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RESEARCH ARTICLE

BAYESIAN ANALYSIS IN AN M/G/S QUEUE WITH OPTIONAL SECOND SERVICE.

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Abstract

The paper proposes Bayesian framework in an M/G/s queuing system with optional second service.The semi-parametric model based on a finite mixture of Gamma distributions is considered to approximate both the general service and re-service times densities in this queuing system. A Bayesian procedure based on birth-death MCMC methodology is proposed to estimate system parameters, predictive densities and some performance measures related to this queuing system such as stationary system size and waiting time. The approach is illustrated with several numerical examples based on various simulation studies.

Introduction:-

Bayesian analysis of queuing systems is somewhat a modern research area. Generally in all queuing systems it is considered that the customers leave the system after taking their service. But it may happen that some customers need to be re-serviced after having their main service. For example, in a manufacturing company, some items might break and require repair. In such types of problems, we must re-service some items. The key plan of this paper is to propose a Bayesian method for an M/G/s queuing system in which some customers with probability p need re-servicing. The service component in the queuing structure in which the arrival pattern and service pattern of customers and also some customers that require re-service follows a poisson distribution. Some researchers studied this queuing system from conventional point of view, they considered three alternatives for re-servicing in this queuing system and obtained the mean busy period, the probability of the idle period and the probability generating function (pgf) of the steady-state system size.

The main contribution of this paper is to introduce a semi-parametric model for the general density of service and re-service based on a mixture of Gamma distributions, providing an alternative Bayesian approach for approximating the general distributions in queuing systems based on former work. Secondly, we will introduce a Bayesian algorithm based on the birth-death MCMC approach in order to fit this model to data. The use of finite mixture distributions is very common and the Bayesian approach provides an important tool in semi-parametric density estimation.

Recently, MCMC methods for fully Bayesian mixture models of unknown dimension have been proposed; see (1). (2) introduced the reversible jump technique (RJ-MCMC) and (3) used this methodology to analyze Normal mixtures. This type of algorithm was used by (4) for mixture of Exponential distributions, (5) for mixture of Gamma distributions and (6) for mixture of Erlang distributions. More recently, in the context with this methodology, (7) rekindled interest in the use of continuous time birth-death methodology (BDMCMC) for variable dimension problems. This type of methodology was used by (8) for mixture of Erlang distributions and (9) for the mixture of

Pareto distributions. Moreover, (10) investigated the similarity between the reversible jump and birth-death methodology.
The paper is structured as follows. Now we illustrate the M/G/s queuing system with optional second service where we consider a mixture of Gamma distributions to approximate the general densities of service and re-service times. Then, we use The M/G/1 queuing system with optional second service results obtained by (11,12)which allow us to estimate the mean number of customers in the system, mean busy period and probability of the idle period for this queuing system.

In another section we explain our Bayesian approach by defining prior distributions, obtaining conditional distributions and propose a birth-death MCMC algorithm to obtain a sample from the posterior distributions of the parameters of the predictive service and re-service times distributions. In this section we explain how to approximate the general densities of service and re-service times by using the data generated from the birth-death MCMC algorithm.

The M/G/s queuing system with optional second service:-
Throughout, we are considering an M/G/s queue, in which some customers with probability p need re-service, with First Come First Serve discipline, and independence between inter-arrival and service times. In this queuing system failed items are stockpiled in a failed queue (FQ) and re-serviced only after all customers in main queue (MQ) are serviced. After completion of re-service of all items in FQ, the server returns to MQ if there are any customers waiting in MQ; otherwise, the server is idle. So, in this queuing system we have two queues and one server.

The variable T is the inter-arrival time with an exponential distribution. For service times, we suppose that service (S) and re-service ($\widetilde{S}$) times are independent and have general distributions, denoted by B1(.) and B2(.) with means μ1, μ2 and variances δ1, δ2 respectively. For these general distributions, we need a model, flexible enough to deal with typical features in service and re-service time distributions (skewness, multimodality, lots of mass near zero, even possibly a mode) and permits usual computations in queuing applications. Thus, we propose a semi-parametric model based on a mixture of Gamma distributions with a Bayesian framework.

If S is a service time, we assume

$$B_1(s|\theta_1) = \sum_{i=1}^{k_1} \pi_{1i}\, G(s|\alpha_{1i}, \beta_{1i})\,, \qquad s > 0$$

Where θ1 = (k1, π1, α1, β1), k1 is the unknown number of mixture components, π1 = (π11, π12, ..., π1k1) are weights and G(s|α1i, β1i) represents the Gamma density function, for i = 1, ..., k1, that is,

$$G(s|\alpha_{1i}, \beta_{1i}) = \frac{(\beta_{1i})^{\alpha_{1i}}}{\Gamma\alpha_{1i}}\, s^{\alpha_{1i}-1} e^{-\beta_{1i} s}\,, \qquad s > 0$$

$\widetilde{S}$ is a re-service time, we have,

$$B_2(\widetilde{S}|\theta_1) = \sum_{i=1}^{k_2} \pi_{2i}\, G(s|\alpha_{2i}, \beta_{2i})\,, \qquad \widetilde{S} > 0$$

Where θ2 = (k2, π2, α2, β2), k2 and π2 = (π21, π22, ..., π2k2) have the same interpretation
as for the service times density. Thus, we have a queuing system with multiple queues and one server with one failure queue.Therefore, all parameters of these queuing systems are (λ, θ1, θ2, p), in which λ is the parameter of inter-arrival times and p is the probability of the items needing re-service.

We assume that the queuing system is in equilibrium. This assumption is equivalent with assuming that the traffic intensity, ρ, is less than one (18). For this queuing system

$$\rho = \rho_1 + p\rho_2,$$

in which $\rho_1 = \lambda\, /\, s\mu 1$ is traffic intensity in MQ and $\rho_1 = \lambda\, /\, s\mu 2$ is traffic intensity in FQ. As a result

$$\rho = \frac{\lambda}{s}\left(\frac{1}{\mu_1} + \frac{p}{\mu_2}\right) \qquad (1)$$

Under this steady state condition, other performance measures will be obtained.

Expectation of busy period and probability of idle period:-
To acquire the expectation of busy period for this queuing system we have:

E[busy period] = E[busy period in MQ] + pE[busy period in FQ],

Therefore, by some computation

$$E[Busy\ period] = \frac{1}{s!}\left[\left(\frac{\lambda_1}{\mu_1}\right)^s \frac{s\mu_1}{s\mu_1 - 1}\right] P_0 + \frac{p}{s!}\left[\left(\frac{\lambda_2}{\mu_2}\right)^s \frac{s\mu_2}{s\mu_2 - 1}\right] (P_0)_1 \quad (2)$$

Here

$$P_0 = \left[\sum_{n=0}^{s-1} \frac{1}{n!} \left(\frac{\lambda_1}{\mu_1}\right)^n + \frac{1}{s!}\left[\left(\frac{\lambda_1}{\mu_1}\right)^s \frac{s\mu_1}{s\mu_1 - 1}\right]\right]^{-1}$$

$$(P_0)_1 = \left[\sum_{n=0}^{s-1} \frac{1}{n!} \left(\frac{\lambda_2}{\mu_2}\right)^n + \frac{1}{s!}\left[\left(\frac{\lambda_2}{\mu_2}\right)^s \frac{s\mu_2}{s\mu_2 - 1}\right]\right]^{-1}$$

Furthermore, the probability of idle period is

$$P_r(idle\ period) = \left(1 - \frac{\lambda_1}{s\mu_1}\right) + p\left(1 - \frac{\lambda_2}{s\mu_2}\right) \quad (3)$$

Expected number of customers in MQ and FQ are as below:-
For our queuing system, suppose that Xn is the number of customers remaining in MQ at the completion of the nth customer's service time also Yn is the number of customers remaining in FQ at the completion of the th customer's service time in the steady state. (14), by using the joint probability general function of (Xn, Yn) obtained the following expression of the mean system size.

$$E(X_n) = \frac{\left[\frac{1}{(s-1)!}\left(\frac{\lambda_1}{\mu_1}\right)^s \frac{\lambda_1 \mu_1}{(s\mu_1 - \lambda_1)^2}\right]}{\left[\sum_{n=0}^{s-1} \frac{1}{n!} \left(\frac{\lambda_1}{\mu_1}\right)^n + \frac{1}{s!}\left[\left(\frac{\lambda_1}{\mu_1}\right)^s \frac{s\mu_1}{s\mu_1 - 1}\right]\right]^{-1}} \quad (4)$$

$$E(Y_n) = \frac{\left[\frac{1}{(s-1)!}\left(\frac{\lambda_2}{\mu_2}\right)^s \frac{\lambda_2 \mu_2}{(s\mu_2 - \lambda_2)^2}\right]}{\left[\sum_{n=0}^{s-1} \frac{1}{n!} \left(\frac{\lambda_2}{\mu_2}\right)^n + \frac{1}{s!}\left[\left(\frac{\lambda_2}{\mu_2}\right)^s \frac{s\mu_2}{s\mu_2 - 1}\right]\right]^{-1}} \quad (5)$$

Conclusion:-

The modeling of Bayesian Analysis pores new challenges for an M/G/s queuing system with optional second service. The proposed study will lead to generate an information regarding density estimation method based on a mixture of Gamma distributions in order to approximate the general service and re-service time distributions. Subsequently the proposed study may lead to modifications in some important measures of queuing system like the system size mean, the mean busy period and probability of idle period. Addressing the existing issues and challenges of such studies with simulation study. Bayesian structure is dissimilar to Gamma mixture and Erlang mixture distributions (16, 17, 18, 19) and its calculation is easier and the results are more accurate. The proposed study will generate information about interrelationship among various parameters and their impact on truncated normal distributions and skewed truncated normal distributions. Also, density estimation problems seem suitable for our approach. Work is currently in progress on these models.An alternative to the BD-MCMC methodology is the RJ-MCMC introduced by (20). This type of algorithm had been used (5) for mixture of Gamma distributions and (17, 18) for the mixture of Erlang distributions. In practice, we have found that both schemes perform similarly. In the

BD-MCMC algorithm, as we have indicated, larger values of the birth rate produce better mixing, but also increase the computational cost. We have experienced some problems of non-convergence of the algorithm if the birth rate is selected too high.Thus, it would be useful to explore methods for selection of this parameter in order to optimize the algorithm.

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