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### RESEARCH ARTICLE

#### BASIC REPRODUCTION NUMBER OF THE COVID-19 EPIDEMIC IN CITIES OF PERU

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#### Abstract

Actually, the Covid-19 is still in force in Peru, due to its complicated understanding of the epidemic's transmission rate, the present research was proposed with the objective of estimating the basic reproduction number  $R_0$ , for the transmission by Covid-19 and simulate the behavior of infections in the cities of Lima, Callao, Piura, Lambayeque, Libertad and Loreto, through the binomial chain model and using the infection reports; the parameters that maximize the chain probability function were estimated. The estimates of  $\hat{R}_0$ , at 95% confidence interval were 1.534(CI, 1.523-1.546), 1.662(CI, 1.623-1.701), 1.829(CI, 1.787-1.873), 1.521(IC, 1.478-1.558), 1.506(IC, 1.454-1.558) and 1.624(IC, 1.570-1.680), in the cities of Lima, Callao, Piura, Lambayeque, Libertad and Loreto, respectively. The projections for the end of the epidemic, the city of Lima will have a longer duration of 35 weeks compared to other cities studied.

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#### Introduction:-

The Coronavirus (Covid-19), initially originating in Wuhan's South China seafood wholesale market [1, 21], is a highly contagious disease; the first reports from Covid-19, in Wuhan (China) at the end of 2019, alerted the world to the existence of the virus. However, the world health organization declared Europe the epicenter of the epidemic and it spread rapidly in several countries causing the collapse of health systems and death. In March 2020, people infected with Covid-19 were confirmed in several South American countries including Peru, reporting the first case on March 6, 2020. The governments have taken drastic decisions to close borders and even declare quarantine to stop the spread of the disease, in Peru the government declares a state of emergency and measures such as quarantine that may help reduce the rate of infection.

The knowledge about contagion between people is still not known and leads many researchers to propose or use old models of epidemic such as Kermack and McKendrick's 1927, SIR and SIER [11, 13], and among others that help to understand and predict the development of the epidemic. Today, the basic reproduction number  $R_0$ , is very much researched by researchers to understand the dynamics of contagion. The epidemiologists define  $R_0$  as the average number of infected cases in a susceptible population by a single infectious individual [10]. The  $R_0$  number has information about how many people an individual with Covid-19 transfers the virus. And the criterion for interpreting  $R_0$ , is the following, when  $R_0 > 1$ , the contagion exists on the susceptible person, while for  $R_0 < 1$ , there is no contagion [6, 7, 15].

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The study is based on the stochastic binomial chain model adapted to the  $R_0$  and  $S_0$  (initial number of susceptibles) parameters, and the objective is to determine the estimates of the  $\widehat{R}_0$  and  $\widehat{S}_0$  parameters with the recording of data confirmed by Covid-19, of the six departments of Peru, Lima, Callao, Piura, Lambayeque, Libertad and Loreto, which have the highest incidence of transmission. From the estimation of the parameters in the cities it is intended to simulate the progress of the epidemic and in the study we have the purpose of knowing the valuable information how the virus is reproducing and these data serve to propose strategies, and effective actions that help defeat the covid-19, soon.

## Methodology:-

### Data:

Data was collected on positive cases of Covid-19, reported at Peru's national center for epidemiology, prevention and disease control, from March 17 to June 29, 2020, in the departments of Lima, Callao, Piura, Lambayeque, Libertad and Loreto [19]. At the beginning of the Covid-19 virus outbreak in Wuhan, it had an average serial time of 7.5 or 7.6 days [16]. The model analysis adopted 7 days and worked in total with data of 15 weeks.

### Model Stochastic binomial chain:

We assume that the population size is constant  $N$  and people go through the three states susceptible, infected, and recovered. In a given time  $t$ , each person in a city of  $N$ , persons are in one state and  $S_t + I^t + R^t = N$ , where  $S_t$ , is the number of susceptible persons,  $I^t$  is the number of infected persons and  $R^t$  is the number of persons recovered in time  $t$ . Let's consider two possibilities [2, 8, 18]:

$$e^{-R_0 \frac{I^t}{S_0}} = \text{the probability that any susceptible individual will} \quad (1)$$

be free from covid-19 infection, at time  $t$ .

In other words, there's no contact with any of the  $I^t$ , infected. So the other option is defined by

$$1 - e^{-R_0 \frac{I^t}{S_0}} = \text{the probability that any susceptible individual will have} \quad (2)$$

contact with infectious individuals during time  $t$  and will be infected.

The conditional probability of transmission of  $I^{t+1} = i_{t+1}$ , new cases of infection in time  $t + 1$  given that  $S_t = s_t$  and  $I^t = i_t$ , susceptible and infectious occurs at time  $t$ , is defined by

$$P(I^{t+1} = i_{t+1} : s_t, i_t) = \binom{s_t}{i_{t+1}} \left(1 - e^{-R_0 \frac{i_t}{S_0}}\right)^{i_{t+1}} e^{-R_0 \frac{i_t}{S_0} (s_t - i_{t+1})} \quad (3)$$

where  $S_t \geq I^{t+1}$ . In addition, updates of the number of susceptible cases and persons recovered are given by the equations:

$$S_{t+1} = S_t - I^{t+1} \quad (4)$$

and

$$R^{t+1} = R^t + I^t. \quad (5)$$

The chain of the epidemic is the form

$$i_0 \rightarrow i_1 \rightarrow i_2 \rightarrow \dots \rightarrow i_T \quad (6)$$

where  $i_0 = 1$  and  $\{i_k\}_{k=1}^T$  the data of infected people. Then the chain probability (6), is given by the product of type (3), conditional binomial probabilities:

$$\text{The chain probability} = P(I^1 = i_1 : s_0, i_0) P(I^2 = i_2 : s_1, i_1) \dots P(I^T = i_T : s_{T-1}, i_{T-1}) \quad (7)$$

The chain probability define

$$L(R_0, S_0) = \text{Log} \left[ \prod_{t=0}^{n-1} \binom{s_t}{i_{t+1}} \left(1 - e^{-R_0 \frac{i_t}{S_0}}\right)^{i_{t+1}} e^{-R_0 \frac{i_t}{S_0} (s_t - i_{t+1})} \right], \quad (8)$$

where  $n \leq T$ . The maximum and minimum properties of a function imply that

$$\max_{(R_0, S_0)} L(R_0, S_0) = - \min_{(R_0, S_0)} \{-L(R_0, S_0)\} \quad (9)$$

For estimation  $(\hat{R}_0, \hat{S}_0)$ , it is sufficient to minimize the function  $-L(R_0, S_0)$ , starting with true values  $R_0$  and  $S_0$ . Using software R version 3.6, with bbme2 package [3], the values  $(\hat{R}_0, \hat{S}_0)$ , which maximize the function given in (8), were estimated for each city studied.

**Simulation:**

To obtain the simulations, the estimates of  $\hat{S}_0$  and  $\hat{R}_0$ , and initial amount  $I_0$  are required. Once we obtain those values we generate the values of the binomial random variable using the methodology based on [14, 20]:

$$\begin{aligned}
 I^0 &:= \text{initial data} \\
 S_0 &:= \hat{S}_0 \\
 R_0 &:= \hat{R}_0 \\
 I^i &:= \text{rbinom}(1, \text{size} = S_{i-1}, \text{prob} = 1 - e^{-R_0 \frac{i-1}{S_0}}) \\
 S_i &:= S_{i-1} - I^i
 \end{aligned}$$

Where  $i = 1, 2, 3, \dots, W$ , ( $W = \inf\{t: S_t I^t = 0, t \geq 0\}$ ), size is number of trials and prob is probability of success on each trial and rbinom R function binomial distribution.

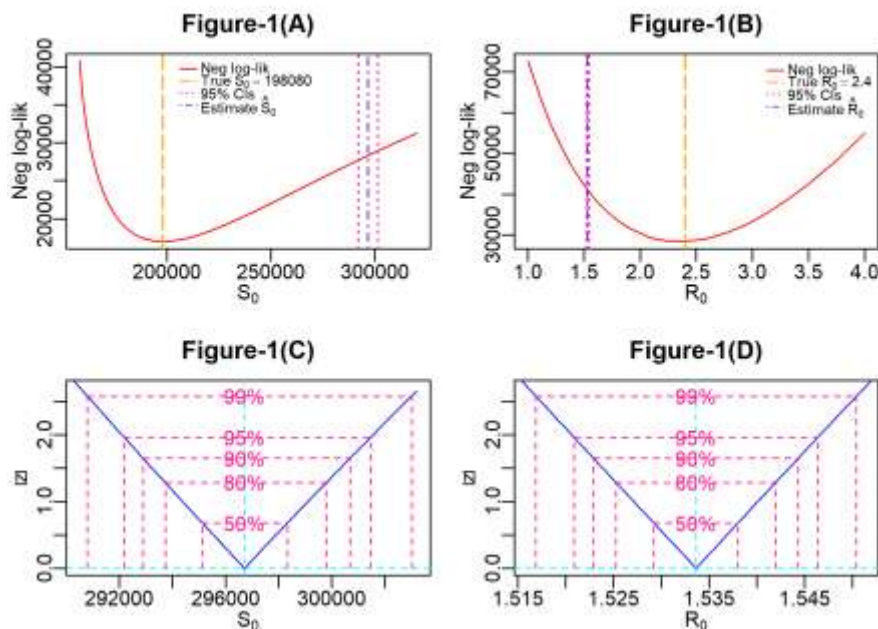
**Results:-**

**Basic Reproduction Number Estimates**

The estimates obtained on susceptible numbers in six Peruvian cities during the quarantine, Lima leads with 296690, followed by the city of Callao with 28049, and next Lambayeque, Piura, Libertad and Loreto with 26817, 24895, 23154 and 15137, people risks to acquire Covid-19, respectively. However, Loreto had fewer susceptible people even though she went through difficult times in the health service. In the case of the results of basic reproduction number of all the studied cities, Piura presented greater value of 1.829, and also revealed less in the cities of Libertad, Lambayeque and Lima with values of 1.506, 1.521 and 1.534. Finally, Loreto and Callao have 1.624 and 1.662. Table 1, summarizes the estimated parameters and confidence intervals (CI).

Next, we obtained the parameter estimation profile graphs: the number of susceptibles and the minimum basic reproduction number of the negative function of the logarithm of  $L(R_0, S_0)$ , in the six cities for data registered during the quarantine, as can be seen in figures 1,2,3,4,5 and 6.

**Results for Lima**



**Figure 1:-** (A) The conditional profile Negative log-likelihood of  $S_0$  for Lima's city assuming  $S_0=198080$ (minimum true value), (B) The conditional profile Negative log-likelihood of  $R_0$  for Lima's city assuming  $R_0 = 2.4$ (minimum true value), (C) Likelihood profile for  $S_0$ , and (D) Likelihood profile for  $R_0$ .

Results for Callao

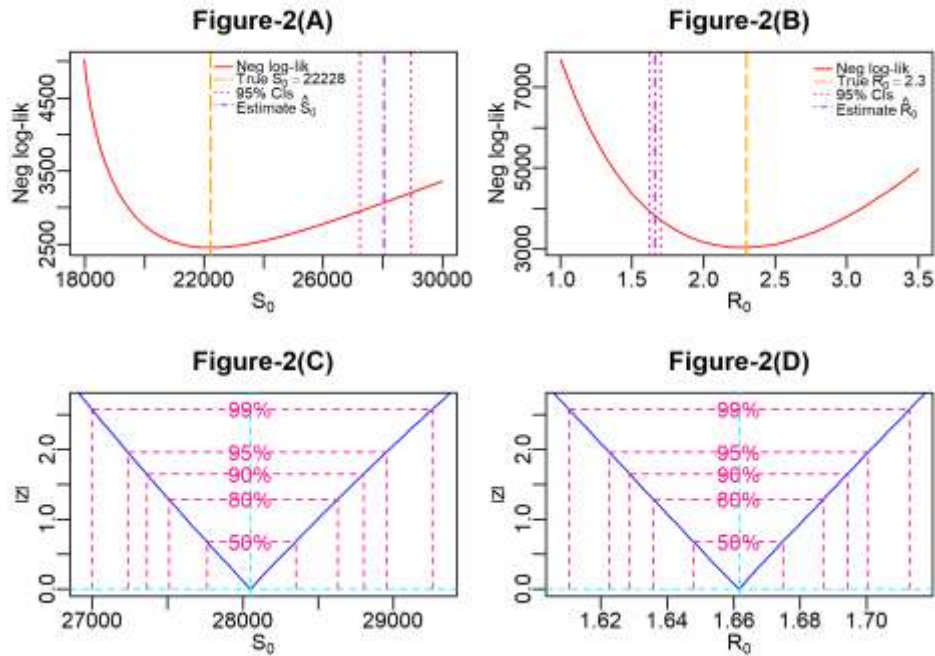


Figure 2:- (A) The conditional profile Negative log-likelihood of  $S_0$  for Callao's city assuming  $S_0 = 22228$ , (B) The conditional profile Negative log-likelihood of  $R_0$  for Callao's city assuming  $R_0 = 2.3$ , (C) Likelihood profile for  $S_0$ , and (D) Likelihood profile for  $R_0$ .

Results for Piura

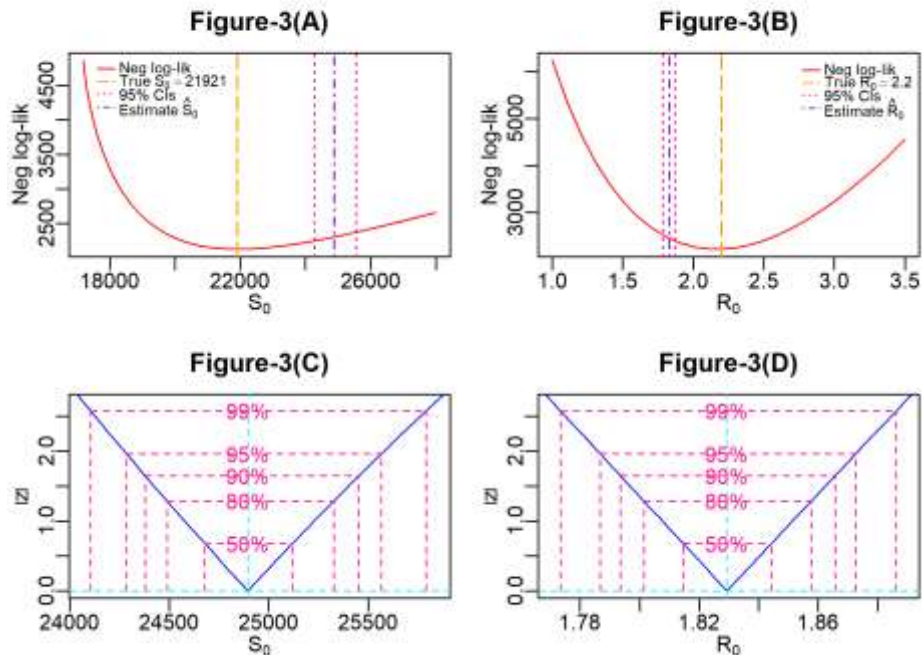


Figure 3:- (A) The conditional profile Negative log-likelihood of  $S_0$  for Piura's city assuming  $S_0 = 21921$ , (B) The conditional profile Negative log-likelihood of  $R_0$  for Piura's city assuming  $R_0 = 2.2$ , (C) Likelihood profile for  $S_0$ , and (D) Likelihood profile for  $R_0$ .

Results for Lambayeque

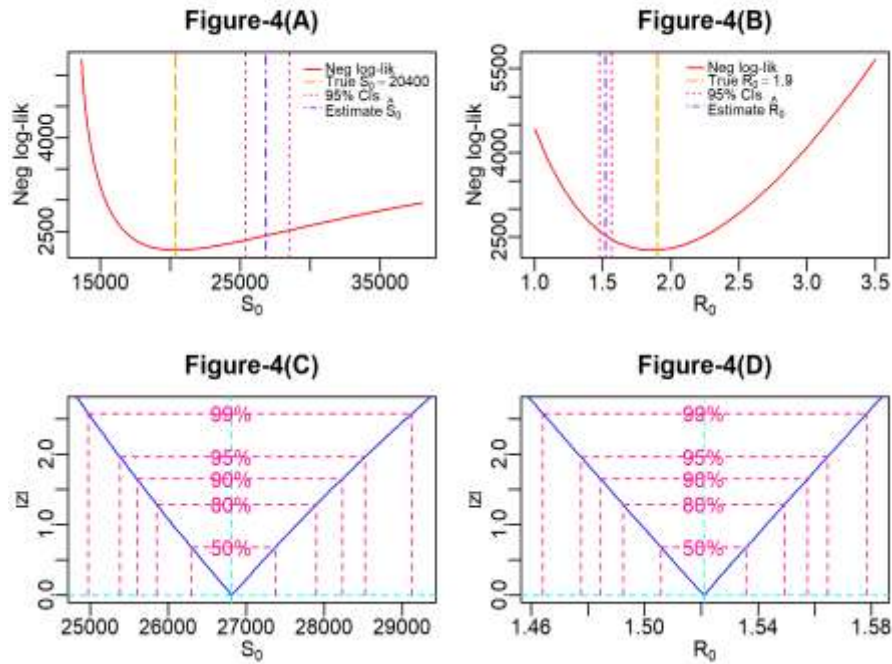


Figure 4:- (A) The conditional profile Negative log-likelihood of  $S_0$  for Lambayeque's city assuming  $S_0 = 20400$ , (B) The conditional profile Negative log-likelihood of  $R_0$  for Lambayeque's city assuming  $R_0 = 1.9$ , (C) Likelihood profile for  $S_0$ , and (D) Likelihood profile for  $R_0$ .

Results for Libertad

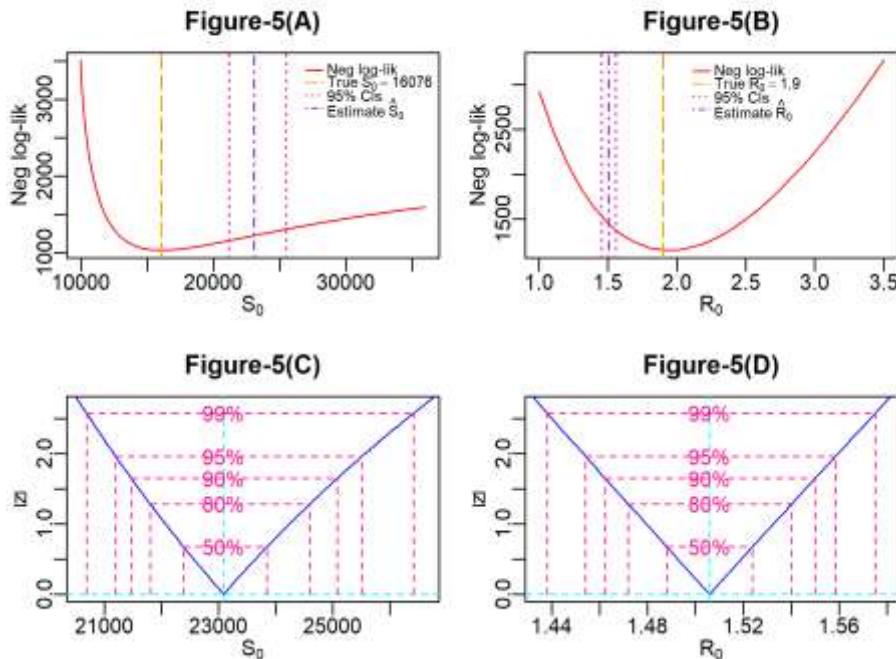
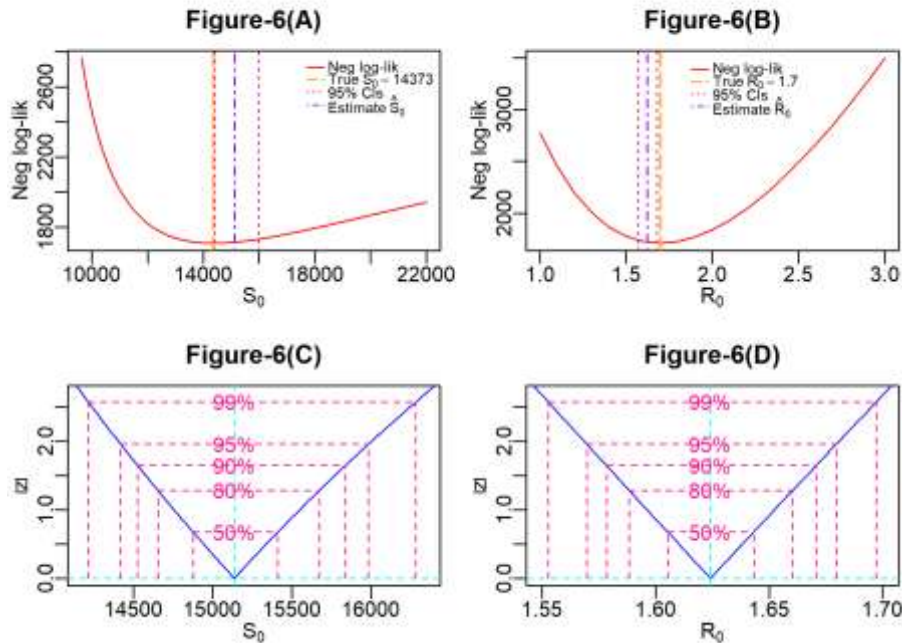


Figure 5:- (A) The conditional profile Negative log-likelihood of  $S_0$  for Libertad's city assuming  $S_0 = 16076$ , (B) The conditional profile log-likelihood of  $R_0$  for Libertad's city assuming  $R_0 = 1.9$ , (C) Likelihood profile for  $S_0$ , and (D) Likelihood profile for  $R_0$ .

Results for Loreto



**Figure 6:-** (A) The conditional profile Negative log-likelihood of  $S_0$  for Loreto's city assuming  $S_0= 14373$ , (B) The conditional profile log-likelihood of  $R_0$  for Loreto's city assuming  $R_0 = 1.7$ , (C) Likelihood profile for  $S_0$ , and (D) Likelihood profile for  $R_0$ .

In summary, we have in the following table the estimated parameters:

**Table 1:-** Estimate of initial number of susceptible and basic reproduction number of six cities in Peru.

Cities	Estimates of initial number of susceptibility $\hat{S}_0$ (95%, CI), (n=15)	Estimates of reproduction number $\hat{R}_0$ (95%, CI), (n=15)
Lima	296690(292192-301464)	1.534 (1.523-1.546)
Callao	28049(27236-28954)	1.662(1.623-1.701)
Piura	24895(24284-25566)	1.829(1.787-1.873)
Lambayeque	26817(25382-28525)	1.521(1.478-1.565)
Libertad	23090(21192-25513)	1.506(1.454-1.558)
Loreto	15137 (14417-15985)	1.624(1.570-1.680)

For the simulation of the number of infected per week, the parameters estimated in Table 1, and values assumed in Table 2, were taken into account.

**Table 2:-** We have assumed values of  $I^0$  and W prediction values, duration of the epidemic in the cities studied.

	Cities					
	Lima	Callao	Piura	Lambayeque	Libertad	Loreto
$I^0$	649	58	19	57	29	27
W(Weeks)	35	27	26	32	33	26

For each city 100, events were simulated and the graphs reflect the real situation of the pandemic, but some data observed do not correspond to the proposed model is due to the number of tests performed during the week greater than others and depends much on the process of issuing the report of the epidemiological center. In figure 7, it reveals that week 13 was the critical phase of the epidemic in Piura and week 11 in the rest of the cities.

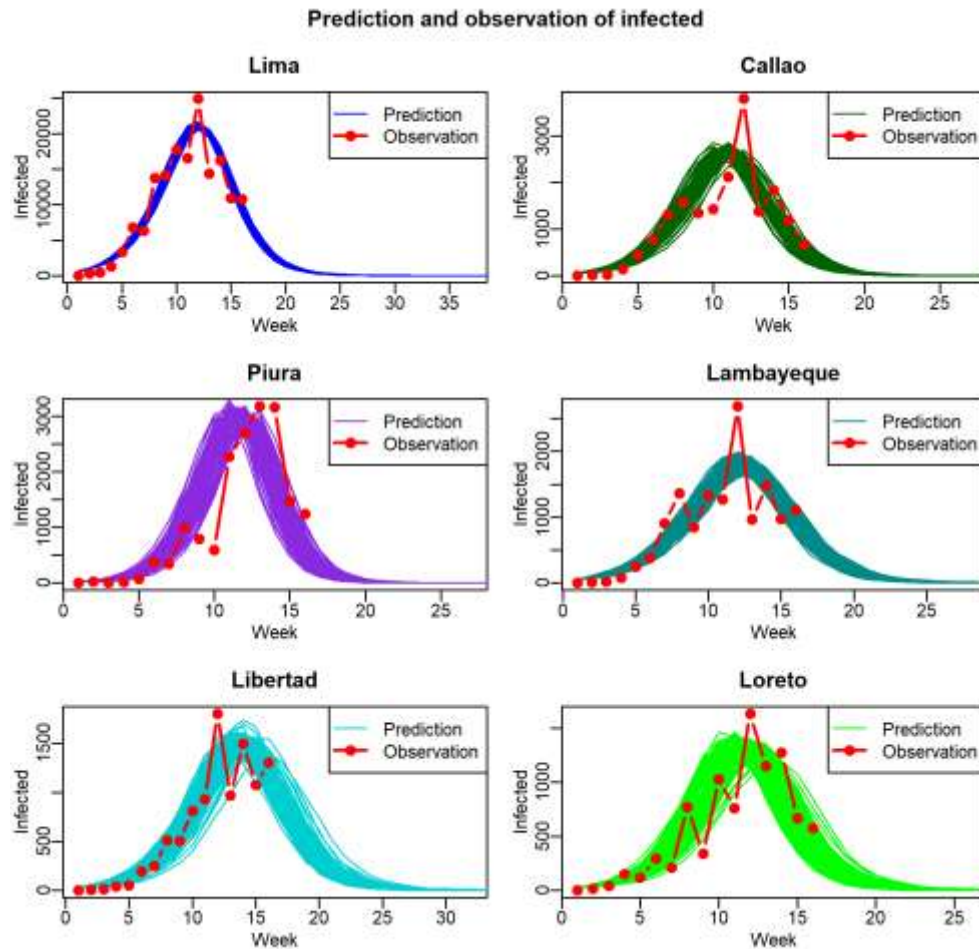
The results show that the city of Lima is 35 weeks long and should end in mid-December, and between 32 and 33 weeks for Lambayeque and Libertad, with a similar end in November. In addition, in the week 26 to 27, in October

the cities of Callao, Piura and Loreto, will be free of Covid 19. These projections should happen during this year 2020.

However, all results depend on strategies adopted in the cities and the responsibility of the population influences the process of the epidemic. Furthermore, the authorities play a very important role in containing contagion, whether by investing in safety protocols, buying medicines, spreading awareness of Covid-19, and among other prevention decisions.

### Discussions:-

The results of [17], suggest that unrestricted mobility significantly accelerates the spread of Covid-19 and the reproduction of the virus is rapid all over the world. The literature support, states that restrictions during the pandemic decrease the number of basic reproduction [12], that justifies the decision of the Peruvian state to declare the quarantine and later post-quarantine because  $R_0$ , is even greater than one and in the same way did the vast majority of South American countries but fewer days of confinement than Peru.



The study of Hao [9], on contagion in China, with data from 21 January to 17 February 2020, under the MSIR and MSEIR models, the basic reproduction number parameter is 1.5, and during 20 to 30 January excluding Hubei other research obtains  $R_0 = 1.5283$ , by SUQC method, considering that there are not forty rigorous and control measures [23] and according to Chu-Chang Ku, and Ta-Chou Ng [5], the basic reproduction de contagio of Covid-19, after the closure of Wuhan most of its provinces oscillate between 1.66 (CI: 0.72, 2.87) in Fujian and 5.51 (CI: 3.87-6.85) in Jilin, during January 24 to February 12 by the SIR model. In addition, in four cities in Korea,  $R_0 = 1.5$  (IC: 1.4-1.6) was obtained by the syncretized probability distribution model using reports between January 20 and February 18 [21] and in the Marche region of Italy the estimates of  $R_0$  calculated by maximum probability, within the four

provinces studied with reports from February 26 to April 20 of this year, the city Ancona has 1.512 (IC:0.75-2.75) [4]. These studies indicate that the parameters estimated in this research are within the range studied; although it is known that one month later the countries mentioned have control over the epidemic due to effective actions taken by their government on the pandemic and their  $R_0$  results are less than 2 before being free of Covid-19. If we consider under the same conditions of restriction made by the countries mentioned in the situation of Peru, the same should happen, but unfortunately they are different events even though the results obtained from  $R_0$  of European countries, are similar to the results of the cities studied in Peru. This different scenario is due to how the epidemic is dealt with and what restrictions are strategic to prevent further contagion. According to the results of the prediction simulation of infected people in Peruvian cities, it will have a duration of more weeks than cities in China, Italy, Korea and other European countries. Figure 7, illustrates the trends of infections, the declines in infections are gradually decreasing and are completed in the following order Piura, Loreto, Callao, Lambayeque, Libertad and Lima.

In the study on the dynamics of Covid-19, virus transmission, the model has limitations in not considering pre-infection periods, incubation, asymptomatic and other factors, which could influence it not to be fulfilled in real scenarios. However, it explains the current situation trend and can be done in other departments to understand the contagious behavior that helps the state to take some future precautions after the quarantine is lifted.

### Conclusion:-

This study provides knowledge about the dynamics of the virus transmission and the characteristics of positive incidences of Covid-19, which are really important to propose alternatives that can stop the disease that is still in progress. Among the results obtained, Piura has a higher number of basic reproduction and less Lambayeque, and the city of Lima will have a longer duration of incidence than the other cities.

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