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RESEARCH ARTICLE

OPERATORS USED IN COMPLEX VALUED HARMONIC UNIVALENT AND MULTIVALENT FUNCTION.

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Abstract

To evaluate the effect of implant platform/abutment design/ crown material combinations on the stress distribution around implant-supported dental restorations. A literature search was made in three databases including PubMed, Cochrane and Web of Science. Inclusion criteria were in vitro studies, switched implant platform versus regular implant platform, titanium implants, internal hex connection and stress values of bone. Two review authors independently screened the articles for inclusion. This was followed by hand searching in the reference lists of all eligible studies for additional studies. Results: the search resulted in 16 eligible studies concerning the effect of platform switching on peri-implant bone stress, however no papers were found studying the effect of different implant platform/ abutment design /crown material complexes on bone stress. From the included studies, platform switching concept can replace conventional platform designs to improve implant survival rate, provided it should be used within its indications.

Harmoic Univalent Function:-

A continuous complex valued function f=u+iv defined in a simply connected complex domain D is said to be harmonic in D if both u and v are real harmonic in D. Let F and G be analytic in D so that F(0)=G(0)=0,

ReF=Ref=u, ReG=Imf=v by writing (F+iG)/2=h, (F-iG)/2 = g, The function f admits the representation $f = h + \overline{g}$, where h and g are analytic in D. h is called the analytic part of f and g, the co-analytic part of f. Clunie and Sheil-Small [15] observe that $f = h + \overline{g}$ is locally univalent and sense-preserving if and only if $|g'(z)| < |h'(z)|, z \in D$. Further if f can be normalize so that f(0)=h(0)= $f_z(0) - 1 = 0$. The SH denotes the family of all hamonic, complex valued, orientation-preserving normalized univalent functions defined on Δ. Thus the function f in SH admits the representation $f = h + \overline{g}$, where,

(1.1.1) $$h(z) = z + \sum_{n=2}^{\infty} a_n z^n, \text{ and } g(z) = \sum_{n=1}^{\infty} b_n z^n; \ |b_1| < 1$$

are analytic functions in Δ.

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It follows from the orientation-preserving property that $|b_1|<1$. Therefore, $(f-\overline{b_1 f})/(1-|b_1|^2)\in SH$ whenever $f\in SH$. Thus a subclass SH^0 of SH is defined by $SH^0=\{f\in SH: g'(0)=b_1=0\}$.

Note that $S\subset SH^0\subset SH$. Both families SH and SH^0 are normal families. That is every sequence of functions in SH (or SH^0) has a subsequence that converges locally uniformly in Δ.

It is noted that $SH\equiv S$ if g=0.
Let TH denote the sub class of SH with negative coefficients whose members $f=h+\overline{g}$ where h and g are of the form

(1.1.2) $h(z)=z-\sum_{n=2}^{\infty}|a_n|z^n$ and $g(z)=\sum_{n=1}^{\infty}|b_n|z^n,\ \ |b_1|<1,\ \ z\in\Delta$.

Complex Valued Harmonoic Multivalent Function:-
Let f be a harmonic function in a Jordan domain D with boundary C. Suppose f is continuous in $\bar{D}$ and $f(z)\neq 0$ on C. Suppose f has no singular zeros in D, and let m to be sum of the orders of the zeros of f in D. Then $\Delta_c \arg(f(z))=2\pi m$, where $\Delta_c \arg(f(z))$ denotes the change in argument of f(z) as z traverses C.

It is also shown that if f is sense-preserving harmonic function near a point z_0, where $f(z_0)=\omega_0$ and if $f(z)-\omega_0$ has a zero of order m $(m\geq 1)$ at z_0, then to each sufficiently small $\in>0$ there corresponds a $\delta>0$ with the property: “for each $\alpha\in N_\delta(\omega_0)=\{\omega:|\omega-\omega_0|<\delta\}$, the function $f(z)-\alpha$ has exactly m zeros, counted according to multiplicity, in $N_\in(z_0)$”. In particular, f has the open mapping property that is, it carries open sets to open sets.

Let Δ be the open unit disc $\Delta=\{z:|z|<1\}$ also let $a_k=b_k=0$ for $0\leq k<m$ and $a_m=1$. Ahuja and Jahangiri [5], [9] introduce and studied certain subclasses of the family SH(m), $m\geq 1$ of all multivalent harmonic and orientation preserving functions in Δ. A function f in SH(m) can be expressed as $f=h+\overline{g}$, where h and g are of the form

(1.2.1) $h(z)=z^m+\sum_{n=2}^{\infty}a_{n+m-1}z^{n+m-1}$

$g(z)=\sum_{n=1}^{\infty}b_{n+m-1}z^{n+m-1},\ \ |b_m|<1$.

According to above argument, functions in SH(m) are harmonic and sense-preserving in Δ if $J_f>0\ \text{in}\ \Delta$. The class SH(1) of harmonic univalent functions was studied in details by Clunie and Sheil Small [15]. It was observed that m-valent mapping need not be orientation-preserving.
Let TH(m) denotes the subclass of SH(m) whose members are of the form

(1.2.2) $h(z)=z^m-\sum_{n=2}^{\infty}|a_{n+m-1}|z^{n+m-1}$

and

$$g(z)=\sum_{n=1}^{\infty}|b_{n+m-1}|z^{n+m-1},\ \ |b_m|<1.$$

Let $SH(m), m \geq 1$ denotes the class of functions $f = h + \overline{g}$ that are m-valent harmonic and orientation-preserving functions in the unit disc $\Delta = \{z : |z| < 1\}$ for which $f(0) = f_z(0) - 1 = 0$. Then f in SH(m) can be expressed as $f = h + \overline{g}$, where h and g are analytic functions of the form

(1.2.3) $h(z) = z^m + \sum_{n=2}^{\infty} a_{n+m-1} z^{n+m-1},\ g(z) = \sum_{n=1}^{\infty} b_{n+m-1} z^{n+m-1}, |b_m| < 1$

Note that $SH^0(m) \subseteq SH(m)$ with $b_m = 0$.
Also TH(m) denote the class of functions $f = h + \overline{g}$ so that h and g are of the form :

(1.2.4) $h(z) = z^m - \sum_{n=2}^{\infty} |a_{n+m-1}| z^{n+m-1}, g(z) = \sum_{n=1}^{\infty} |b_{n+m-1}| z^{n+m-1}, |b_m| < 1$

Hardmard Product:-
The Hadamard product (or convolution) of two analytic functions $f_1(z)$ and $f_2(z)$ is defined by

$$(f_1 * f_2)(z) = (f_2 * f_1)(z) = \sum_{n=0}^{\infty} c_n d_n z^n$$

where $f_1(z) = \sum_{n=0}^{\infty} c_n z^n$ and $f_2(z) = \sum_{n=0}^{\infty} d_n z^n, z \in \Delta$.
The Pochhammer symbol $(\lambda)_n$ is given by

$$(\lambda)_n := \frac{\Gamma(\lambda+n)}{\Gamma(\lambda)} = \begin{cases} 1 (n=0) \\ \lambda(\lambda+1) - -(\lambda+n-1)(n \in N), \end{cases}$$

Consider a function $\phi_m(a, c; z)$, defined as

(1.3.1) $\phi_m(a, c; z) = z^m F(a, 1; c; z) = \sum_{n=0}^{\infty} \frac{(a)_n}{(c)_n} z^{n+m}$

$$= z^m + \sum_{n=2}^{\infty} \frac{(a)_{n-1}}{(c)_{n-1}} z^{n+m-1}$$

$(a \in R; c \in R \setminus Z_0^-, Z_0^- := \{0, -1, -2, \ldots\}; z \in \Delta)$.
where $F(a, 1; c; z)$ is well known Gauss hypergeometric function.

Linear Operator:-
Corresponding to the function $\phi_m(a, c; z)$ a linear operator $L_m(a, c)$ on the analytic functions of the form (1.1.1) is considered which is defined by means of the following Hadamard product :
(1.4.1) $L_m(a, c) h(z) = \phi_m(a, c; z) * h(z)$.
The linear operator of the harmonic function $f = h + \overline{g}$, where h and g are given by (1.1.1) is defined as
(1.4.1) $L_m(a, c) f(z) = L_m(a, c) h(z) + \overline{L_m(a, c) g(z)}$
where,

$$L_m(a, c) h(z) = z^m + \sum_{n=2}^{\infty} \frac{(a)_{n-1}}{(c)_{n-1}} a_{n+m-1} z^{n+m-1}$$

and

$$L_m(a,c)g(z)=\sum_{n=1}^{\infty}\frac{(a)_n}{(c)_n}b_{n+m-1}z^{n+m-1};\quad a\,|\,b_m\,|<c.$$

Salagean Operator:-

For analytic function $h(z)\in S(m)$ Salagean [33] introduced an operator D_m^{ν} defined as follows:

$D_m^0 h(z)=h(z)$, $D_m^1 h(z)=D_m(h(z))=\dfrac{z}{m}h'(z)$ and

$$D_m^{\nu}h(z)=D_m(D_m^{\nu-1}h(z))=\frac{z(D_m^{\nu-1}h(z))'}{m}$$

$$=z+\sum_{n=2}^{\infty}\left(\frac{n+m-1}{m}\right)^{\nu}a_{n+m-1}z^{n+m-1},\ \nu\in N.$$

Whereas, Jahangiri et al. [22] defined the Salagean operator $D_m^{\nu}f(z)$ for multivalent harmonic function as follows:

(1.5.1) $$D_m^{\nu}f(z)=D_m^{\nu}h(z)+(-1)^{\nu}\overline{D_m^{\nu}g(z)}$$

where,

$$D_m^{\nu}h(z)=z^m+\sum_{n=2}^{\infty}\left(\frac{n+m-1}{m}\right)^{\nu}a_{n+m-1}z^{n+m-1}$$

$$D_m^{\nu}g(z)=\sum_{n=1}^{\infty}\left(\frac{n+m-1}{m}\right)^{\nu}b_{n+m-1}z^{n+m-1}.$$

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