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RESEARCH ARTICLE

VORTEX EXCITATIONS UNDER COLLISION BETWEEN A SUPERFLOWING RING-SHAPED BEC AND A CENTERED STATIC BEC.

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Abstract

By performing numerical calculations of the quasi-two-dimensional GP equation, the dynamical generation and evolution of vortex excitations under collision between a superflowing ring-shaped BEC and a centered static BEC are investigated. We study the collision between a superflowing ring-shaped BEC and a static centered BEC. Symmetrically positioned singly quantized vortices can be created and their time evolution mainly depends on the atomic interactions.

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Introduction:-

Quantized vortex is a topological singularity and one of the hallmarks in a superfluid or superconductor, where the phase of the order parameter varies by integral multiples of 2π when following a closed path around the defect.

Since the quantized vortices have been successfully created in a trapped Bose-Einstein Condensate(BEC) of ultracold atomic due to their experimental versatility [1], more and more theoretical and experimental researches are attracted by BEC to investigate the characteristics of quantum vortices. The research progress of the ultracold atomic gases in recent years have also demonstrated BEC is an ideal platform for studying the static and dynamical properties of vortex. In experiment, several methods have been mainly employed to create quantum vortices such as stirring BEC with a laser beam [2-3], trap rotation [4-5], and phase imprintings [6-7].

Except for the studies on static vortices[8-9], numerous studies have been performed to evaluate the dynamical properties including the nucleation [10] and decay of an off-centered vortex[11-12]. However, there still have little studies on the dynamical behaviors of BEC collision. In Ref[13-14], the behavior of collision and interference dynamics of BEC is found to be related with the initial displacement and phase difference. The collision dynamics of vortices in a two-dimensional spinor Bose-Einstein condensate and vortex dipoles propagating in opposite directions are also investigated [15].

In this paper, by solving the time-dependent Gross-Pitaevskii (GP) equations, we study both the dynamical generation and evolution of vortex excitations under collision between a superflowing ring-shaped BEC and a centered static BEC. We take two cases to perform our calculations according to the atoms being identical or not. Quantized vortices are found to be created symmetrically and their evolution with mainly depends on the atomic interactions.

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Model:-

We consider a single BEC described by the macroscopic wave function $\psi(\vec{r}, t)$. In the mean-field framework, the dynamics of a system with N identical atoms close to thermo-dynamic equilibrium and subject to weak dissipation can be described by the dissipative Gross-pitaevskii(GP) equation[16]:

$$i\eta\partial_t\psi(\vec{r}, t) = (1-\gamma)\left[\frac{\eta^2\nabla^2}{-2m} + V(\vec{r}) + g|\psi(\vec{r}, t)|^2\right]\psi(\vec{r}, t), \quad (1)$$

where $V(\vec{r}) = \frac{1}{2}m(\omega_x^2x^2 + \omega_y^2y^2 + \omega_z^2z^2)$ is the axially symmetric harmonic trap potential. $\omega_x = \omega_y = \omega_t$ is the radial trap potential frequency and ω_z presents the axial trap potential frequency. A small dimensionless parameter γ gives a phenomenological description of the dissipation processes due to interactions and thermal excitations.

We focus in this paper on a pancake-shaped situation with $\omega_z \gg \omega_t$. In this extreme limit, the axial dimension is sufficiently thin that the motion along z direction can be neglected and atoms can move only within the xy plane. The wave function of BEC $\psi(\vec{r}, t)$ is normalized according to $\int d\vec{r}\psi^*(\vec{r}, t)\psi(\vec{r}, t) = N$. The last term in equation is the atom-atom contact interaction which characterized by $g = \frac{4\pi\eta^2a_s}{m}$ with a_s the s -wave scattering length and m the atom mass.

In actual computation, we discrete the x - y plane into a square lattice points. The space scale is much larger than the TF radius of the condensate. a is assumed to be the lattice constant whose value must be much less than the characteristic length $l = \frac{\eta}{m\omega_t}$ of the axial harmonic oscillator so that the dynamics are independent of the grid. For the time evolution forth-order Runge-kutta method is used at each time step. The central-difference formula is used mainly to calculate the diffusion term (kinetic term). Introducing dimensionless $\psi(i, j)$ in the GP equations, by substituting $\psi(\vec{r}, t)$ with $\frac{1}{\sqrt{a^2a_z}}\psi(i, j, t)$, where $a_z = \frac{\eta}{m\omega_z}$ is the characteristic length of longitudinal harmonic oscillator, we will thus obtain the following lattice-version of the GP equations:

$$i\eta\partial_t\psi(i, j, t) = (1-\gamma)\{t_0[(\psi(i+, j) + \psi(i-, j) + \psi(i, j+1) + \psi(i, j-1) - 4\psi(i, j))] + [V(i^2 + j^2) + g|\psi(i, j)|^2]\psi(i, j, t)\}, \quad (2)$$

where $t_0 = \frac{\eta^2}{2ma^2}$, $V = \frac{1}{2}m\omega_t^2a^2$, $g = \frac{4\pi\eta^2a_s}{ma^2a_z}$. Now all the parameters t_0, V, g and $\frac{\eta}{t_0}$ have the scale of energy.

It is convenient to introduce dimensionless parameters $V' = \frac{V}{t_0}$, $g' = \frac{g}{t_0}$, $t' = \frac{t}{\eta/t_0}$, measured in unit of t_0 . The

parameters in the above equations are all dimensionless and they are actually only dependent of $\frac{a}{l}$ and $\frac{a_s}{a_z}$. A

straightforward calculation leads to the following expressions of $V' = (\frac{a}{l})^4$ and $g' = 4\pi\frac{a_s}{a_z}$. Note that g' is essentially independent of the artificial lattice constant a .

In the following discussions and calculations, We assum the system consists of ^{87}Rb atoms,. The trap frequency is chosen to be $\omega_t = 2\pi \times 10s^{-1}$, then l is estimated to be about $3.4 \mu m$. When the square lattice we study takes a typical size of 256×256 system and $(\frac{a}{l})^2 = 0.0064$. This means the system has a size of about $69m \times 69\mu m$ and $\frac{\eta\omega_t}{t_0} = 2(\frac{a}{l})^2 = 0.0128$. Therefore, we have $V' = 0.00004$. In addition, to guarantee both the convergence and efficiency of iteration of the GP equations, dt' is chosen to be between 0.0001 and 0.01 in numerical calculations.

Results and disscussion:-

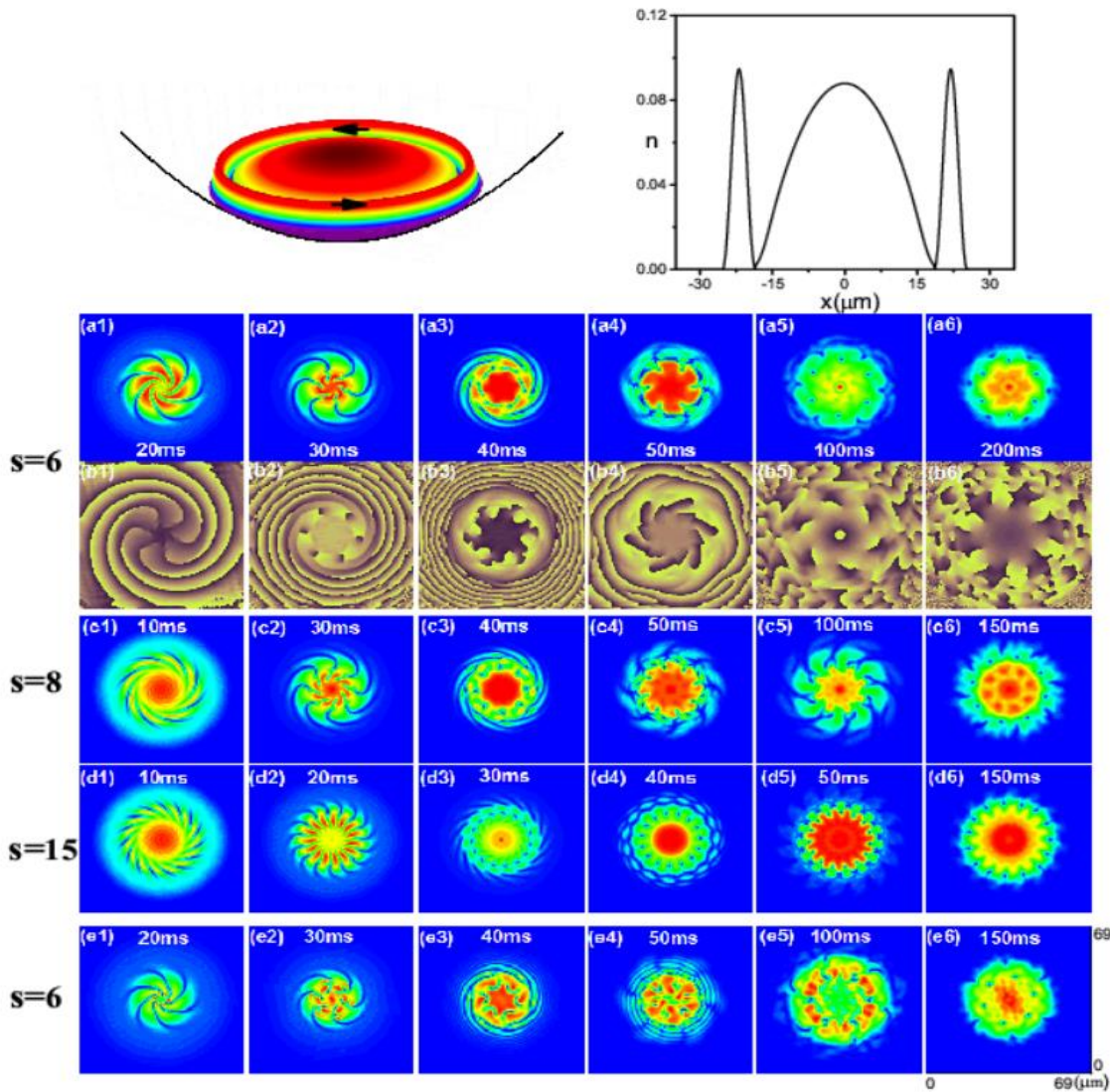


Fig 1:-The sketch and the central crosssections of the density profiles for a superflowing ring-shaped BEC colliding with another centered static BEC when both are confined in a harmonic trap and the ratio of atomic numbers in the initial state $N1/N2=1$. The arrow denotes the direction of the superfluid velocity. (bottom)Time evolution of the dynamics for the collision. The upper two rows give the density and phase profiles for $s=6$, while the third and

fourth rows gives the density profiles for $s=8$ and $s=15$ with $N_1/N_2=1$. The last row gives the density profiles for $s=6$ with $N_1/N_2=3$. Here $Ng'=2400$, and the strength of fluctuations is $\eta = 3 \times 10^{-5}$.

Since vortices are topological defects, they can be only created in pairs or can enter a system from its boundary. Generally, collisions between BECs can result in the creation and motion of vortices or vortex-antivortex pairs. In this section, we study another dynamical process of vortex generation, where a superflowing ring-shaped condensate with N_1 atoms collides with a static trapped BEC at center with N_2 atoms. This is schematically shown in the inset of Fig.1. The superflowing condensate can be realized by a rotating weak link or obstacle[17]. The wave function for it is characterized by its winding number s , and can be given by $\psi(r) = \psi_0(\rho)e^{is\theta}$. Here θ is the polar phase of

ψ and $\psi_0(\rho) \propto \sin(\pi \frac{\rho - r_1}{r_2 - r_1})$ is the ground state wave function for the condensate in a toroidal trap which can

be simplified as an ring-shaped infinite potential with internal radii r_1 and r_2 . If the condensate is then released into the harmonic trap, which can be assumed that the infinite well is suddenly switched off, it would collide with a static trapped BEC at center. The centered static BEC is assumed to be in the ground state of the harmonic trap, with a TF radius slightly less than r_1 .

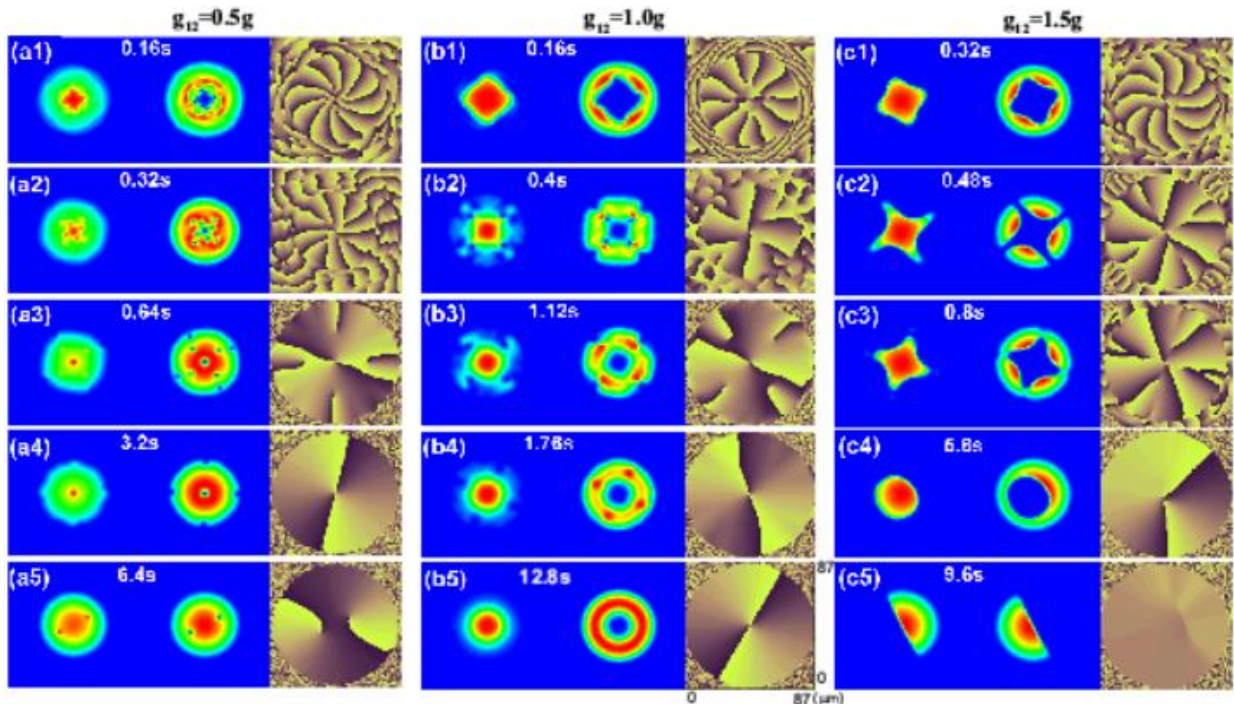


Figure 2:-(color online) Time evolution of the dynamics for the collision between a superflowing ring-shaped BEC with winding number $s = 10$ and a centered static BEC, where the two BECs are composed of different identical atoms. Here $N_1 = N_2 = N$, $g_1 = g_2 = g$, and $Ng' = 2400$. The left panels are the density profiles of the two BECs and the phase profiles of the ring-shaped BEC for the phase-mixing regime with $g_{12} = 0.5g$, while the middle and right panels show the corresponding profiles for the critical regime with $g_{12} = g$ and phase-separation regime with $g_{12} = 1.5g$. Here the strength of fluctuations is $\eta = 3 \times 10^{-5}$.

To simplify the discussion, we first assume that the two BECs consist of the same identical atoms, and so both two should be described by a single condensate wave function. When the collision happens, the superflowing atoms from the ring-shaped region tend to move inside, and by merging with the inner atoms, also tend to form a new condensate confined by the trap potential. Under this competition, s symmetric snake-like density valleys are formed quickly, each extending from the trap center to the condensate edge. Each density valley contains one singly quantized vortex, which can be seen from the phase profiles in Fig.1. Consider a circle in the BEC centered at the

trap center with a radius slightly larger than r_1 . During a relatively long time of the evolution process, the circulation $\oint \vec{v} \cdot d\vec{r}$ along the circle with \vec{v} the superfluid velocity, always keeps the quantized value $s \frac{2\pi\eta}{m}$, as long as no

vortex or antivortex passes across the circle. Soon each density valley will be broken into pieces, many of which are composed of vortex-antivortex pairs. Within a relatively short period of time, these antivortices will evolve out of the condensate, leaving s symmetrically positioned singly quantized vortices near the original interface of the two BECs. Finally, these vortices will move gradually in a spiral way to the condensate edge and then disappear, leaving behind a new static ground state of BEC confined by the trap. These are illustrated in the upper panels of Fig.1, where the ratio between the numbers of atoms of the two BECs N_1/N_2 is fixed to be 1.

The dynamical evolution processes for s from 1 to 20 have been investigated. It is found that the above dynamical behavior is generically unchanged regardless of the s value. Quantitatively, upon increasing s , the time intervals for the existence of density valleys become shorter and the relaxation time needed to the new ground state also become shorter. Fluctuation effect has also been taken into account, and the above behaviors are preserved against small fluctuations so long as $\eta < 1 \times 10^{-4}$.

Now we consider the situation where the two BECs consist of different identical atoms. The two BECs are thus described by two different condensate wave functions. Let the interactions between atoms in the same BECs be identical and denoted by g and that between atoms in the different BECs denoted by g_{12} . Generically, the ratio g_{12}/g determines the phase regimes of the condensate. In the phase-mixing regime $g_{12}/g < 1$, when the collision happens, the atoms of the ring-shaped BEC first move inside towards the trap center, accompanied by vortices formation. Then the time evolution process takes on 4-fold symmetry. In the phase-separation regime $g_{12}/g > 1$, the two BECs always occupy complementary spaces. While the ring-shaped BEC is broken into several pieces with most of the vortices escaping out, the inner BEC has sharp corners accordingly. Eventually, the two BECs evolve into a phase-separated state composed of two semicircular parts, which is the ground state of the system. In the critical regime $g_{12}/g = 1$, the dynamical process is more like that of the phase-separation regime, apart from that the final state of the system is stabilized at an excited state, where the original ring-shaped BEC still preserve the shape and is superflowing with a smaller winding number. These are demonstrated in Fig.2. The calculation is done by the square-lattice discretization and the comparison is also made with that by the triangular-lattice discretization. We find that apart from the 4-fold symmetry related behavior shown in Fig.2(a), these phenomena are independent of the lattice discretization.

Conclusion:-

We have performed numerical calculations of the quasi-two-dimensional GP equation to investigate the creation and dynamical evolution of vortices in a harmonic trap potential. As two GOPs move and collide head on, we find extra vortex dipoles and a pair of solitons are nucleated symmetrically after the two GOPs separating from each other which enrich the phase profiles of vortex excitations. when a trapped SO coupling BEC is in a plane-wave phase, it is favorable for vortex creation under the influence of SO coupling effect. The velocity of a vortex-antivortex is much smaller than that without SO coupling.

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