Journal Homepage: - www.journalijar.com

INTERNATIONAL JOURNAL OF ADVANCED RESEARCH (IJAR)

Article DOI: 10.21474/IJAR01/12332
DOI URL: http://dx.doi.org/10.21474/IJAR01/12332

ISSN NO. 2320-5407

RESEARCH ARTICLE

GENERALISED EXPONENTIAL RATIO-TYPE ESTIMATOR FOR FINITE POPULATION VARIANCE UNDER RANDOM NON-RESPONSE

Alisha Mittal and Manoj Kumar
Department of Mathematics and Statistics, COBS, CCS HAU, Hisar-125004, Haryana.

Manuscript Info

Manuscript History
Received: 15 November 2020
Final Accepted: 18 December 2020
Published: January 2021

Key words:-
Random Non-Response, Quartiles, Exponential Estimator, Properties of the Estimators, Percent Relative Efficiency

Abstract

In this research paper an effort has been made for the estimation of population variance of the study variable by using information on certain known parameters of the auxiliary variable under non-response for scheme I and II given by Singh and Joarder (1998). Generalized exponential ratio-type estimator has been proposed and their properties have been studied under non response techniques and conditions were found when the family of proposed estimators identified by using different choices for (P, Q) performed better than the usual unbiased estimator. It was also observed that for different values of α ∈ (0.0, 1.0), the estimators T_4 and $T_4^{'}$ were found to be best under numerical illustration.

Introduction:-

Non-response is defined as when information from one or more than one sampling unit(s) on study variable or auxiliary variable(s) or both is not available. There always exists non-response when sample survey has been done on human population. Non-response rate is affected by various factors like the official position of the surveying agency, the lawful obligations of the respondents, the degree of publicity, the time of visit by enumerator and length of the schedule etc. Non-response reduces the sample size and consequently the accuracy of estimators will be smaller and the chances of error will be larger. A more serious outcome of non-response is that it can be selective. The application of auxiliary information in sample surveys leads to substantial expansion in the precision of the population parameters. By applying auxiliary information in different forms, estimators for population parameters generally population mean and variance are studied and are existing in the literature. For example, the concept of auxiliary information to form the ratio estimators was given by Cochran (1940). After that a number of estimators have been prepared by utilizing the information on auxiliary variables for estimating the population parameters i.e. population mean or population variance under simple random sampling and stratified sampling. But, the difficulty of non-response while estimating the population mean by utilizing the information on auxiliary variable or study variable was considered for the first time by Hansen and Hurwitz (1946).

The concept of evaluation of population variance has a great importance in real life situation such as industry, agriculture, biology and medical studies. A number of estimators have been developed by various researchers and suggested better estimator for estimating S_y^2 under simple random sampling by utilizing the auxiliary information, for some work along these lines, see (Isaki, 1983; Kadilar and Cingi, 2006; Singh *et al*., 2014). The properties of estimator for estimating the population mean and finite population variance under scheme-I and scheme-II in presence of non-response given by Singh and Joarder (1998) are available in literature. A lot of work has been for estimating the population mean under non-response, see (Kumar *et al*., 2018; Mohamed *et al*., 2018; Irfan, 2018).

Corresponding Author:- Alisha Mittal
Address:- Department of Mathematics and Statistics, COBS, CCS HAU, Hisar-125004, Haryana.

In present study, an effort has been made to modify the estimators of Solanki *et al*. (2015) under non-response. Novelty of present work lies in the fact that we have introduced the modified and generalized version of the exponential estimators proposed by Solanki *et al*. (2015) in presence of non response under scheme-I and scheme-II given by Singh and Joarder (1998). The section 1 introduced the notations and expected values where as in section 2 & 3 proposed estimator and their properties have been studied. Section 4 dealt with the comparison of family of the proposed estimator under scheme-I & II with the typical ratio estimator due to Isaki (1983) and with each other. Empirical study has been done in section 5 by taking the actual data set from Singh (2003).

Notations:

Let us define

$\varepsilon = (s_y^{*2}/S_y^2) - 1$, $\delta = (s_x^{*2}/S_x^2) - 1$, and $\eta = (s_x^2/S_x^2) - 1$

where

$s_y^{*2} = (n-r-1)^{-1}\sum_{i=1}^{n-r}(y_i - \bar{y}^*)^2$ and $s_x^{*2} = (n-r-1)^{-1}\sum_{i=1}^{n-r}(x_i - \bar{x}^*)^2$ are conditionally unbiased estimators of S_y^2 and S_x^2 respectively, and $\bar{y}^* = (n-r)^{-1}\sum_{i=1}^{n-r} y_i$ and $\bar{x}^* = (n-r)^{-1}\sum_{i=1}^{n-r} x_i$.

Thus under the probability model, $P(r) = \frac{n-r}{nq+2p}\binom{n-2}{r}p^r q^{n-2-r}$ (1.1)

where $p+q=1$; $r= 0,1,2,\ldots, n-2$; and $\binom{n-2}{r}$ denotes the total number of ways of r non responses out of total possible $(n-2)$ responses, we have following results,

$E(\varepsilon) = E(\delta) = E(\eta) = 0$,

$E(\varepsilon^2) = \left[\frac{1}{(nq+2p)} - \frac{1}{N}\right](\lambda_{40} - 1)$, $E(\delta^2) = \left[\frac{1}{(nq+2p)} - \frac{1}{N}\right](\lambda_{04} - 1)$,

$E(\eta^2) = \left[\frac{1}{n} - \frac{1}{N}\right](\lambda_{04} - 1)$, $E(\varepsilon\delta) = \left[\frac{1}{(nq+2p)} - \frac{1}{N}\right](\lambda_{22} - 1)$,

$E(\varepsilon\eta) = \left[\frac{1}{n} - \frac{1}{N}\right](\lambda_{22} - 1)$, and $E(\delta\eta) = \left[\frac{1}{n} - \frac{1}{N}\right](\lambda_{04} - 1)$

where $\lambda_{ls} = \frac{\mu_{ls}}{\mu_{20}^{l/2}\mu_{02}^{s/2}}$ and $\mu_{ls} = \frac{1}{N-1}\sum_{i=1}^{N}(Y_i - \bar{Y})^l (X_i - \bar{X})^s$

Suggested estimators and properties of generalised exponential ratio-type estimator for finite population variance under scheme-I:

The proposed estimators have been modified version of Solanki *et al*. (2015). The generalized exponential estimator of finite population variance under non-response for scheme-I & II have been proposed. The estimator (T_{PQ}) has been given under scheme-I when random non-response exists on both the study variable and the auxiliary variable which are denoted by y and x respectively and the population variance S_x^2 of the auxiliary variable is known. The proposed exponential estimator under the scheme-I is as follows:

$$T_{PQ} = s_y^{*2} exp\left(\frac{PS_x^2 + \alpha Q^2}{Ps_x^{*2} + \alpha Q L^2}\right) \quad (2.1)$$

where $(PS_x^2 + \alpha Q^2) > 0$, $(Ps_x^{*2} + \alpha Q^2) > 0$ and (P, Q) are either real constants or function of known parameters of an auxiliary variable x with $0 \leq \alpha \leq 1$.

Theorem2.1:-The properties of the estimator T_{PQ} i.e. bias and mean square error to first degree of approximation is given by

$$Bias(T_{PQ}) = S_y^2\left[\frac{5}{3} + \frac{1}{2}\tau^*\left(\frac{1}{(nq+2p)} - \frac{1}{N}\right)\left(7\tau^*(\lambda_{04} - 1) - 5(\lambda_{22} - 1)\right)\right] \quad (2.2)$$

$$MSE(T_{PQ}) = S_y^4\left[\frac{25}{9} + \left(\frac{1}{(nq+2p)} - \frac{1}{N}\right)\left(\frac{25}{4}\tau^{*2}(\lambda_{04} - 1) + \frac{64}{9}(\lambda_{40} - 1) - \frac{40}{3}\tau^*(\lambda_{22} - 1)\right)\right] \quad (2.3)$$

where $\tau^* = PS_x^2(PS_x^2 + \alpha Q^2)^{-1}$

Proof:

Expressing the proposed exponential estimator T_{PQ} in terms of ε, δ and η, we have

$$(T_{PQ} - S_y^2) = S_y^2\left[\frac{5}{3} + \frac{8}{3}\varepsilon - \frac{5}{2}\tau^*\delta - \frac{5}{2}\tau^*\varepsilon\delta + \frac{7}{2}\tau^{*2}\delta^2\right] \quad (2.4)$$

Taking expectation of both sides of (2.4) and using the results on the expectations from section 1.1, we get the bias in the proposed exponential estimator T_{PQ} to the first degree of approximation as given in (2.2).

Squaring both sides of equation (2.4) and taking expectations to the first degree of approximation, as given below

$$MSE(T_{PQ}) = E(T_{PQ} - S_y^2)^2 = S_y^4 E\left[\frac{5}{3} + \frac{8}{3}\varepsilon - \frac{5}{2}\tau^*\delta\right]^2$$

$MSE(T_{PQ}) = S_y^4 E\left[\frac{25}{9} + \frac{64}{9}\mathcal{E}^2 + \frac{25}{4}\tau^{*2}\delta^2 + 2\times\frac{5}{3}\times\frac{8}{3}\mathcal{E} - 2\times\frac{8}{3}\mathcal{E}\times\frac{5}{2}\tau^*\delta - 2\times\frac{5}{3}\times\frac{5}{2}\tau^*\delta\right]$

We obtain the mean square error (*MSE*) of the proposed exponential estimator T_{PQ} to the first degree of approximation given in equation (2.3).
An estimator of the MSE (T_{PQ}) is given by

$\widehat{MSE}\ (T_{PQ}) = s_y^{*4}\left[\frac{25}{9} + \left(\frac{1}{(n\hat{q}+2\hat{p})} - \frac{1}{N}\right)\left(\frac{25}{4}\hat{\tau}^{*2}\left(\hat{\lambda}_{04}^* - 1\right) + \frac{64}{9}\left(\hat{\lambda}_{40}^* - 1\right) - \frac{40}{3}\hat{\tau}^*\left(\hat{\lambda}_{22}^* - 1\right)\right)\right]$

where $\hat{\tau}^* = Ps_x^{*2}(Ps_x^{*2} + \alpha Q^2)^{-1}$; $\hat{\lambda}_{ls}^* = \frac{\hat{\mu}_{ls}^*}{(\hat{\mu}_{20}^*)^{l/2}(\hat{\mu}_{02}^*)^{s/2}}$; $\hat{\mu}_{ls}^* = \frac{1}{(n-r-1)}\sum_{i=1}^{n-r}(y_i - \bar{y}^*)^l\,(x_i - \bar{x}^*)^s$

$\hat{p} = \frac{(n-1+r)-\sqrt{(n-1+r)^2-4nr\,(n-3)/(n-2)}}{2(n-3)}$ is a maximum likelihood estimator (m. l. e) of p obtained from the distribution given by (1.1) and $\hat{q} = 1 - \hat{p}$.

Now, some members of proposed exponential ratio type estimator T_{PQ} for different choices of (P, Q).
(i) The estimator based on coefficient of variation C_x and quartile Q_1:
If we set (P, Q) = (C_x, Q_1) in (2.1), we get the estimator of S_y^2 as,
$T_1 = s_y^{*2} exp\left(\frac{C_x S_x^2 + \alpha Q_1^2}{C_x s_x^{*2} + \alpha Q_1^2}\right)$
(ii) The estimator based on coefficient of kurtosis $\beta_2(x)$ and median Q_2:
If we set (P, Q) = $(\beta_2(x), Q_2)$ in (2.1), we get the estimator of S_y^2 as,
$T_2 = s_y^{*2} exp\left(\frac{\beta_2(x) S_x^2 + \alpha Q_2^2}{\beta_2(x) s_x^{*2} + \alpha Q_2^2}\right)$
(iii) The estimator based on population mean $\bar{X}$ and quartile Q_3:
If we set (P, Q) = $(\bar{X}, Q_3)$ in (2.1), we get the estimator of S_y^2 as,
$T_3 = s_y^{*2} exp\left(\frac{\bar{X}\, S_x^2 + \alpha Q_3^2}{\bar{X}\, s_x^{*2} + \alpha Q_3^2}\right)$

(iv) The estimator based on coefficient of kurtosis $\beta_2(x)$ and interquartile range Q_r:
If we set (P, Q) = $(\beta_2(x), Q_r)$ in (2.1), we get the estimator of S_y^2 as,
$T_4 = s_y^{*2} exp\left(\frac{\beta_2(x) S_x^2 + \alpha Q_r^2}{\beta_2(x) s_x^{*2} + \alpha Q_r^2}\right)$
(v) The estimator based on correlation coefficient ρ and semi-quartile range Q_d:
If we set (P, Q) = (ρ, Q_d) in (2.1), we get the estimator of S_y^2 as,
$T_5 = s_y^{*2} exp\left(\frac{\rho S_x^2 + \alpha Q_d^2}{\rho s_x^{*2} + \alpha Q_d^2}\right)$

(vi) The estimator based on correlation coefficient ρ and semi-quartile average Q_a:
If we set (P, Q) = (ρ ,Q_a) in (2.1), we get the estimator of S_y^2 as,
$T_6 = s_y^{*2} exp\left(\frac{\rho S_x^2 + \alpha Q_a^2}{\rho s_x^{*2} + \alpha Q_a^2}\right)$
In this way we have obtained six estimators for different choice of P and Q for scheme-I. More estimators can be obtained for different choice of P and Q. These six estimators are denoted by T_k, where , k = 1,2,...,6.

Corollary 2.1.1.:-To the first degree of approximation the biases and mean squared errors (MSEs) of these six exponential ratio-type estimator T_k, (k = 1,2,...,6) are respectively given by

Bias$(T_k) = S_y^2\left[\frac{5}{3} + \frac{1}{2}\tau_k^*\left(\frac{1}{(nq+2p)} - \frac{1}{N}\right)\left(7\tau_k^*(\lambda_{04} - 1) - 5(\lambda_{22} - 1)\right)\right]$ (2.5)

MSE$(T_k) = S_y^4\left[\frac{25}{9} + \left(\frac{1}{(nq+2p)} - \frac{1}{N}\right)\left(\frac{25}{4}\tau_k^{*2}(\lambda_{04} - 1) + \frac{64}{9}(\lambda_{40} - 1) - \frac{40}{3}\tau_k^*(\lambda_{22} - 1)\right)\right]$ (2.6)

where $\tau_1^* = C_x S_x^2(C_x S_x^2 + \alpha Q_1^2)^{-1}$, $\tau_2^* = \beta_2(x) S_x^2(\beta_2(x) S_x^2 + \alpha Q_2^2)^{-1}$, $\tau_3^* = \bar{X}\, S_x^2(\bar{X}\, S_x^2 + \alpha Q_3^2)^{-1}$,
$\tau_4^* = \beta_2(x) S_x^2(\beta_2(x) S_x^2 + \alpha Q_r^2)^{-1}$, $\tau_5^* = \rho S_x^2(\rho S_x^2 + \alpha Q_d^2)^{-1}$, $\tau_6^* = \rho S_x^2(\rho S_x^2 + \alpha Q_a^2)^{-1}$.

Suggested estimators and pproperties of generalised exponential ratio-type estimator for finite population variance under scheme-II:

The proposed estimators have been modified version of Solanki *et al*. (2015). The estimator $T_{PQ}{'}$ has been given under scheme-II i.e. when information on study variable *y* could not be obtained for *r* units while information on the auxiliary variable x is available and population variance S_x^2 of the auxiliary variable is known. The proposed exponential estimator under the scheme-II is as follows:

$$T_{PQ}{'} = s_y^{*2} exp\left(\frac{PS_x^2+\alpha Q^2}{Ps_x^2+\alpha Q^2}\right) \quad (3.1)$$

Theorem 3.1: The properties of the estimator $T_{PQ}{'}$ i.e. bias and mean square error to first degree of approximation is given by:

$$Bias\ (T_{PQ}{'}) = S_y^2\left[\frac{5}{3}+\frac{1}{2}\tau^*\left(\frac{1}{n}-\frac{1}{N}\right)\left(7\tau^*(\lambda_{04}-1)-5(\lambda_{22}-1)\right)\right] \quad (3.2)$$

$$MSE(T_{PQ}{'}) = S_y^4\left[\frac{25}{9}+5\tau^*\left(\frac{1}{n}-\frac{1}{N}\right)\left(\frac{5}{4}\tau^*(\lambda_{04}-1)-\frac{8}{3}(\lambda_{22}-1)\right)+\frac{64}{9}\left(\frac{1}{(nq+2p)}-\frac{1}{N}\right)(\lambda_{40}-1)\right] \quad (3.3)$$

where $\tau^* = PS_x^2(AS_x^2+\ \alpha Q^2)^{-1}$

Proof: Expressing the proposed exponential estimator $T_{PQ}{'}$ in terms of ε, δ and ŋ, we have

$$(T_{PQ}{'} - S_y^2) = S_y^2\left[\frac{5}{3}+\frac{8}{3}\varepsilon-\frac{5}{2}\tau^*\eta-\frac{5}{2}\tau^*\varepsilon\eta+\frac{7}{2}\tau^{*2}\eta^2\right] \quad (3.4)$$

Taking expectation of both sides of (3.4) and using the results on the expectations, we get the bias in the proposed exponential estimator $T_{PQ}{'}$ to the first degree of approximation as given in (3.2).

Squaring both sides of equation (3.4) and taking expectations to the first degree of approximation as given below

$$MSE(T_{PQ}{'}) = E(T_{PQ}{'} - S_y^2)^2 = S_y^4 E\left[\frac{5}{3}+\frac{8}{3}\varepsilon-\frac{5}{2}\tau^*\eta\right]^2$$

$$MSE(T_{PQ}{'} = E\left[\frac{25}{9}+\frac{64}{9}\varepsilon^2+\frac{25}{4}\tau^{*2}\eta^2+2\times\frac{5}{3}\times\frac{8}{3}\varepsilon-2\times\frac{8}{3}\varepsilon\times\frac{5}{2}\tau^*\eta-2\times\frac{5}{3}\times\frac{5}{2}\tau^*\eta\right]$$

The mean square error of the proposed exponential estimator $T_{PQ}{'}$, up to terms of order $O(n^{-1})$ is as given in (3.3).

An estimator of the MSE ($T_{PQ}{'}$) is given by

$$\widehat{MSE}(T_{PQ}{'}) = s_y^{*4}\left[\frac{25}{9}+5\hat{\tau}^*\left(\frac{1}{n}-\frac{1}{N}\right)\left(\frac{5}{4}\hat{\tau}^*(\hat{\lambda}_{04}-1)-\frac{8}{3}(\hat{\lambda}_{22}^*-1)\right)+\frac{64}{9}\left(\frac{1}{(n\hat{q}+2\hat{p})}-\frac{1}{N}\right)(\hat{\lambda}_{40}^*-1)\right]$$

where $\hat{\tau}^* = Ps_x^{*2}(Ps_x^{*2}+\ \alpha Q^2)^{-1}$; $\hat{\lambda}_{04}=\frac{\hat{\mu}_{04}}{(\hat{\mu}_{02})^2}$ and $\hat{\mu}_{0s} = \frac{1}{(n-1)}\sum_{i=1}^{n}(x_i-\bar{x})^s$

Now, some members of proposed exponential estimator $T_{PQ}{'}$ for different choices of (P,Q).

(i) The estimator based on coefficient of variation C_x and quartile Q_1:

If we set (P, Q) = (C_x, Q_1) in (3.1), we get the estimator of S_y^2 as,

$$T_1' = s_y^{*2} exp\left(\frac{C_xS_x^2+\alpha Q_1^2}{C_xs_x^2+\alpha Q_1^2}\right)$$

(ii) The estimator based on coefficient of kurtosis $\beta_2(x)$ and median Q_2:

If we set (P, Q) = ($\beta_2(x)$, Q_2) in (3.1), we get the estimator of S_y^2 as,

$$T_2' = s_y^{*2} exp\left(\frac{\beta_2(x)S_x^2+\alpha Q_2^2}{\beta_2(x)s_x^2+\alpha Q_2^2}\right)$$

(iii) The estimator based on population mean $\bar{X}$ and quartile Q_3:

If we set (P, Q) = ($\bar{X}$, Q_3) in (3.1), we get the estimator of S_y^2 as,

$$T_3' = s_y^{*2} exp\left(\frac{\bar{X}S_x^2+\alpha Q_3^2}{\bar{X}s_x^2+\alpha Q_3^2}\right)$$

(iv) The estimator based on coefficient of kurtosis $\beta_2(x)$ and interquartile range Q_r:

If we set (P, Q) = ($\beta_2(x)$, Q_r) in (3.1), we get the estimator of S_y^2 as,

$$T_4' = s_y^{*2} exp\left(\frac{\beta_2(x)S_x^2+\alpha Q_r^2}{\beta_2(x)s_x^2+\alpha Q_r^2}\right)$$

(v) The estimator based on correlation coefficient ρ and semi-quartile range Q_d:

If we set (P, Q) = (ρ ,Q_d) in (3.1), we get the estimator of S_y^2 as,

$T_5^{'}= s_y^{*2} exp\left(\frac{\rho S_x^2+\alpha Q_d^2}{\rho s_x^{*2}+\alpha Q_d^2}\right)$

(vi) The estimator based on correlation coefficient ρ and semi-quartile average Q_a:
If we set (P, Q) = (ρ ,Q_a) in (3.1), we get the estimator of S_y^2 as,
$T_6^{'}= s_y^{*2} exp\left(\frac{\rho S_x^2+\alpha Q_a^2}{\rho s_x^2+\alpha Q_a^2}\right)$
In this way we have obtained six estimators for different choice of P and Q for scheme-II. More estimators can be obtained for different choice of P and Q. The six estimators are denoted by $T_k^{'}$ where, k = 1, 2,..., 6.

Corollary 3.1.1.:-
To the first degree of approximation the biases and mean squared errors (MSEs) of these six exponential ratio-type estimator $T_k^{'}$, (k = 1,2,...,6) are respectively given by

$$\text{Bias}(T_k^{'}) = S_y^2\left[\frac{5}{3}+\frac{1}{2}\tau_k^*\left(\frac{1}{n}-\frac{1}{N}\right)\left(7\tau_k^*(\lambda_{04}-1)-5(\lambda_{22}-1)\right)\right] \quad (3.5)$$

$$\text{MSE}(T_k^{'})= S_y^4\left[\frac{25}{9}+5\tau_k^*\left(\frac{1}{n}-\frac{1}{N}\right)\left(\frac{5}{4}\tau_k^*(\lambda_{04}-1)-\frac{8}{3}(\lambda_{22}-1)\right)+\frac{64}{9}\left(\frac{1}{(nq+2p)}-\frac{1}{N}\right)(\lambda_{40}-1)\right] \quad (3.6)$$

where $\tau_1^* = C_x S_x^2(C_x S_x^2 + \alpha Q_1^2)^{-1}$, $\tau_2^* = \beta_2(x) S_x^2(\beta_2(x) S_x^2 + \alpha Q_2^2)^{-1}$, $\tau_3^* = \bar{X} S_x^2(\bar{X} S_x^2 + \alpha Q_3^2)^{-1}$,$\tau_4^* = \beta_2(x) S_x^2(\beta_2(x) S_x^2 + \alpha Q_r^2)^{-1}$, $\tau_5^* = \rho S_x^2(\rho S_x^2 + \alpha Q_d^2)^{-1}$,$\tau_6^* = \rho S_x^2(\rho S_x^2 + \alpha Q_a^2)^{-1}$.

Comparison of mean square error of proposed estimators under scheme-I and II with the typical ratio estimator and with each other:

As it is already established in the literature, the mean square error of typical ratio estimator of population variance is given by $Var(s_y^{*2}) = S_y^4\left(\frac{1}{(nq+2p)}-\frac{1}{N}\right)(\lambda_{40}-1)$ (4.1)

On comparing (2.6) with (4.1), the family of proposed estimator T_k for scheme-I has less mean square error than typical ratio estimator of population variance under non-response, if
$Var(s_y^{*2}) - MSE\ (T_k) \geq 0$
i.e. if $\lambda_{40} \leq 1-\frac{1}{11}\left[5\left(\frac{1}{(nq+2p)}-\frac{1}{N}\right)^{-1}-\frac{45}{4}\tau_k^{*2}(\lambda_{04}-1)-24\tau_k^*(\lambda_{22}-1)\right]$ (4.2)

Similarly on comparing (3.6) with the (4.1), the family of proposed estimator $T_k^{'}$ scheme-II has less mean square error than typical ratio estimator of population variance under non-response, if
$Var(s_y^{*2}) - MSE\ (T_k^{'}) \geq 0$
i.e.if $\lambda_{40} \leq 1-\frac{1}{11}\left(\frac{1}{(nq+2p)}-\frac{1}{N}\right)^{-1}\left[5-\frac{45}{4}\tau_k^{*2}\left(\frac{1}{n}-\frac{1}{N}\right)(\lambda_{04}-1)-24\tau_k^*\left(\frac{1}{n}-\frac{1}{N}\right)(\lambda_{22}-1)\right]$ (4.3)

On the same line if we compare the mean square error of the family of proposed estimator T_k for scheme-I with the family of proposed estimator $T_k^{'}$ for scheme-II
$MSE\ (T_k) - MSE\ (T_k^{'}) \geq 0$
$\frac{1}{(nq+2p)}-\frac{1}{n} \geq 0$ (4.4)

It follows from the above expressions that the family of proposed estimators T_k and $T_k^{'}$ are most efficient than the typical unbiased estimator s_y^{*2} under the condition given in (4.2 and (4.3). It is further observed that the family of the proposed estimator $T_k^{'}$ i.e. under scheme-II is more efficient than family of the proposed estimators T_k i.e. under scheme-I only if (4.4) condition holds.

Empirical study:

The typical unbiased ratio-type estimator and the family of proposed generalized exponential estimators of population variance of study variable in presence of non-response have been compared empirically by using the data considered by Singh (2013). The parameter values and constants required for comparison of estimators are given in the table 1.

Table 1:- Description of parametric values and constant.

N	n	r	$\hat{\rho}$	$\bar{x}$	$\bar{x}^*$	$\bar{y}^*$	s_x^{*2}	s_y^{*2}	$\hat{\lambda}_{40}^*$	$\hat{\lambda}_{04}^*$
50	20	4	0.234	1148.98	1123.62	735.77	1622949.414	4327775.816	2.078	2.814

$\hat{\lambda}_{22}^*$	$\hat{\lambda}_{04}$	C_x	$\beta_2(x)$	Q_1	Q_2	Q_3	Q_r	Q_d	Q_a	ρ
1.178	2.613	1.134	0.496	168.121	507.034	1569.017	1400.897	700.448	868.569	0.788

The mean square errors of family of estimators T_k, (k = 1,2,...,6) and $T_k^{'}$, (k = 1,2,...,6) are obtained using the formulae given by equation (2.6) and (3.6) respectively as shown in Table 2 and Table 3.

Table 2:- MSEs of T_k, (k = 1,2,...,6) with respect to various values of α.

α	$MSE\ (T_1)$	$MSE\ (T_2)$	$MSE\ (T_3)$	$MSE\ (T_4)$	$MSE\ (T_5)$	$MSE\ (T_6)$
0.0	6.552×10^{11}	6.552×10^{11}	6.552×10^{11}	6.552×10^{11}	6.552×10^{11}	6.552×10^{11}
0.1	6.549×10^{11}	6.502×10^{11}	6.552×10^{11}	6.265×10^{11}	6.492×10^{11}	6.463×10^{11}
0.2	6.547×10^{11}	6.456×10^{11}	6.552×10^{11}	6.111×10^{11}	6.439×10^{11}	6.388×10^{11}
0.3	6.544×10^{11}	6.415×10^{11}	6.551×10^{11}	6.021×10^{11}	6.392×10^{11}	6.325×10^{11}
0.4	6.542×10^{11}	6.377×10^{11}	6.551×10^{11}	5.963×10^{11}	6.349×10^{11}	6.271×10^{11}
0.5	6.540×10^{11}	6.343×10^{11}	6.551×10^{11}	5.925×10^{11}	6.311×10^{11}	6.225×10^{11}
0.6	6.537×10^{11}	6.311×10^{11}	6.551×10^{11}	5.898×10^{11}	6.276×10^{11}	6.184×10^{11}
0.7	6.535×10^{11}	6.282×10^{11}	6.550×10^{11}	5.879×10^{11}	6.245×10^{11}	6.149×10^{11}
0.8	6.532×10^{11}	6.255×10^{11}	6.550×10^{11}	5.865×10^{11}	6.216×10^{11}	6.119×10^{11}
0.9	6.530×10^{11}	6.230×10^{11}	6.550×10^{11}	5.855×10^{11}	6.190×10^{11}	6.092×10^{11}
1.0	6.527×10^{11}	6.207×10^{11}	6.550×10^{11}	5.847×10^{11}	6.166×10^{11}	6.068×10^{11}

Table 3:- MSEs of $T_k^{'}$, (k = 1,2,...,6) with respect to various values of α.

α	$MSE\ (T_1^{'})$	$MSE\ (T_2^{'})$	$MSE\ (T_3^{'})$	$MSE\ (T_4^{'})$	$MSE\ (T_5^{'})$	$MSE\ (T_6^{'})$
0.0	5.730×10^{11}	5.730×10^{11}	5.730×10^{11}	5.730×10^{11}	5.730×10^{11}	5.730×10^{11}
0.1	5.728×10^{11}	5.696×10^{11}	5.730×10^{11}	5.537×10^{11}	5.690×10^{11}	5.671×10^{11}
0.2	5.726×10^{11}	5.666×10^{11}	5.729×10^{11}	5.432×10^{11}	5.655×10^{11}	5.621×10^{11}
0.3	5.725×10^{11}	5.638×10^{11}	5.729×10^{11}	5.369×10^{11}	5.623×10^{11}	5.578×10^{11}
0.4	5.723×10^{11}	5.613×10^{11}	5.729×10^{11}	5.328×10^{11}	5.594×10^{11}	5.542×10^{11}
0.5	5.721×10^{11}	5.590×10^{11}	5.729×10^{11}	5.300×10^{11}	5.568×10^{11}	5.510×10^{11}
0.6	5.720×10^{11}	5.569×10^{11}	5.729×10^{11}	5.280×10^{11}	5.545×10^{11}	5.483×10^{11}
0.7	5.718×10^{11}	5.549×10^{11}	5.729×10^{11}	5.266×10^{11}	5.524×10^{11}	5.459×10^{11}
0.8	5.716×10^{11}	5.531×10^{11}	5.728×10^{11}	5.255×10^{11}	5.504×10^{11}	5.437×10^{11}
0.9	5.715×10^{11}	5.514×10^{11}	5.728×10^{11}	5.246×10^{11}	5.486×10^{11}	5.419×10^{11}
1.0	5.713×10^{11}	5.498×10^{11}	5.728×10^{11}	5.239×10^{11}	5.470×10^{11}	5.402×10^{11}

The performance of the family of estimators T_k and $T_k^{'}$, (k = 1,2,...,6) which are members of the proposed generalized exponential ratio-type estimator T_{PQ} and T_{PQ} respectively are evaluated against the typical unbiased estimator s_y^{*2} for the population data set described in Table 1.

The percent relative efficiencies (PREs) of T_k and $T_k^{'}$, (k = 1,2,...,6) have been computed with respect to the typical unbiased estimator s_y^{*2} in certain range of α ∈ (0.0,1.0) by using following formulae respectively as

$$PRE\left(T_k, s_y^{*2}\right) = 100 \times \frac{MSE\left(s_y^{*2}\right)}{MSE(T_k)}$$

$$= \frac{100 \times \left(\frac{1}{(nq+2p)} - \frac{1}{N}\right)(\lambda_{40} - 1)}{\frac{25}{9} + \left(\frac{1}{(nq+2p)} - \frac{1}{N}\right)\left(\frac{25}{4}\tau_k^{*2}(\lambda_{04} - 1) + \frac{64}{9}(\lambda_{40} - 1) - \frac{40}{3}\tau_k^{*}(\lambda_{22} - 1)\right)}$$

$$PRE\left(T_k^{'}, s_y^{*2}\right) = 100 \times \frac{MSE\left(s_y^{*2}\right)}{MSE(\ T_k^{'})}$$

$$= \frac{100 \times \left(\frac{1}{(nq+2p)} - \frac{1}{N}\right)(\lambda_{40} - 1)}{\frac{25}{9} + 5\tau_k^{*}\left(\frac{1}{n} - \frac{1}{N}\right)\left(\frac{5}{4}\tau_k^{*}(\lambda_{04} - 1) - \frac{8}{3}(\lambda_{22} - 1)\right) + \frac{64}{9}\left(\frac{1}{(nq+2p)} - \frac{1}{N}\right)(\lambda_{40} - 1)}$$

and findings are shown by Table 4 and Table 5. It is observed that the generalized exponential ratio-type estimator T_k, (k = 1,2,...,6) which are members of proposed exponential ratio-type estimator under scheme-I performed better than the $T_k^{'}$, (k = 1,2,...,6) which are members of proposed exponential ratio-type estimator under scheme-II, for all α ∈ (0.0,1.0).

Table 4:- *PRE*s of estimator T_k, (k = 1,2,...,6) with respect to s_y^{*2}.

Percent relative efficiency (PRE)						
α	(T_1, s_y^{*2})	(T_2, s_y^{*2})	(T_3, s_y^{*2})	(T_4, s_y^{*2})	(T_5, s_y^{*2})	(T_6, s_y^{*2})
0.0	1.335	1.335	1.335	1.335	1.335	1.335
0.1	1.335	1.345	1.335	1.396	1.347	1.353
0.2	1.336	1.354	1.335	1.431	1.358	1.369
0.3	1.336	1.363	1.335	1.452	1.368	1.382
0.4	1.337	1.371	1.335	1.466	1.377	1.394
0.5	1.337	1.379	1.335	1.476	1.386	1.405
0.6	1.338	1.385	1.335	1.482	1.393	1.414
0.7	1.338	1.392	1.335	1.487	1.400	1.422
0.8	1.339	1.398	1.335	1.491	1.407	1.429
0.9	1.339	1.403	1.335	1.493	1.413	1.435
1.0	1.340	1.409	1.335	1.495	1.418	1.441

Table 5:- *PRE*s of estimator $T_k^{'}$, (k = 1,2,...,6) with respect to s_y^{*2}.

Percent relative efficiency (PRE)						
α	$(T_1^{'}, s_y^{*2})$	$(T_2^{'}, s_y^{*2})$	$(T_3^{'}, s_y^{*2})$	$(T_4^{'}, s_y^{*2})$	$(T_5^{'}, s_y^{*2})$	$(T_6^{'}, s_y^{*2})$
0.0	1.526	1.526	1.526	1.526	1.526	1.526
0.1	1.526	1.535	1.526	1.579	1.537	1.542
0.2	1.527	1.543	1.526	1.610	1.546	1.556
0.3	1.527	1.551	1.526	1.629	1.555	1.567
0.4	1.528	1.558	1.526	1.641	1.563	1.578
0.5	1.528	1.564	1.526	1.650	1.570	1.587
0.6	1.529	1.570	1.526	1.656	1.577	1.595
0.7	1.529	1.576	1.526	1.661	1.583	1.602
0.8	1.530	1.581	1.526	1.664	1.589	1.608
0.9	1.530	1.586	1.526	1.667	1.594	1.614
1.0	1.530	1.590	1.526	1.669	1.598	1.619

The estimators $T_4^{'}$ and T_4 which utilize the information on ($\beta_2(x)$, Q_r) are the best in the terms of having largest percent relative efficiency among all the estimators discussed here for α = 1.

Conclusions:-

Generalized exponential ratio-type estimators of population variance S_y^2 of the study variable y using information on certain known parameters of the auxiliary variable under non-response has been developed by modifying the estimators of Solanki *et al*. (2015) under scheme-I and scheme-II. Properties of the proposed generalized exponential ratio-type estimators under non response were also obtained under studied schemes. Conditions were developed when family of estimators T_k and $T_k^{'}$ performed better than the usual unbiased ratio-type estimator. The performance of the family of estimators T_k and $T_k^{'}$ estimators were also studied under numerical illustration. It was found that T_4 and $T_4^{'}$ estimators are better than the other usual existing estimator for different value of α.

References:-

1. Cochran, W. G. (1940). Some properties of estimators based on sampling scheme with varying probabilities. Australian Journal of Statistics, 17: 22-28.
2. Hansen, M. H. and Hurwitz, W. N. (1946). The problem of non-response in sample surveys. Journal of the American Statistical Association, 41: 517-529.

3. Isaki, C. T. (1983). Variance estimation using auxiliary information. Journal of the American Statistical Association, 78: 117-123.
4. Irfan, M., Javed, M. and Lin, Z. (2018). Optimized estimation for population mean using conventional and non-conventional measures under the joint influence of measurement error and non-response. J Stat Comput Simul., 88(12): 2385-2403.
5. Kadilar, C. and Cingi, H. (2006). Improvement in variance estimation using auxiliary information. Hacettepe Journal of Mathematics and Statistics, 35(1): 111-115.
6. Kumar, S., Trehan, M. and Joorel, J. P. S. (2018). A simulation study: estimation of population mean using two auxiliary variables in stratified random sampling. J Stat Comput Simul., 88(18): 3694-3707.
7. Mohamed, C., Sedory, S. A. and Singh, S. (2018). A fresh imputing survey methodology using sensible constraints on study and auxiliary variables: dubious random non-response. J Stat Comput Simul., 88(7): 1273-1294.
8. Singh, H. P., Singh, A. K. and Solanki, R. S. (2014). Estimation of finite population variance using auxiliary information in sample surveys. Statistica, 74: 95-112.
9. Singh, S. (2003). Advanced Sampling Theory with Applications: How Michael 'selected' Amy, Kluwer Academic Publisher, The Netherlands.
10. Singh, S. and Joarder, A. H. (1998). Estimation of finite population variance using random non-response in survey sampling. Metrika, 47: 241-249.
11. Solanki, R. S., Singh, H. P. and Pal, S. K. (2015). Improved ratio-type estimators of finite population variance using quartiles. Hacettepe Journal of Mathematics and Statistics, 44(3): 747-754.