

$\sigma_1(x) = \sigma_2[h(x)]$ and

$\mu_1(x, y) \leq \mu_2[h(x), h(y)]\ \forall\ x, y \in S$

from the definition of FLG,

$\omega_1(S_x) = \mu_1(x)$ and $\omega_2(S_x) = \mu_2(x)$

$\lambda_1(S_x, S_y) = \mu_1(x) \wedge \mu_1(y)$ and

$\lambda_2(S_x, S_y) = \mu_2(x) \wedge \mu_2(y)$

Consider, $\omega_1(S_x) = \mu_1(x) \leq \mu_2[h(x)] = \omega_2(S_x)$

$\Rightarrow \boldsymbol{\omega_1(S_x) \leq \omega_2(S_x)}$

$\lambda_1(S_x, S_y) = \mu_1(x) \wedge \mu_1(y)$

$\leq \mu_2[h(x)] \wedge \mu_2[h(y)] = \lambda_2(S_x, S_y)$

$\Rightarrow \boldsymbol{\lambda_1(S_x, S_y) \leq \lambda_2(S_x, S_y)}$

Hence if G_1 and G_2 are the weak isomorphic fuzzy graphs then $L(G_1)$ and $L(G_2)$ are the homomorphism of fuzzy line graphs.

Example:-

(B). Weak isomorphism of fuzzy graphs

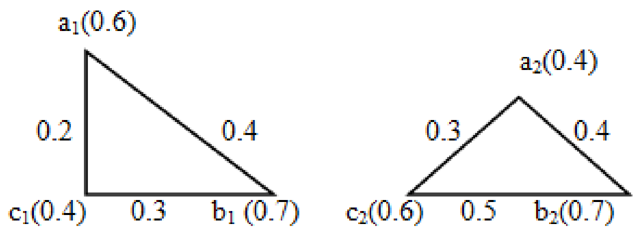


Fig (a) $G_1 : (\sigma_1, \mu_1)$ Fig (b) $G_2 : (\sigma_2, \mu_2)$

Homomorphism of fuzzy line graph
from (B) when $\mu(x,y) \leq \min(\sigma(x), \sigma(y))$

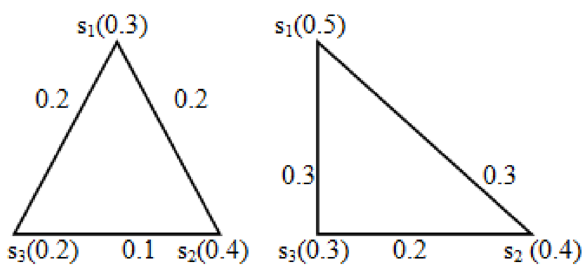


Fig (a) $L(G_1) : (\omega, \lambda)$ Fig (b) $L(G_2) : (\omega, \lambda)$

Co - Weak isomorphism of Fuzzy Line Graph:-

Co – weak isomorphism of fuzzy line graphs is exist only when the given fuzzy graph is weak isomorphism by the condition ***μ (x,y) ≤ min(σ(x), σ(y).***

Example:-

Co – weak isomorphism of fuzzy line graph from (B) when μ (x,y) ≤ min(σ(x), σ(y))

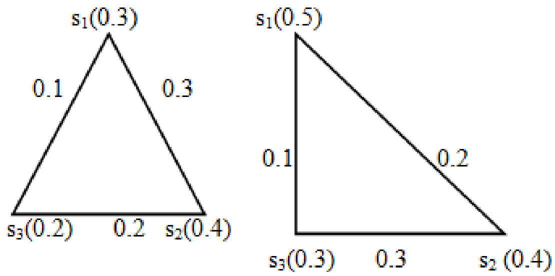


Fig (a) $L(G_1) :(\omega, \lambda)$ Fig (b) $L(G_2) :(\omega, \lambda)$

Conclusion:-

This paper analyzed the properties of fuzzy line graphs L(G) when the given fuzzy graph is isomorphism, weak isomorphism, homomorphism and co – weak isomorphism. The following results were concluded from this analysis.

- Isomorphism and weak isomorphism of fuzzy line graphs can be derived from the isomorphism or co – weak isomorphism of fuzzy graphs.
- Homomorphism of fuzzy line graphs exist only when the corresponding fuzzy graphs are weak isomorphism.
- Co – weak isomorphism of fuzzy line graph is derived only from the weak isomorphism of fuzzy graph only when the edges satisfies μ (x,y) ≤ min(σ(x), σ(y)).

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