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RESEARCH ARTICLE

METHOD OF TAYLOR'S SERIES FOR THE PRIMITIVE OF LINEAR FIRST KIND VOLTERRA INTEGRAL EQUATION

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Abstract

With the remarkable advancement in engineering, science, and technology, today more than ever before, the study of integral equations has become essential. These integral equations may be linear or nonlinear. In this paper, authors have solved linear first kind Volterra integral equations (V.I.E.) using Taylor series method. Authors have been considered four numerical examples for explaining the complete methodology. Results of numerical examples show that Taylor series method is very useful and effective numerical method for handling the problem of obtaining the primitives of linear first kind V.I.E.

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Introduction:-

Integral equations are playing an increasingly important role in obtaining the solution of many scientific and engineering problems such as determination of potentials, seismic travel time, optical fibers and system identification. To have an exhaustive understanding of subjects like waves and electromagnetic, chemistry, fluid dynamics, physics, statistics, mechanics, heat transfer, chemical science, mathematical biology, aerodynamics, electricity, the knowledge of determining the solution to integral equations is absolutely necessary. Finding and interpreting the solutions of these integral equations is therefore a central part of applied mathematics and a thorough understanding of integral equations is essential for any scholars. Aggarwal with others [1-5] used different integral transformations for obtaining the solutions of V.I.E. of second kind. The primitives of first kind V.I.E. were obtained by Aggarwal et al. [6-11] by applying Laplace; Kamal; Mahgoub; Aboodh; Elzaki; Shehu integral transformations on them. Aggarwal and others scholars [12-18] determined the exact solution of famous problem of mechanics (Abel's problem) by applying Laplace; Kamal; Mohand; Aboodh; Sumudu; Shehu; Sadik integral transformations on it. This problem was a special case of V.I.E.

The goal of this paper is to determine the solutions of linear first kind V.I.E. by applying Taylor series method on them.

Power series (Taylor series) of frequently used functions in engineering and mathematical science:

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$$\begin{aligned}
 e^\tau &= \left\{ 1 + \tau + \frac{\tau^2}{2!} + \frac{\tau^3}{3!} + \frac{\tau^4}{4!} + \frac{\tau^5}{5!} + \dots \dots \dots \right\} \\
 e^{-\tau} &= \left\{ 1 - \tau + \frac{\tau^2}{2!} - \frac{\tau^3}{3!} + \frac{\tau^4}{4!} - \frac{\tau^5}{5!} + \dots \dots \dots \right\} \\
 e^{a\tau} &= \left\{ 1 + a\tau + \frac{(a\tau)^2}{2!} + \frac{(a\tau)^3}{3!} + \frac{(a\tau)^4}{4!} + \dots \dots \right\} \\
 e^{-a\tau} &= \left\{ 1 - a\tau + \frac{(a\tau)^2}{2!} - \frac{(a\tau)^3}{3!} + \frac{(a\tau)^4}{4!} - \dots \dots \right\} \\
 \sin\tau &= \left\{ \tau - \frac{\tau^3}{3!} + \frac{\tau^5}{5!} - \frac{\tau^7}{7!} + \dots \dots \dots \right\} \\
 \cos\tau &= \left\{ 1 - \frac{\tau^2}{2!} + \frac{\tau^4}{4!} - \frac{\tau^6}{6!} + \dots \dots \dots \right\} \\
 \tan\tau &= \left\{ \tau + \frac{\tau^3}{3} + \frac{2\tau^5}{15} + \dots \dots \dots \right\} \\
 \sinh\tau &= \left\{ \tau + \frac{\tau^3}{3} + \frac{\tau^5}{5!} + \frac{\tau^7}{7!} + \dots \dots \dots \right\} \\
 \cosh\tau &= \left\{ 1 + \frac{\tau^2}{2!} + \frac{\tau^4}{4!} + \frac{\tau^6}{6!} + \dots \dots \dots \right\} \\
 \sin^{-1}\tau &= \left\{ \tau + \frac{1}{2}\left(\frac{\tau^3}{3}\right) + \frac{1.3}{2.4}\left(\frac{\tau^5}{5}\right) + \dots \dots, \tau^2 < 1 \right\} \\
 \tan^{-1}\tau &= \left\{ \tau - \frac{\tau^3}{3} + \frac{\tau^5}{5} - \dots \dots \dots \right\} \\
 \log(1 + \tau) &= \left\{ \tau - \frac{\tau^2}{2} + \frac{\tau^3}{3} - \frac{\tau^4}{4} + \dots \dots, -1 < \tau \leq 1 \right\} \\
 \log(1 - \tau) &= \left\{ -\tau - \frac{\tau^2}{2} - \frac{\tau^3}{3} - \frac{\tau^4}{4} - \dots \dots, -1 \leq \tau < 1 \right\} \\
 \frac{1}{(1 - \tau)} &= \{ 1 + \tau + \tau^2 + \tau^3 + \dots \dots \dots, |\tau| < 1 \} \\
 \frac{1}{(1 + \tau)} &= \{ 1 - \tau + \tau^2 - \tau^3 + \dots \dots \dots, |\tau| < 1 \} \\
 \frac{1}{(1 - \tau)^2} &= \{ 1 + 2\tau + 3\tau^2 + 4\tau^3 + \dots \dots \dots, |\tau| < 1 \} \\
 \frac{1}{(1 - \tau)^3} &= \{ 1 + 3\tau + 6\tau^2 + 10\tau^3 + \dots \dots \dots, |\tau| < 1 \} \\
 (1 + \tau)^{\frac{1}{2}} &= \left\{ 1 + \frac{\tau}{2} - \frac{\tau^2}{8} + \frac{\tau^3}{16} - \dots \dots \dots, |\tau| < 1 \right\} \\
 (1 + \tau)^{-\frac{1}{2}} &= \left\{ 1 - \frac{\tau}{2} + \frac{3\tau^2}{8} - \frac{5\tau^3}{16} + \dots \dots \dots, |\tau| < 1 \right\}
 \end{aligned}$$

Method of Taylor’s series for the primitive of linear first kind V.I.E.:

The linear first kind V.I.E. is given by [19-21]

$$f(\tau) = \delta \int_0^\tau K(\tau, t)\varphi(t)dt \tag{1}$$

where

$$\left. \begin{aligned}
 &\varphi(t) = \text{unknown function} \\
 f(\tau) &= \text{known function (perturbation function)} \\
 &\delta = \text{non - zero parameter} \\
 &K(\tau, t) = \text{kernel of integral equation}
 \end{aligned} \right\}$$

Suppose the solution $\varphi(\tau)$ of equation (1) is analytic so it can be represent in the form of Taylor’s series as

$$\varphi(\tau) = \sum_{n=0}^\infty \beta_n \tau^n \tag{2}$$

Use equation (2) in equation (1), we have

$$T(f(\tau)) = \delta \int_0^\tau K(\tau, t) \left(\sum_{n=0}^\infty \beta_n t^n \right) dt \tag{3}$$

where $T(f(\tau))$ is the Taylor series expansion of the function $f(\tau)$.

Equation (3) can be written as

$$[T(f(\tau)) = \delta \int_0^\tau K(\tau, t) (\beta_0 + \beta_1 t + \beta_2 t^2 + \beta_3 t^3 \dots \dots \dots) dt] \tag{4}$$

On simplification, (4) gives a system of algebraic equations in terms of $(\beta_0, \beta_1, \beta_2, \beta_3, \dots \dots \dots)$. After solving this system, we get a chain of coefficients namely $(\beta_0, \beta_1, \beta_2, \beta_3, \dots \dots \dots)$. The required solution of equation (1) may be obtained by using these coefficients in equation (2).

Example:

Consider the following linear first kind V.I.E.

$$1 + \sin \tau - \cos \tau = \int_0^\tau (\tau - t + 1) \phi(t) dt \tag{5}$$

Suppose the solution $\phi(\tau)$ of equation (5) is analytic so it can be represent in the form of Taylor's series as

$$\phi(\tau) = \sum_{n=0}^\infty \beta_n \tau^n \tag{6}$$

Use equation (6) in equation (5), we have

$$\left\{ 1 + \left[\tau - \frac{\tau^3}{3!} + \frac{\tau^5}{5!} - \dots \dots \dots \right] - \left[1 - \frac{\tau^2}{2!} + \frac{\tau^4}{4!} - \dots \dots \dots \right] = \int_0^\tau (\tau - t + 1) (\sum_{n=0}^\infty \beta_n t^n) dt \right\} \tag{7}$$

Equation (7) can be written as

$$\left\{ 1 + \left[\tau - \frac{\tau^3}{3!} + \frac{\tau^5}{5!} - \dots \dots \dots \right] - \left[1 - \frac{\tau^2}{2!} + \frac{\tau^4}{4!} - \dots \dots \dots \right] = \int_0^\tau (\tau - t + 1) \left(\begin{matrix} \beta_0 + \beta_1 t + \beta_2 t^2 \\ + \beta_3 t^3 + \beta_4 t^4 \\ + \beta_5 t^5 + \dots \dots \dots \end{matrix} \right) dt \right\}$$

$$\Rightarrow \left\{ \left[\begin{matrix} \tau + \frac{\tau^2}{2!} - \frac{\tau^3}{3!} \\ - \frac{\tau^4}{4!} + \frac{\tau^5}{5!} - \dots \dots \dots \end{matrix} \right] = \left[(\tau + 1) \left(\begin{matrix} \beta_0 \tau + \beta_1 \frac{\tau^2}{2} + \beta_2 \frac{\tau^3}{3} \\ + \beta_3 \frac{\tau^4}{4} + \beta_4 \frac{\tau^5}{5} + \beta_5 \frac{\tau^6}{6} \\ + \dots \dots \dots \end{matrix} \right) \right] - \left[\begin{matrix} \beta_0 \frac{\tau^2}{2} + \beta_1 \frac{\tau^3}{3} + \beta_2 \frac{\tau^4}{4} \\ + \beta_3 \frac{\tau^5}{5} + \beta_4 \frac{\tau^6}{6} + \beta_5 \frac{\tau^7}{7} + \dots \dots \dots \end{matrix} \right] \right\} \tag{8}$$

Now on simplification, (8) gives a system of following algebraic equations

$$\left. \begin{matrix} 1 = \beta_0 \\ \frac{1}{2!} = \beta_0 - \frac{\beta_0}{2} + \frac{\beta_1}{2} \\ -\frac{1}{3!} = \frac{\beta_1}{2} - \frac{\beta_1}{3} + \frac{\beta_2}{3} \\ -\frac{1}{4!} = \frac{\beta_2}{3} - \frac{\beta_2}{4} + \frac{\beta_3}{4} \\ \frac{1}{5!} = \frac{\beta_3}{4} - \frac{\beta_3}{5} + \frac{\beta_4}{5} \\ -\frac{1}{6!} = \frac{\beta_4}{5} - \frac{\beta_4}{6} + \frac{\beta_5}{6} \end{matrix} \right\} \tag{9}$$

After solving the system (9), we get

$$\left. \begin{matrix} \beta_0 = 1 \\ \beta_1 = 0 \\ \beta_2 = -\frac{1}{2} \\ \beta_3 = 0 \\ \beta_4 = \frac{1}{24} \\ \beta_5 = 0 \end{matrix} \right\} \tag{10}$$

Using equation (10) in equation (6), we get the required solution of equation (5) given by

$$\phi(\tau) = \left\{ 1 + 0. \tau + \left(-\frac{1}{2}\right). \tau^2 + 0. \tau^3 + \left(\frac{1}{24}\right) \tau^4 + 0. \tau^5 + \dots \dots \dots \right\}$$

$$= \left\{ 1 - \frac{\tau^2}{2} + \frac{\tau^4}{24} - \dots \dots \dots \right\} = \cos \tau.$$

Example:

Consider the following linear first kind V.I.E.

$$\tau = \int_0^\tau (\tau - t + 1) \phi(t) dt \tag{11}$$

Suppose the solution $\phi(\tau)$ of equation (11) is analytic so it can be represent in the form of Taylor's series as

$$\phi(\tau) = \sum_{n=0}^\infty \beta_n \tau^n \tag{12}$$

Use equation (12) in equation (11), we have

$$\tau = \int_0^\tau (\tau - t + 1) (\sum_{n=0}^\infty \beta_n t^n) dt \tag{13}$$

Equation (13) can be written as

$$\begin{aligned} \tau &= \int_0^\tau (\tau - t + 1) (\beta_0 + \beta_1 t + \beta_2 t^2 + \beta_3 t^3 + \beta_4 t^4 + \beta_5 t^5 + \dots) dt \\ \Rightarrow \tau &= \left[(\tau + 1) \left(\beta_0 \tau + \beta_1 \frac{\tau^2}{2} + \beta_2 \frac{\tau^3}{3} + \beta_3 \frac{\tau^4}{4} + \beta_4 \frac{\tau^5}{5} + \beta_5 \frac{\tau^6}{6} + \dots \right) - \left[\beta_0 \frac{\tau^2}{2} + \beta_1 \frac{\tau^3}{3} + \beta_2 \frac{\tau^4}{4} + \beta_3 \frac{\tau^5}{5} + \beta_4 \frac{\tau^6}{6} + \beta_5 \frac{\tau^7}{7} + \dots \right] \right] \end{aligned} \tag{14}$$

Now on simplification, (14) gives a system of following algebraic equations

$$\left. \begin{aligned} 1 &= \beta_0 \\ 0 &= \beta_0 - \frac{\beta_0}{2} + \frac{\beta_1}{2} \\ 0 &= \frac{\beta_1}{2} - \frac{\beta_1}{3} + \frac{\beta_2}{3} \\ 0 &= \frac{\beta_2}{3} - \frac{\beta_2}{4} + \frac{\beta_3}{4} \\ 0 &= \frac{\beta_3}{4} - \frac{\beta_3}{5} + \frac{\beta_4}{5} \\ 0 &= \frac{\beta_4}{5} - \frac{\beta_4}{6} + \frac{\beta_5}{6} \end{aligned} \right\} \tag{15}$$

After solving the system (15), we get

$$\left. \begin{aligned} \beta_0 &= 1 \\ \beta_1 &= -1 \\ \beta_2 &= \frac{1}{2} \\ \beta_3 &= -\frac{1}{6} \\ \beta_4 &= \frac{1}{24} \\ \beta_5 &= -\frac{1}{120} \end{aligned} \right\} \tag{16}$$

Using equation (16) in equation (12), we get the required solution of equation (11) given by

$$\begin{aligned} \varphi(\tau) &= \left\{ 1 + (-1) \cdot \tau + \left(\frac{1}{2}\right) \cdot \tau^2 + \left(-\frac{1}{6}\right) \cdot \tau^3 + \left(\frac{1}{24}\right) \tau^4 + \left(-\frac{1}{120}\right) \cdot \tau^5 + \dots \dots \dots \right\} \\ &= \left\{ 1 - \tau + \frac{\tau^2}{2} - \frac{\tau^3}{6} + \frac{\tau^4}{24} - \frac{\tau^5}{120} + \dots \dots \dots \right\} = e^{-\tau}. \end{aligned}$$

Example:

Consider the following linear first kind V.I.E.

$$e^\tau - \tau - 1 = \int_0^\tau (\tau - t + 1) \varphi(t) dt \tag{17}$$

Suppose the solution $\varphi(\tau)$ of equation (17) is analytic so it can be represent in the form of Taylor's series as

$$\varphi(\tau) = \sum_{n=0}^\infty \beta_n \tau^n \tag{18}$$

Use equation (18) in equation (17), we have

$$\left\{ \left[\begin{aligned} 1 + \frac{\tau}{1!} + \frac{\tau^2}{2!} + \frac{\tau^3}{3!} \\ + \frac{\tau^4}{4!} + \frac{\tau^5}{5!} + \frac{\tau^6}{6!} + \dots \dots \dots \end{aligned} \right] - \tau - 1 = \int_0^\tau (\tau - t + 1) (\sum_{n=0}^\infty \beta_n t^n) dt \right\} \tag{19}$$

Equation (19) can be written as

$$\begin{aligned} \left\{ \left[\frac{\tau^2}{2!} + \frac{\tau^3}{3!} + \frac{\tau^4}{4!} + \frac{\tau^5}{5!} + \frac{\tau^6}{6!} + \dots \dots \dots \right] = \int_0^\tau (\tau - t + 1) \left(\begin{aligned} \beta_0 + \beta_1 t + \beta_2 t^2 \\ + \beta_3 t^3 + \beta_4 t^4 + \beta_5 t^5 + \dots \dots \dots \end{aligned} \right) dt \right\} \\ \Rightarrow \left\{ \left[\begin{aligned} \frac{\tau^2}{2!} + \frac{\tau^3}{3!} \\ + \frac{\tau^4}{4!} + \frac{\tau^5}{5!} \\ + \frac{\tau^6}{6!} + \dots \dots \dots \end{aligned} \right] = \left[(\tau + 1) \left(\begin{aligned} \beta_0 \tau + \beta_1 \frac{\tau^2}{2} \\ + \beta_2 \frac{\tau^3}{3} + \beta_3 \frac{\tau^4}{4} \\ + \beta_4 \frac{\tau^5}{5} + \beta_5 \frac{\tau^6}{6} + \dots \dots \dots \end{aligned} \right) \right] - \left[\begin{aligned} \beta_0 \frac{\tau^2}{2} + \beta_1 \frac{\tau^3}{3} + \beta_2 \frac{\tau^4}{4} \\ + \beta_3 \frac{\tau^5}{5} + \beta_4 \frac{\tau^6}{6} + \beta_5 \frac{\tau^7}{7} + \dots \dots \dots \end{aligned} \right] \right\} \end{aligned} \tag{20}$$

Now on simplification, (20) gives a system of following algebraic equations

$$\left. \begin{aligned} 0 &= \beta_0 \\ \frac{1}{2} &= \beta_0 - \frac{\beta_0}{2} + \frac{\beta_1}{2} \\ \frac{1}{6} &= \frac{\beta_1}{2} - \frac{\beta_1}{3} + \frac{\beta_2}{3} \\ \frac{1}{24} &= \frac{\beta_2}{3} - \frac{\beta_2}{4} + \frac{\beta_3}{4} \\ \frac{1}{120} &= \frac{\beta_3}{4} - \frac{\beta_3}{5} + \frac{\beta_4}{5} \\ \frac{1}{720} &= \frac{\beta_4}{5} - \frac{\beta_4}{6} + \frac{\beta_5}{6} \end{aligned} \right\} \tag{21}$$

After solving the system (21), we get

$$\left. \begin{aligned} \beta_0 &= 0 \\ \beta_1 &= 1 \\ \beta_2 &= 0 \\ \beta_3 &= \frac{1}{6} \\ \beta_4 &= 0 \\ \beta_5 &= \frac{1}{120} \end{aligned} \right\} \tag{22}$$

Using equation (22) in equation (18), we get the required solution of equation (17) given by

$$\begin{aligned} \varphi(\tau) &= \left\{ 0 + 1 \cdot \tau + 0 \cdot \tau^2 + \left(\frac{1}{6}\right) \cdot \tau^3 + 0 \cdot \tau^4 + \left(\frac{1}{120}\right) \cdot \tau^5 + \dots \dots \dots \right\} \\ &= \left\{ \tau + \frac{\tau^3}{6} + \frac{\tau^5}{120} + \dots \dots \dots \right\} = \text{sint} \dots \end{aligned}$$

Example:

Consider the following linear first kind V.I.E.

$$5\tau^4 + \tau^5 = \int_0^\tau (\tau - t + 1)\varphi(t)dt \tag{23}$$

Suppose the solution $\varphi(\tau)$ of equation (23) is analytic so it can be represent in the form of Taylor’s series as

$$\varphi(\tau) = \sum_{n=0}^\infty \beta_n \tau^n \tag{24}$$

Use equation (24) in equation (23), we have

$$5\tau^4 + \tau^5 = \int_0^\tau (\tau - t + 1)(\sum_{n=0}^\infty \beta_n t^n)dt \tag{25}$$

Equation (25) can be written as

$$\begin{aligned} &\{ [5\tau^4 + \tau^5] = \int_0^\tau (\tau - t + 1)(\beta_0 + \beta_1 t + \beta_2 t^2 + \beta_3 t^3 + \beta_4 t^4 + \beta_5 t^5 + \dots \dots \dots) dt \} \\ \Rightarrow &\left\{ 5\tau^4 + \tau^5 = \left[(\tau + 1) \left(\begin{aligned} &\beta_0 \tau + \beta_1 \frac{\tau^2}{2} + \beta_2 \frac{\tau^3}{3} \\ &+ \beta_3 \frac{\tau^4}{4} + \beta_4 \frac{\tau^5}{5} + \beta_5 \frac{\tau^6}{6} + \dots \dots \dots \end{aligned} \right) - \left[\begin{aligned} &\beta_0 \frac{\tau^2}{2} + \beta_1 \frac{\tau^3}{3} + \beta_2 \frac{\tau^4}{4} \\ &+ \beta_3 \frac{\tau^5}{5} + \beta_4 \frac{\tau^6}{6} + \beta_5 \frac{\tau^7}{7} + \dots \dots \dots \end{aligned} \right] \right] \right\} \end{aligned} \tag{26}$$

Now on simplification, (26) gives a system of following algebraic equations

$$\left. \begin{aligned} 0 &= \beta_0 \\ 0 &= \beta_0 - \frac{1}{2}\beta_0 + \frac{1}{2}\beta_1 \\ 0 &= \frac{1}{2}\beta_1 - \frac{1}{3}\beta_1 + \frac{1}{3}\beta_2 \\ 5 &= \frac{1}{3}\beta_2 - \frac{1}{4}\beta_2 + \frac{1}{4}\beta_3 \\ 1 &= \frac{1}{4}\beta_3 - \frac{1}{5}\beta_3 + \frac{1}{5}\beta_4 \\ 0 &= \frac{1}{5}\beta_4 - \frac{1}{6}\beta_4 + \frac{1}{6}\beta_5 \end{aligned} \right\} \tag{27}$$

After solving the system (27), we get

$$\left. \begin{aligned} \beta_0 &= 0 \\ \beta_1 &= 0 \\ \beta_2 &= 0 \\ \beta_3 &= 20 \\ \beta_4 &= 0 \\ \beta_5 &= 0 \end{aligned} \right\} \tag{28}$$

Using equation (28) in equation (24), we get the required solution of equation (23) given by

$$\varphi(\tau) = \{0 + 0.\tau + 0.\tau^2 + 20.\tau^3 + 0.\tau^4 + 0.\tau^5 + \dots \dots \dots\} = 20\tau^3.$$

Conclusions:-

In the present paper, authors fruitfully discussed the Taylor series method for determining the primitives of linear first kind V.I.E. The complete methodology explained by taking numerical examples. Results of numerical examples depict that Taylor series method is very effective method for determining the primitives of linear first kind V.I.E. without large computational work.

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