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RESEARCH ARTICLE

VIBRATIONAL ANALYSIS OF CIRCULAR CYLINDERUSING RITZ METHOD.

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Abstract

This study deals with free vibration analysis of circular cylinders. The analysis procedure uses of small-strain, linear elasticity theory. By considering algebraic polynomials multiplied by a boundary function to satisfy the geometric boundary conditions, the Ritz method is applied to derive the governing eigenvalue problem of cylinders. According to the axisymmetric geometrical property of circular cylinders, the vibration modes are divided into three categories viz, axisymmetric mode, torsional and flexural mode. Furthermore, for a cylinder with the same boundary conditions at the two ends, the vibration modes are classified into two types, viz anti-symmetric and symmetric modes. Finally, vibration frequencies of circular cylinders with free – free boundary conditions are computed. The results are then compared with those presented in literature.

Introduction:-

Circular cylinders are classified as hollow and solid cylinders. They are used as/in gas pipes, boilers, shafts etc. Cylinders in practical use may be subjected to vibration if the conditions, specific to each domain of use, are favorable for setting the cylinder into oscillatory motion. For example, flow induced vibration in boilers occurs when the vortex shed by the passing flow coincides with the natural frequency of the component or gas column.

Study of vibration of circular cylinders has wide variety of engineering applications such as in civil, aerospace, marine engineering etc. For three-dimensional analysis, researchers have used different methods. To name a few, there are numerical methods such as finite differences, finite element methods, and experimental methods.

A.W Leissa and Jinyoung So [1] proposed a methodology to solve the three-dimensional vibration analysis of circular cylinders. The paper demonstrates how the Ritz method may be used to accurately compute frequencies for circular cylinders of finite length having arbitrary boundary conditions. The paper also explains in detail about choosing proper set of displacement functions in the form of algebraic polynomials.

Zhou et al [2] explained in detail about the vibration analysis of circular cylinders based on the small – strain, linear elasticity theory. The authors have used the Chebyshev polynomial series as admissible functions in the Ritz method.

A.W.Leissa [3] has presented in detail about the Ritz method and how is it different from Rayleigh method. The Ritz method called as the direct method is applied directly to the variational formulation of a problem. To get a clear idea, method of Rayleigh and the method of Ritz are explained in depth in the paper.

Based on literature review, the preliminary objective of the present study is set as review and reproduction of results related to torsional and axisymmetric modes of vibration of solid circular cylinders. The study uses algebraic polynomials as admissible function in the Ritz method based on determination of frequencies.

Formulation:-

A solid circular cylinder with radiusR and length L is considered, as shown in Figure 1. An orthogonal cylindrical coordinate system (r, θ, and z) is defined with r in the radial direction, θ in circumferential direction and z in length direction.

The energy function used in Ritz method are elastic strain energy and kinetic energy.

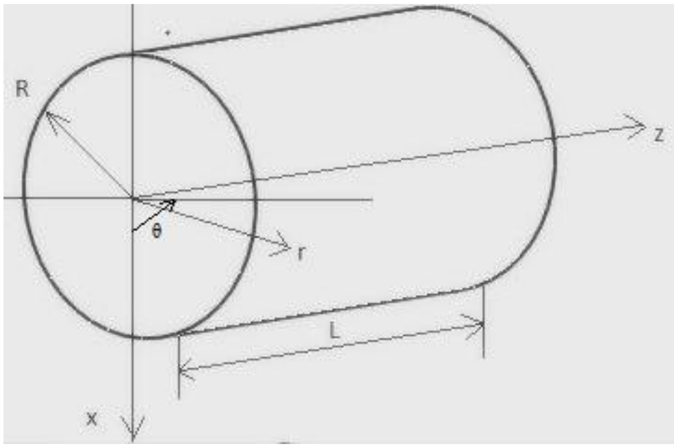

Figure 1:circular cylinder.

The elastic strain energy V of a cylinder is given by

$$V = \frac{G}{2}\int_0^R \int_0^{2\Pi} \int_0^L [\frac{2\nu}{1-2\nu}(\varepsilon_{rr} + \varepsilon_{\theta\theta} + \varepsilon_{zz})^2 + 2({\varepsilon_{rr}}^2 + {\varepsilon_{\theta\theta}}^2 + {\varepsilon_{zz}}^2) + ({\varepsilon_{r\theta}}^2 + {\varepsilon_{\theta z}}^2 + {\varepsilon_{rz}}^2)] r\, dz\, d\theta\, dr \quad (1)$$

where G is shear modulus and ν is Poisson’s ration. The strain components are defined as follow:

$$\varepsilon_{rr} = \frac{\partial u}{\partial r};\ \varepsilon_{\theta\theta} = \frac{\partial v}{r\partial\theta} + \frac{u}{r};\ \varepsilon_{zz} = \frac{\partial w}{\partial z};\ \varepsilon_{r\theta} = \frac{\partial v}{r\partial\theta} + \frac{\partial u}{\partial r} - \frac{u}{r};\ \varepsilon_{rz} = \frac{\partial w}{\partial r} + \frac{\partial u}{\partial z};\ \varepsilon_{\theta z} = \frac{\partial w}{r\partial\theta} + \frac{\partial v}{\partial z} \quad (2)$$

where u, v, z are displacement components along r, θ, z directions, respectively

Now the Kinetic energy T of a cylinder is given by

$$T = \frac{\rho}{2}\int_0^R \int_0^{2\pi} \int_0^L [(\frac{\partial u}{\partial t})^2 + (\frac{\partial v}{\partial t})^2 + (\frac{\partial w}{\partial t})^2] r\, dz\, d\theta\, dr \quad (3)$$

where ρ is the mass density per unit volume

Non - dimensionalization of coordinates

For the sake of convenience, non – dimensionalization the coordinates (r, z) into (ξ, ζ). The transformation may be given by

$$\xi = \frac{r}{R}$$

$$\zeta = \frac{z}{L}$$

Then the strain energy and kinetic energy may be rewritten as

$$V_{max} = \frac{GL}{2}\int_A^B \int_0^1 [(\frac{\partial U}{\partial\xi} + \frac{U}{\xi} + \frac{nV}{\xi} + \frac{R}{L}\frac{\partial W}{\partial\zeta})^2 \Gamma_1 + 2\left[\left(\frac{\partial U}{\partial\xi}\right)^2 + \left(\frac{U}{\xi} + \frac{nV}{\xi}\right)^2 + \left(\frac{R}{L}\frac{\partial W}{\partial\zeta}\right)^2\right]\Gamma_1 + \left[\left(\frac{nU}{\xi} - \frac{\partial V}{\partial\xi} + \frac{V}{\xi}\right)^2 + \left(\frac{R}{L}\frac{\partial V}{\partial\zeta} - \frac{nW}{\xi}\right)^2\right]\Gamma_2 + (\frac{R}{L}\frac{\partial U}{\partial\zeta} + \frac{\partial W}{\partial\xi})^2\Gamma_1]\xi\ d\xi\, d\zeta \quad (4)$$

$$T_{max} = \frac{\rho R^2 L}{16}\omega^2 \int_A^B \int_0^1 (\Gamma_1 U^2 + \Gamma_2 V^2 + \Gamma_1 W^2)\ \xi\ d\xi\, d\zeta \quad (5)$$

where

$$\varGamma_1 = \int_0^{2\pi} \cos(n\theta)^2 \, d\theta = \begin{cases} 2\pi \ \ if \ n = 0; \\ \pi \ if \ n > 0; \end{cases}$$

$$\varGamma_2 = \int_0^{2\pi} \sin(n\theta)^2 \, d\theta = \begin{cases} o \ \ if \ n = 0; \\ \pi \ if \ n > 0; \end{cases}$$

The integral limits are given by [0, 1] for ξ and [A, B] for ξ, where [A, B] may have different values, i.e. [-1/2, 1/2] for the free-free boundary case.

In this present analysis, each of the displacement amplitude functions $U(\xi,\zeta), V(\xi,\zeta), and\ W(\xi,\zeta)$are written in the form of algebraic polynomials multiplied by a boundary function as follows;

$$U(\xi,\zeta) = f_1(\xi,\zeta) \sum_i^I \sum_j^J A_{ij} \, \xi^i \zeta^j,$$

$$V(\xi,\zeta) = f_2(\xi,\zeta) \sum_k^K \sum_l^L B_{kl} \, \xi^k \zeta^l,$$

$$W(\xi,\zeta) = f_3(\xi,\zeta) \sum_p^P \sum_q^Q C_{pq} \, \xi^p \zeta^q,$$

where i, j, k, l, p and q are integers andA_{ij}, B_{kl}, C_{pq} are coefficients yet to be determined. The $f_i(\xi,\zeta)$ is a function to fit the prescribed geometrical boundary condition. Thus, each displacement may have different fitting function. If the 3D circular cylinder has completely free boundaries, then $f_i(\xi,\zeta)$ is just unity.

Then for completely free case, the assumed displacements function are as follow:

1) For the torsional modes(n=0), the displacements are given by

$$V(\xi,\zeta) = \sum_k^K \sum_l^L B_{kl} \, \xi^k \, \zeta^l$$

2) For the axisymmetric modes (n=0), the displacements are given by

$$U(\xi,\zeta) = \sum_i^I \sum_j^J A_{ij} \, \xi^i \zeta^j,$$

$$W(\xi,\zeta) = \sum_p^P \sum_q^Q C_{pq} \, \xi^p \zeta^q$$

Ritz method and its mathematical solvers

The minimizing conditions are follows:

$$\frac{\partial (V_{max} - T_{max})}{\partial A_{ij}} = 0$$

$$\frac{\partial (V_{max} - T_{max})}{\partial B_{kl}} = 0$$

$$\frac{\partial (V_{max} - T_{max})}{\partial C_{pq}} = 0$$

By applying the minimizing conditions in terms of the natural polynomials, the Ritz method results in an algebraic set of equations as given below:

For torsional mode

$$([K^{vv}]-\omega^2)[B]=[0], n=0$$

For axisymmetric mode

$$\begin{bmatrix}[K^{uu}] & [K^{uw}]\\ [K^{uw}] & [K^{ww}]\end{bmatrix}-\omega^2\begin{bmatrix}[M^{uu}] & 0\\ 0 & [M^{ww}]\end{bmatrix}\begin{bmatrix}[A]\\ [C]\end{bmatrix}=\begin{bmatrix}[0]\\ [0]\end{bmatrix}, n=0$$

where $[K^{ij}]$ and $[M^{ij}]$ (i, j=u, v, w) are the stiffness sub-matrices and the mass sub-matrix, respectively. The column vectors [A], [B], [C] are composed of the unknown coefficients as follows;

$$[A]=(A_{1j}\ \ldots..A_{1J}\ \ldots.A_{21}\ \ldots..A_{2j}\ \ldots..A_{I1}\ \ldots..A_{IJ})^T;$$
$$[B]=(B_{1j}\ \ldots..B_{1J}\ \ldots.B_{21}\ \ldots..B_{2j}\ \ldots..B_{I1}\ \ldots..B_{IJ})^T;$$
$$[C]=(C_{1j}\ \ldots..C_{1J}\ \ldots.C_{21}\ \ldots..C_{2j}\ \ldots..C_{I1}\ \ldots..C_{IJ})^T$$

For a nontrivial solution the determinant of the square coefficient matrix of these equations is set equal to zero. The zeros of this determinant are the nondimensional frequency parameters.

Evaluation of the integrands was carried out by symbolic computing in MATLAB.

Convergence and comparison:-

Table I show the results of dimensionless frequencies for the symmetric and torsional modes with free- free boundary case with $\frac{L}{D}=5$. In Table 1, there are five columns of data corresponding to the five dimensionless frequencies and six groups of rows, each indicating that different set of polynomial terms has been used. TR, TZ, DET represent the numbers of polynomial terms used along ξ, ζ and the size of determinant, respectively , in the displacements function. In torsional modes, $f_2(\xi,\zeta)$=1, and i=1,2,3….; j=0,2,4,…;(even numbers only). The convergence show in Table 1 start with small values of TR =1, TZ = 2, and increases TZ until the frequencies are converged to six significant figures. Then TR=2, 3, 4, 5..: is increased and process is repeated.

Table II show the results of dimensionless frequencies for the antisymmetric and torsional modes with free- free boundary case with $\frac{L}{D}=5$. In antisymmetric and torsional modes case $f_2(\xi,\zeta)$=1, and i=1, 2, 3…; j=1, 3, 5...; (odd number only). The same process is repeated as explained before.

Table 1: Dimensionless frequencies for the five lowest symmetric and torsional modes, where $\frac{L}{D}=5$ with free – free boundary case for solid cylinder.

TR	TZ	DET	1	2	3	4	5
1	2	2	0.77460	-	-	-	-
1	4	4	0.62833	1.29583	3.61193	-	-
1	6	6	0.62832	1.25665	1.88497	2.95793	8.45369
1	8	8	0.62832	1.25664	1.88497	2.51990	3.33490
2	2	4	0.77460	6.12372	6.17252	-	-
2	4	8	0.62833	1.29583	3.61193	6.12372	6.15588
2	6	12	0.62832	1.25665	1.90047	2.95793	6.12372
2	8	16	0.62832	1.25664	1.88497	2.51990	3.33490
3	2	6	0.77460	5.20034	5.25771	11.4217	11.4479
3	4	12	0.62833	1.29583	3.61193	520034	5.23816
3	6	18	0.62832	1.25665	1.90047	2.95793	5.20034
4	2	8	0.77460	5.14665	5.20461	8.68071	8.71520
4	4	16	0.62833	1.29583	3.61193	5.14665	5.18486
5	2	10	0.77460	5.13596	5.19404	8.51234	8.54751
5	4	20	0.62833	1.29583	3.61193	5.13596	5.17425
6	2	12	0.7746	5.13565	5.19374	8.42258	8.45812

Table 2: Dimensionless frequencies for the five lowest antisymmetric and torsional modes, where $\frac{L}{D} = 5$ with free – free boundary case for solid cylinder

TR	TZ	DET	1	2	3	4	5
1	2	2	0.31425	1.30432	-	-	-
1	4	4	0.31416	0.94268	1.66063	4.63195	-
1	6	6	0.31416	0.94248	1.57093	2.23464	3.46884
1	8	8	0.31416	0.94248	1.57080	2.19919	2.84288
1	10	10	0.31416	0.94248	1.57080	2.19911	2.82747
2	2	4	0.31425	1.30432	6.13178	6.26109	-
2	4	8	0.31416	0.94268	1.66063	4.63195	6.13178
2	6	12	0.31416	0.94268	1.57093	2.23464	3.46884
2	8	16	0.31416	0.94248	1.5080	2.19919	2.84288
3	2	6	0.31425	1.30432	5.20982	5.36141	11.4260
3	4	12	0.31416	0.94268	1.66063	4.63195	5.20982
3	6	18	0.31416	0.94248	1.57093	2.23464	3.46884
4	2	8	0.31425	1.30432	5015623	5.30935	8.68640
4	4	16	0.31416	0.94268	1.66063	4.63195	5.15622
5	2	10	0.31425	1.30432	5.14556	5.29899	8.51814
5	4	20	0.31416	0.94268	1.66063	4.63195	5.14556
6	2	12	0.31425	1.30432	5.1426	5.29869	8.42844
6	4	24	0.31416	0.94268	1.66063	4.63195	5.14525

Table III show the results of dimensionless frequencies for the five lowest symmetric and axisymmetric modes with free- free boundary case with$\frac{L}{D} = 5$. In this case the powers of ζ in U, W should be even and odd, respectively. The same process is repeated as explained before.

Table IV show the results of dimensionless frequencies for the five lowest antisymmetric and axisymmetric modes with free- free boundary case with$\frac{L}{D} = 5$. In this case the powers of ζ in U, W should be odd and even, respectively. The same process is repeated as explained before.

Table 3: Dimensionless frequencies for the five lowest symmetric and axisymmetric modes, where$\frac{L}{D} = 5$ and ν=0.3 with free – free boundary case for solid cylinders

TR	TZ	DET	1	2	3	4	5
2	2	8	0.50562	2.00163	3.79125	4.03860	4.57181
2.	4	16	0.50547	1.48639	2.47842	3.03776	3.73760
2.	6	24	0.50547	1.48604	2.33501	2.94336	3.05168
2	8	32	0.50547	1.48603	2.33465	2.90491	3.02468
3	2	12	0.50556	2.00057	3.73661	4.02134	4.38840
3	4	24	0.50542	1.48499	2.46172	3.02324	370269
3	6	36	0.50542	1048462	2.32722	2.93028	3.02818
3	8	48	0.50542	1.48462	2.32693	2.88644	3.01082
4	2	16	0.50556	2.00034	3.62360	4.00464	4.27210
4	4	32	0.50542	1.48498	2.45966	2.96124	3.61607
4	6	48	0.50542	1.48461	2032676	2.91794	2.97994
5	2	20	0.50556	2.00034	3.62292	4.00453	4.26969
5	4	40	0.50542	1.48461	2.45927	2.96068	3.61571
6	2	24	0.50556	2.00034	3.62249	4.00448	4.26914
6	4	48	0.50542	1.48497	2.45922	2.96049	3.61541
7	2	28	0.50556	2.00034	3.62249	4.00448	4.26913
7	4	56	0.50542	1.48497	2.45921	2.96048	3.61541

Table 4: Dimensionless frequencies for the five lowest antisymmetric and axisymmetric modes, where $\frac{L}{D} = 5$ and ν=0.3 with free – free boundary case for solid cylinder

TR	TZ	DET	1	2	3	4	5
2	2	8	1.23083	3.60345	3.92150	4.40029	4.90298
2.	4	16	1.00395	1.99695	3.02243	3.69982	3.79375
2.	6	24	1.00394	1.93704	2.67589	3.00968	3.53125
2	8	32	1.00394	1.93700	2.65810	3.00137	3.16450
3	2	12	1.23068	3.58327	3.86305	4.23845	4.65875
3	4	24	1.00356	1.99093	3.01480	3.67519	3.67519
3	6	36	1.00354	1.93339	2.66186	3.00206	3.47631
3	8	48	1.00354	1.93337	2.64419	2.99481	3.12353
4	2	16	1.23068	3.50186	3.74564	4.16121	4.54985
4	4	32	1.00356	1.99057	2.96173	3.60239	3.69113
4	6	48	1.00354	1.93328	2.66073	2.94799	3.45645
5	2	20	1.23068	3.50171	3.74457	4.15966	4.54385
5	4	40	1.00356	1.99053	2.96146	3.60223	3.69058
6	2	24	1.23068	3.50147	3.74406	4.15935	4.54309
6	4	48	1.00356	1.99053	2.96134	3.60202	3.69029
7	2	28	1.23068	3.50147	3.74406	4.15935	4.54307

The frequency values presented in the above four tables are compare well, as is supposed to since the same process is used, with those given in reference [1].

Conclusion:-

The frequencies of a completed free circular cylinders is determined with the help of 3D method of analysis. The lowest five frequencies of symmetric and torsional modes, anti-symmetric and torsional modes, where$\frac{L}{D} = 5$and the lowest five frequencies of axisymmetric and symmetric modes, axisymmetric and antisymmetric modes, for $\frac{L}{D} = 5$ and ν=0.3 are calculated and tabulated. From the above mentioned results, it can be concluded that these frequency values match with published results confirming meeting of the objective of the present study.

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