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RESEARCH ARTICLE

COUETTE FLOW WITH TRANSVERSE MAGNETIC FIELD AND ITS APPLICATION TO JOURNAL BEARING.

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Abstract

In this paper, influence of transverse magnetic field has been studied on a steady, incompressible and fully developed laminar flow between two infinitely large non-conducting parallel plates. The fluid between the two plates is viscous and electrically conducting in nature. A mathematical model has been developed and the governing differential equations developed are solved using conventional methods.The essence of this work lies in analytical solution obtained which has been compared with differential equation solver and found correct. Effect of Hartman number (M) for different Reynolds number(R) has been studied on velocity profile and coefficient of skin friction. The results thus obtained have been used to solve the problem of variable loading in a journal bearing by using electrically conducting fluid as a lubricant.

Introduction:-

In recent years a lot of study has been carried out regarding the flow characteristic of electrically conducting fluid due to its many practical applications. A great deal of effort has been made to understand the interaction between fluid flow characteristic and magnetic field. The simplest example of electrically conducting fluid is a liquid metal like mercury, liquid sodium. The flow of electrically conducting fluids in the presence of applied transverse magnetic field has application in many devices such as magneto hydro dynamic(MHD) pumps ,accelerators , generators, petroleum industries, purification of molten metal from non-metallic inclusion. It has found application in MHD casting of steels. It is also used to develop pulsed magneto plasma thrusters for rocket propulsion system which seem to be very effective for long range missions. Hartmann (1937) has done pioneer work in the study of steady MHD flow. Verma and Mathur (1968) have also studied MHD couette flow. In this paper, we have analyzed the flow of conducting fluid with a transverse magnetic field and a constant pressure gradient.

In case of journal bearing, the flow between the shaft and housing is nearly similar to the flow between two parallel plates. High load journal bearing undergoes severe loading conditions .For these type of bearings the same lubricant cannot be used for both high and low loads because during high load and high temperature conditions lubrication may fail and during light load conditions the same lubricant may be very resistive for the motion of the shaft. Thus the lubricant needs to be changed for every time the load varies. This problem can be resolved by using a MHD bearing. The load carrying capacity can be varied by controlling the magnitude of magnetic field. So Couette flow with transverse magnetic field is analyzed in this paper.

Formulation of problem:-

Let us consider a shear flow between two plates A and B. B is moving with uniform velocity U as shown in Fig.1. Let the plates be separated by a distance L. A uniform transverse magnetic field of strength B has been applied. When a conducting fluid moves through the magnetic field $\overline{B}$ and electric field $\overline{E}$, the particles move in such a way that they give rise to electric current density

$\bar{J}=\sigma(\bar{E}+\bar{V}\times\bar{B}')$ (1)

Where σ is electrical conductivity of fluid, $\bar{J}$ is current density and $\bar{B}'=\bar{B}+\bar{b}$,where the quantity $\bar{b}$ is the magnetic induction induced due to the electric current density $\bar{J}$. In this analysis $\bar{b}$ has been considered as a perturbation on basic field strength and is neglected in comparison with $\bar{B}$, i.e. $|\bar{B}| >> |\bar{b}|$. In our case externally applied electric field is assumed to be zero. However in any small but finite volume the number of particles with negative and positive charges are assumed to be nearly equal. Thus there is no net charge density.

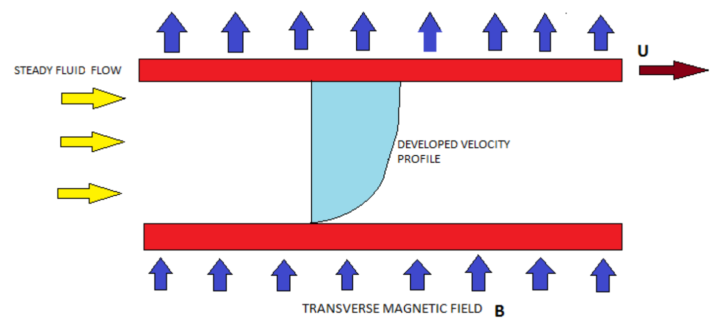

Fig.1

For an incompressible, viscous flow with magnetic field the Navier-Stokes equation (N.S.E) is

$$\rho\frac{D\bar{V}}{Dt} = -\bar{\nabla}p + \mu\nabla^2\bar{V} + \bar{F}_m + \rho\bar{g} \quad (2)$$

Here, $\bar{F}_m=\bar{J}\times\bar{B}$ is magnetic body force per unit volume.In our case $\bar{B}= B\bar{j}$ and $\bar{V}= u\bar{i}$ and $\bar{E}=0$, so

$$\bar{F}_m = \sigma\,(u\bar{i}\times B\bar{j})\times B\bar{j} = -(\sigma B^2 u)\bar{i} \quad (3)$$

Thus x and y components of NSE become

$$\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right) = -\frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right) - \sigma B^2 u \quad (4)$$

$$\rho\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right) = -\frac{\partial p}{\partial y} + \mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right) - \rho g \quad (5)$$

We know that the Continuity equation

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad (6)$$

In our case v=0, flow is fully developed and steady so

$\frac{\partial u}{\partial x}=0$ and $\frac{\partial u}{\partial t}=0$

Thus (4) and (5) become

$$\mu\frac{d^2 u}{dy^2} - \sigma B^2 u - \frac{\partial p}{\partial x}=0 \quad (7)$$

$$\frac{\partial p}{\partial y} = -\rho g \quad (8)$$

From (8), $p=-\rho gy+f(x)+C \Rightarrow \frac{\partial p}{\partial x}=f'(x)$.Thus we can say that $\frac{\partial p}{\partial x}$ is a function of x only and can be taken as a constant to evaluate (7).

Method of solution

Equation (7) obtained is a second order linear differential equation

$$\frac{d^2u}{dy^2}-\frac{\sigma u B^2}{\mu}=\frac{1}{\mu}\frac{\partial p}{\partial x} \quad (9)$$

Primary solution of (9) is

$$u_p = c_1 e^{\sqrt{\frac{\sigma}{\mu}}By} + e^{-\sqrt{\frac{\sigma}{\mu}}By} \qquad (10)$$

And its particular integral is

$$u_{pi} = -\frac{\frac{1}{\mu}\frac{\partial p}{\partial x}}{\left(D^2-\frac{\sigma B^2}{\mu}\right)} \quad (11)$$

Where, $D=\frac{d^2}{dy^2}$. After solving this we get

$$u_{pi} = -\frac{\frac{1}{\mu}\frac{\partial p}{\partial x}}{\frac{\sigma B^2}{\mu}} \quad (12)$$

So our solution becomes

$$u = c_1 e^{\sqrt{\frac{\sigma}{\mu}}By} + c_2 e^{-\sqrt{\frac{\sigma}{\mu}}By} - \frac{\frac{1}{\mu}\frac{\partial p}{\partial x}}{\frac{\sigma B^2}{\mu}} \qquad (13)$$

Boundary conditions are y=0, u=0 and y=L, u=U.The term $\sqrt{\frac{\sigma}{\mu}}By$ is non-dimensional and can also be written as $\left(\sqrt{\frac{\sigma}{\mu}}BL\right)\left(\frac{y}{L}\right)$

Where $\sqrt{\frac{\sigma}{\mu}}BL$ is the Hartmann number M,$\frac{y}{L}=Y$

After substituting the boundary conditions in (13) we get

$$u=\frac{\left(U+\frac{L^2}{M^2}\frac{1}{\mu}\frac{\partial p}{\partial x}\right)\sinh\left(M\frac{y}{L}\right)+\frac{L^2}{M^2}\frac{1}{\mu}\left(\frac{\partial p}{\partial x}\right)\sinh M\left(1-\frac{y}{L}\right)}{\sinh M} - \frac{L^2}{M^2}\frac{1}{\mu}\left(\frac{\partial p}{\partial x}\right) \quad (14)$$

Where the term $\frac{L^2}{M^2}\frac{1}{\mu}\left(\frac{\partial p}{\partial x}\right)$ can also be written as $U\left(\frac{\rho UL}{\mu}\right)\left(\frac{\frac{\partial p}{\partial x}}{\frac{\rho U^2}{L}}\right)$ in which $\frac{\rho UL}{\mu}$= Reynolds number R, $\frac{\frac{\partial p}{\partial x}}{\frac{\rho U^2}{L}}$= Pressure gradient parameter P.

So (14) can be written as

$$\frac{u}{U}=\frac{\left(1+\frac{RP}{M^2}\right)\sinh(MY)+\frac{RP}{M^2}\sinh M(1-Y)}{\sinh M} - \frac{RP}{M^2} \quad (15)$$

Equation (15) is a non-dimensional solution for velocity profile. For coefficient of skin friction we need shear stress at upper plate given by

$\tau_w = \mu(\frac{du}{dy})_{y=L}$(16)

From (15)

$$(\frac{du}{dy})_{y=L} = U\left(1+\frac{RP}{M^2}\right)\frac{M}{L}\cos hM - U\frac{RP}{ML}(17)$$

Thus

$$\tau_w = \mu U\left(\left(1+\frac{RP}{M^2}\right)\frac{M}{L}\cos hM - \frac{RP}{ML}\right)(18)$$

So the coefficient of skin friction is

$$C_f = \frac{2\tau_w}{\rho U^2} = \frac{2\mu U}{\rho U^2}\left(\left(1+\frac{RP}{M^2}\right)\frac{M}{L}\cos hM - \frac{RP}{ML}\right)$$

$$C_f = 2\left(\left(1+\frac{RP}{M^2}\right)\frac{M}{R}\cos hM - \frac{P}{M}\right)(19)$$

Equation (19) is a non-dimensional solution for coefficient of skin friction

Results and discussion:-

The variation of velocity profile with Hartmann number M, Reynolds number R and pressure gradient parameter P is shown in Fig.2, Fig.3 and Fig.4. As M approaches zero, (15) can be simplified into classical Couette flow solution. This nature can also be observed from Fig. 2. As M is increased, influence of magnetic field also increases and it starts to dominate over the pressure gradient and hence resist the flow. From Fig. 5, we can observe that for a particular R, coefficient of skin friction increases with the increase of M. Thus the viscous nature of the fluid gets enhanced. From Fig.2 it is clear that at higher M velocity becomes very small near the stationary plate which gives rise to very large velocity gradient at the upper plate. Thus the coefficient of skin friction increases. This concept can be used for the proper working of the journal bearing with varying load condition. So we can vary load carrying capacity of journal bearing with magnetic field. Fig. 4 shows the variation of velocity profile with variation of pressure gradient parameter P. It can be seen that as long as the pressure gradient is negative it opposes the domination of magnetic force. But if the pressure gradient is applied in reverse direction, it starts to support the magnetic field and chances of back flow increase. From Fig. 5, as the pressure gradient increases for a particular Hartmann number the coefficient of friction is found to decrease.

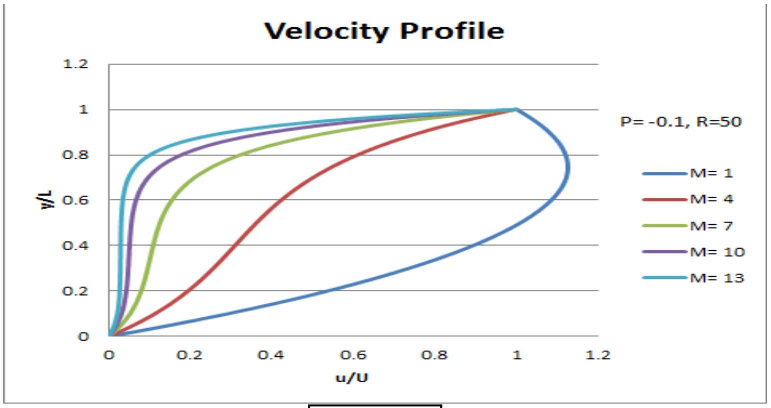

Fig 2

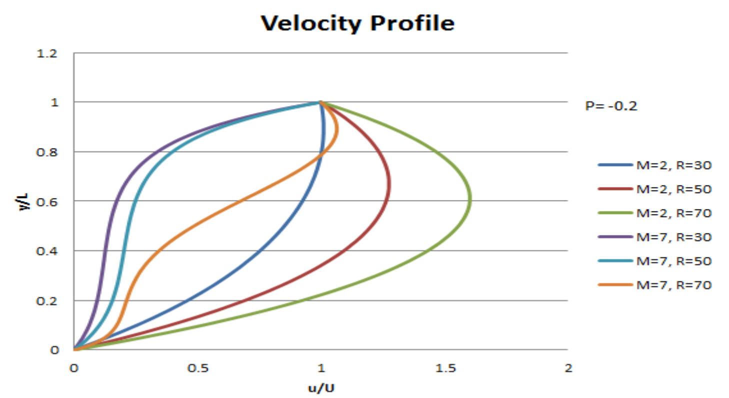

Fig. 3

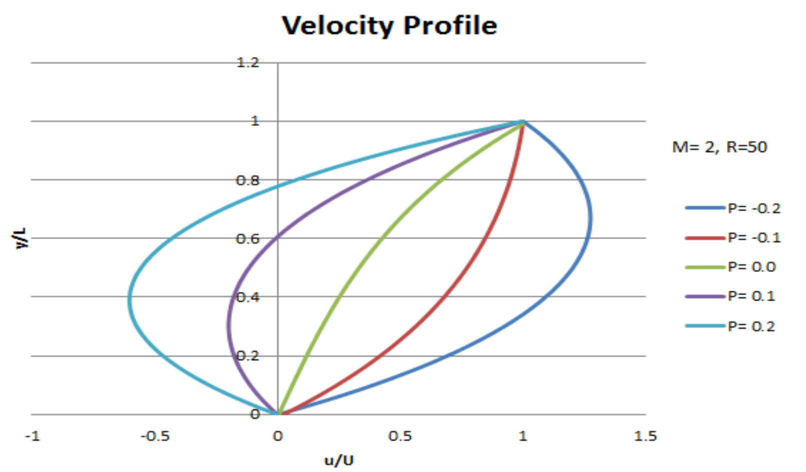

Fig.4

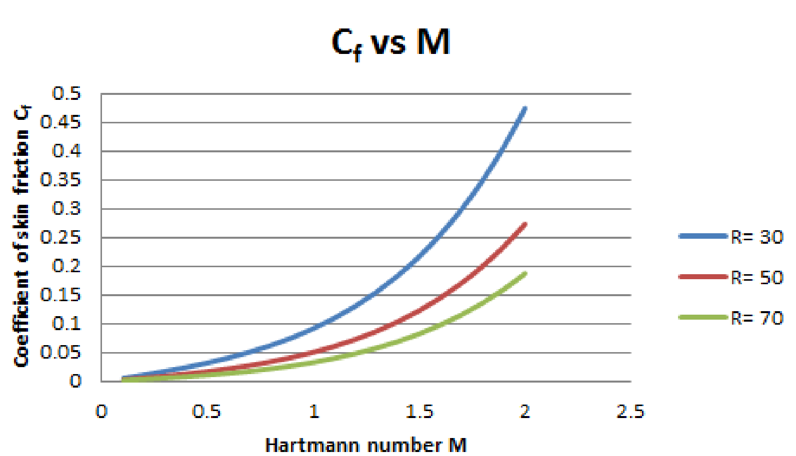

Fig. 5

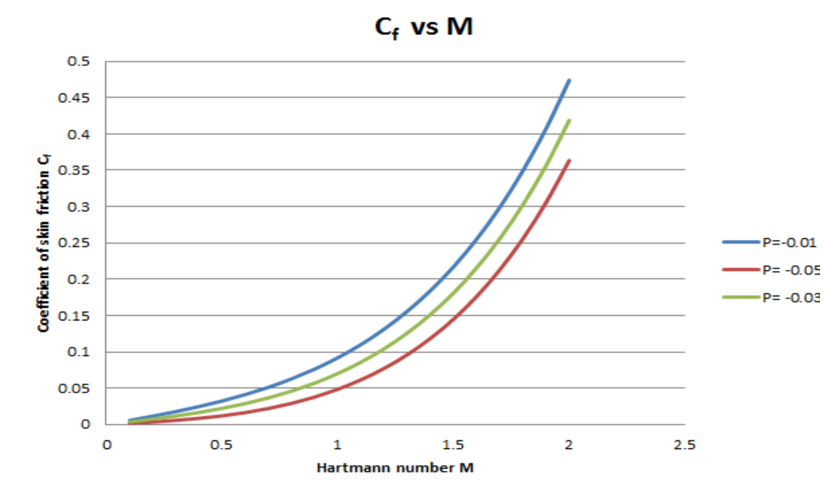

Fig. 6

Approximateanalysis for journal bearing:-

For a designer it is very important to know the value of Sommerfeld number S to design a journal bearing because it includes all the parameter which are important for the designing of the journal bearing. The designer knows the variation of Sommerfeld number of bearing with the load. If the lubricant used is an electrically conducting fluid and designer is told the variation of Sommerfeld number with Hartmann number M, he can design the bearing accordingly.

The Sommerfeld number of a bearing is given by

$$S = \frac{\mu N}{P}\left(\frac{r}{c}\right)^2 \quad (20)$$

Rearranging the terms of (20), we get

$$S = \frac{\frac{1}{4\pi} C_f \frac{r}{c}}{\frac{P}{\rho U^2}} \quad (21)$$

Where, $\frac{r}{c}$ = clearance ratio, $\frac{P}{\rho U^2}$ = P' which is a non- dimensional term.Substituting the value of C_f from 19, we get

$$S = \frac{\frac{1}{4\pi}\left(2\left(1+\frac{RP}{M^2}\right)\frac{M}{R}\cosh M - \frac{P}{M}\right)\frac{r}{c}}{\frac{P}{\rho U^2}} \quad (22)$$

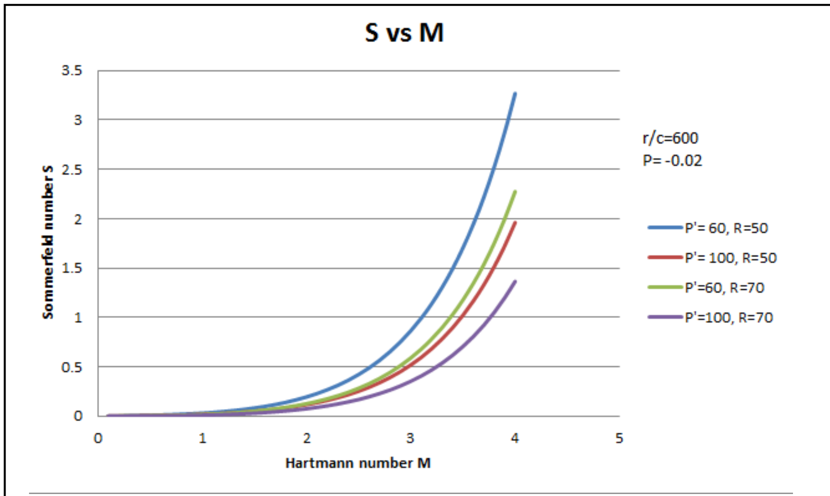

Fig. 7

From Fig. 7, we notice that Sommerfeld number increases with increase of Hartmann number. Thus, the designer can choose the value of Hartmann number for a given value of Sommerfeldnumber and magnetic field can be set accordingly.

For example, a journal bearinghaving clearance ratio of 600, R = 50, P' = 60 is running at a particular load with S= 0.8. Suddenly load becomes double at the same running speed. Then from (20), S becomes 0.4. Thus viscous nature of lubricant must be enhanced to sustain the load. This can be done by increasing the Sommerfeld number and hence the value of Hartmann number.

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