Journal Homepage: - www.journalijar.com

INTERNATIONAL JOURNAL OF ADVANCED RESEARCH (IJAR)

ISSN NO. 2320-5407

Article DOI: 10.21474/IJAR01/13126
DOI URL: http://dx.doi.org/10.21474/IJAR01/13126

RESEARCH ARTICLE

STABILITY OF NON-LINEAR DYNAMICAL SYSTEM

Dr. Anas Salim Youns
College of Basic Education University of Mosul/ Mosul/ Iraq.

Manuscript Info

Manuscript History
Received: 15 July 2021
Final Accepted: 17 July 2021
Published: July 2021

Key words:-
Linearization, Fixed Point, Jacobian Matrix, Dynamical Systems

Abstract

The mainobjective of this research is to study the stability of thenon-linear"dynamical system" by using the "linearization" technique of three dimension systems toobtain an approximate linear system and find it's stability. We apply this technique to reaches to the stability of the public non linear dynamical systems of dimension. Finally, some proposed examples (example (1) and example (2)) are given to explain this technique and used the corollary.

Introduction:-

We shall study the "stability" of thenon-linear"dynamical system" by using the technique of "linearization" to transform the non-linear "dynamical system" to be aan approximate "linear dynamical system" and by using the theorem or the corollary for linear systems we find the "stability conditions" of the systems. In [1] Arrowsmith and Place are gave the method of linearizationfor(non-linear)"dynamical system"in two dimensions, we develop this method for the system of three dimensionsand finally we reached to the public non-linear "dynamical system"for n dimension with examples to clarify this technique. The "linearization method"ofa "dynamical system" needs first to find the non-zero "fixed points" that satisfies the non-linear "dynamical system", second wefind the "Jacobian matrix"and substitute the "fixed points" in the "Jacobian matrix" [2],[3]third wefind the eigenvalues of a "linear system" and apply the stability conditions to know the stability forthe non-linear "dynamical systems"and finally,wegave examples to explain this technique.

Thefundamentalsconceptsfordynamical Systems

Definition2.1: The linear "homogeneous dynamical system" with steady coefficients of n dimensional satisfies the equations that:

$$\dot{X}_1 = \frac{dX_1}{dt} = a_{11}x_1 + ... + a_{1n}x_n$$

.

.

.

$$\dot{X}_n = \frac{dX_n}{dt} = a_{n1}x_1 + ... + a_{nn}x_n$$, which is denoted by $\dot{X} = A.X$

Corresponding Author:- Dr. Anas Salim Younis
Address**:-** College of Basic Education University of Mosul/ Mosul/Iraq.

Where , $A = \begin{bmatrix} a_{11} & . & . & . & a_{1n} \\ . & . & . & . & . \\ . & . & . & . & . \\ . & . & . & . & . \\ a_{n1} & . & . & . & a_{nn} \end{bmatrix}$, $X = \begin{bmatrix} x_1 \\ . \\ . \\ . \\ x_n \end{bmatrix}$ are the steady coefficients matrix for the"linear system", where $a_{ij} \in R; \forall i, j = 1,..., n$ are a real constant, and X is vertical vector [3],[7],[8],[9].

Definition2.2: The "Jacobian matrix"fordefinition (2.1) is denoted by $J(x_1,..., x_n)$ that is defined asthe nxn matrix, consisting of all the first-order partial derivatives forthe functions $[X_1,..., X_i,..., X_n]^T$, with respect to the dependent variable $[x_1,..., x_j,..., x_n]^T$.

That is meaning that:

$J_{ij} = \left[\frac{\partial X_i}{\partial x_j} \right]; \forall i, j = 1,..., n$, or equivalently we have that

$$J(x_1,..., x_n) = \begin{bmatrix} \frac{\partial X_1}{\partial x_1} & . & . & . & . & \frac{\partial X_1}{\partial x_n} \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ \frac{\partial X_n}{\partial x_1} & . & . & . & . & \frac{\partial X_n}{\partial x_n} \end{bmatrix}_{(x_1,......,x_n)}$$ [1],[2],[3],[4],[10].

Theorem: The stability forthe linear systems [4],[5],[6]

Let λ_1 and λ_2 be the eigenvalues forsteady coefficients matrix A of definition (2.1) in two dimensional linear system that

$$\dot{X}_1 = \frac{dX_1}{dt} = ax_1 + bx_2$$

$$\dot{X}_2 = \frac{dX_2}{dt} = cx_1 + dx_2$$

Where, $ad - bc \neq 0$, then the trivial critical point (0,0) satisfies that:

1- Asymptotically stable while the real portion of the eigenvalues λ_1 and λ_2 are both negative.
2- Stable but however not asymptotically stable when the real portionof the eigenvalues λ_1 and λ_2 are both zero.
3- Unstable if either the eigenvalues λ_1 or λ_2 are both positive.

From the theorem we can get immediately the next corollary.

Corollary: Stability for the public case of (linear) systems [5],[6]

If $\lambda_1, \lambda_2, \dots, \lambda_n$ are the eigenvalues for the steady coefficients matrix A of n dimensional linear system such that: $\dot{X} = A.X; |A| \neq 0$.

Then, the trivial critical point (0,0) satisfies that:

1- Asymptotically stable when the real portion of the eigenvalues $\lambda_1, \lambda_2, \dots, \lambda_n$ are both negative;

2- Stable but however not asymptotically stable when the real portion of the eigenvalues $\lambda_1, \lambda_2, \ldots, \lambda_n$ are both zero and the critical point is center.
3- Unstable if either any eigenvalues $\lambda_1, \lambda_2, \ldots, \lambda_n$ are positive.

Results:-

(Derivationofthelinearization Near Fixedpoint)

The linearization near fixed point of three dimension system

The dynamical system(non-linear)with a "fixed point" in three dimension that:

$$\dot{X} = X(\bar{x}), \bar{x} = (x_1, x_2, x_3) \in R^3$$

can written as the form:

$$\dot{X}_1 = a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + g_1(x_1, x_2, x_3)$$
$$\dot{X}_2 = a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + g_2(x_1, x_2, x_3) \quad (1)$$
$$\dot{X}_3 = a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + g_3(x_1, x_2, x_3)$$

Where, $g_i(x_1, x_2, x_3); \forall i = 1,2,3$ are any non-linear functions and we have that: $[g_i(x_1, x_2, x_3)/r] \to 0; \forall i = 1,2,3$, as $r = \sqrt{x_1^2 + x_2^2 + x_3^2} \to 0$.

Since $Lim_{r\to 0}[g_i(x_1, x_2, x_3)/r] = 0; \forall i = 1,2,3$

Then the "linear system" of equations (1) is that:

$$\dot{X}_1 = a_{11}x_1 + a_{12}x_2 + a_{13}x_3$$
$$\dot{X}_2 = a_{21}x_1 + a_{22}x_2 + a_{23}x_3 \quad (2)$$
$$\dot{X}_3 = a_{31}x_1 + a_{32}x_2 + a_{33}x_3$$

The equations system (2) is called the "linearization"for the equations system (1).

Let (ζ, η, γ) be fixed point for the(non-linear) dynamical system that satisfy $\dot{X} = X(x) = 0$, define the other's variables such that:

$$y_1 = x_1 - \zeta \to x_1 = y_1 + \zeta$$
$$y_2 = x_2 - \eta \to x_2 = y_2 + \eta \quad (3)$$
$$y_3 = x_3 - \gamma \to x_3 = y_3 + \gamma$$

Then substitutethenew variables of the equations system (3) in theequations system (1) such that:

$$\dot{y}_i = \dot{x}_i = X_i(x_1, x_2, x_3) = X_i(y_1 + \zeta, y_2 + \eta, y_3 + \gamma); i = 1,2,3 \quad (4)$$

Where, X_1, X_2, X_3 are component functions of the function X .

Let we define that $Y_i(y_1, y_2, y_3) = X_i(x_1, x_2, x_3) = X_i(y_1 + \zeta, y_2 + \eta, y_3 + \gamma); i = 1,2,3$ (5)

Finally, $\dot{y}_i = Y_i(y_1, y_2, y_3), i = 1,2,3$ or $\dot{y} = Y(\bar{y})$ (6)

If the component functions $X_i(x_1, x_2, x_3); i = 1,2,3$ are continuous differentiable in some of the neighborhood ofthe fixed point (ζ, η, γ) then we have that:

$$X_i(x_1, x_2, x_3) = X_i(\zeta, \eta, \gamma) + (x_1 - \zeta)\frac{\partial X_i}{\partial x_1}(\zeta, \eta, \gamma) + (x_2 - \eta)\frac{\partial X_i}{\partial x_2}(\zeta, \eta, \gamma)$$
$$+ (x_3 - \gamma)\frac{\partial X_i}{\partial x_3}(\zeta, \eta, \gamma) + R_i(x_1, x_2, x_3) \quad (7)$$

Since (ζ, η, γ) is a "fixed point" for non-linear "dynamical system" which satisfy $\dot{X} = X(\bar{x}) = O$, then $X_i(\zeta, \eta, \gamma) = 0; i = 1,2,3$.

Byusing the equations (3)that: $y_1 = x_1 - \zeta$, $y_2 = x_2 - \eta$, $y_3 = x_3 - \gamma$.

Also, the remainder functions $R_i(x_1, x_2, x_3); \forall i = 1,2,3$ are satisfies that:

$$Lim_{r \to 0}\{R_i(x_1, x_2, x_3)/r\} = 0; i = 1,2,3 \quad (8)$$

Where, $r = \sqrt{(x_1 - \zeta)^2 + (x_2 - \eta)^2 + (x_3 - \gamma)^2}$.

Therefore the equations system (7) becomes as that:

$$\dot{y}_1 = y_1\frac{\partial X_1}{\partial x_1}(\zeta, \eta, \gamma) + y_2\frac{\partial X_1}{\partial x_2}(\zeta, \eta, \gamma) + y_3\frac{\partial X_1}{\partial x_3}(\zeta, \eta, \gamma)$$
$$\dot{y}_2 = y_1\frac{\partial X_2}{\partial x_1}(\zeta, \eta, \gamma) + y_2\frac{\partial X_2}{\partial x_2}(\zeta, \eta, \gamma) + y_3\frac{\partial X_2}{\partial x_3}(\zeta, \eta, \gamma) \quad (9)$$
$$\dot{y}_3 = y_1\frac{\partial X_3}{\partial x_1}(\zeta, \eta, \gamma) + y_2\frac{\partial X_3}{\partial x_2}(\zeta, \eta, \gamma) + y_3\frac{\partial X_3}{\partial x_3}(\zeta, \eta, \gamma)$$

Since

$$a_{11} = \frac{\partial X_1}{\partial x_1}(\zeta, \eta, \gamma), a_{12} = \frac{\partial X_1}{\partial x_2}(\zeta, \eta, \gamma), a_{13} = \frac{\partial X_1}{\partial x_3}(\zeta, \eta, \gamma)$$
$$a_{21} = \frac{\partial X_2}{\partial x_1}(\zeta, \eta, \gamma), a_{22} = \frac{\partial X_2}{\partial x_2}(\zeta, \eta, \gamma), a_{23} = \frac{\partial X_2}{\partial x_3}(\zeta, \eta, \gamma) \quad (10)$$
$$a_{31} = \frac{\partial X_3}{\partial x_1}(\zeta, \eta, \gamma), a_{32} = \frac{\partial X_3}{\partial x_2}(\zeta, \eta, \gamma), a_{33} = \frac{\partial X_3}{\partial x_3}(\zeta, \eta, \gamma)$$

Therefore the dynamical system is (linear) such that:

$$\dot{y}_1 = a_{11}y_1 + a_{12}y_2 + a_{13}y_3$$
$$\dot{y}_2 = a_{21}y_1 + a_{22}y_2 + a_{23}y_3 \quad (11)$$
$$\dot{y}_3 = a_{31}y_1 + a_{32}y_2 + a_{33}y_3$$

Or, equation (11) is written as the form

$$\dot{Y} = A\bar{Y} \text{, where } A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} \frac{\partial X_1}{\partial x_1} & \frac{\partial X_1}{\partial x_2} & \frac{\partial X_1}{\partial x_3} \\ \frac{\partial X_2}{\partial x_1} & \frac{\partial X_2}{\partial x_2} & \frac{\partial X_2}{\partial x_3} \\ \frac{\partial X_3}{\partial x_1} & \frac{\partial X_3}{\partial x_2} & \frac{\partial X_3}{\partial x_3} \end{bmatrix}_{(\zeta,\eta,\gamma)} \quad (12)$$

The linearization near fixed point of n dimension system

Let we have a (non-linear)dynamical system with fixed point in dimension that: $\dot{X} = X(\bar{x}), \bar{x} = (x_1,..., x_n) \in R^n$. can written as the form:

$$\dot{X}_1 = a_{11}x_1 + ... + a_{1n}x_n + g_1(x_1,..., x_n)$$

.

.(13)

.

$$\dot{X}_n = a_{n1}x_1 + ... + a_{nn}x_n + g_n(x_1,..., x_n)$$

Where, $g_i(x_1,..., x_n); \forall i = 1,..., n$ are any non-linear functions and we have $[g_i(x_1,..., x_n)/r] \to 0$ as $r = \sqrt{x_1^2 + ... + x_n^2} \to 0$.

Then the "linear dynamical system" of the equations (13) is that:

$$\dot{X}_1 = a_{11}x_1 + ... + a_{1n}x_n$$

.

.(14)

.

$$\dot{X}_n = a_{n1}x_1 + ... + a_{nn}x_n$$

The equations system (14) is called the linearization for (non-linear) dynamical system(13), let $(\zeta_1,..., \zeta_n)$ is fixed point for(non-linear) dynamical system of the equations system (13) which satisfy that: $\dot{X} = X(\bar{x}) = 0$, define the new other's variables such that:

$$y_1 = x_1 - \zeta_1 \to x_1 = y_1 + \zeta_1$$

.

. (15)

.

$$y_n = x_n - \zeta_n \to x_n = y_n + \zeta_n$$

Then, substitute above newvariablesof the equations (15)in the equations system (13) such that:

$$\dot{y}_i = \dot{x}_i = X_i(x_1,..., x_n) = X_i(y_1 + \zeta_1,..., y_n + \zeta_n); i = 1,..., n \quad (16)$$

Where that $X_1,..., X_n$ are component functions of the function X .

Let us define that $Y_i(y_1,..., y_n) = X_i(x_1,..., x_n) = X_i(y_1 + \zeta_1,..., y_n + \zeta_n); i = 1,..., n$ (17)

Finally, $\dot{y}_i = Y_i(y_1,..., y_n), i = 1,..., n$, or $\dot{y} = Y(\bar{y})$ (18)

If the component functions $X_i(x_1,..., x_n); i=1,..., n$ are continuous differentiable in some of the neighborhood ofthe fixed point $(\zeta_1,..., \zeta_n)$ then we have that:

$$X_i(x_1,..., x_n) = X_i(\zeta_1,..., \zeta_n) + (x_1-\zeta_1)\frac{\partial X_i}{\partial x_1}(\zeta_1,..., \zeta_n) + (x_2-\zeta_2)\frac{\partial X_i}{\partial x_2}(\zeta_1,..., \zeta_n) +$$

$$... + (x_n-\zeta_n)\frac{\partial X_i}{\partial x_3}(\zeta_1,..., \zeta_n) + R_i(x_1,..., x_n) \quad (19)$$

Since, $(\zeta_1,..., \zeta_n)$ is fixed point for the (non-linear) dynamical system which satisfies that $X_i(\zeta_1,..., \zeta_n) = 0; \forall i = 1,..., n$.

Byusing the equations (15)that: $y_1 = x_1 - \zeta_1$,…, $y_n = x_n - \zeta_n$.

Also, $R_i(x_1,..., x_n)$ are the remainder functions that satisfies $Lim_{r\to 0}\{R_i(x_1,..., x_n)/r\} = 0$ (20)

Where, $r = \sqrt{(x_1-\zeta_1)^2 + ... + (x_n-\zeta_n)^2}$.

Therefore, the equations system (19) becomes as the form:

$$\dot{y}_1 = y_1\frac{\partial X_1}{\partial x_1}(\zeta_1,..., \zeta_n) + ... + y_n\frac{\partial X_1}{\partial x_n}(\zeta_1,..., \zeta_n)$$

.(21)

.

$$\dot{y}_n = y_1\frac{\partial X_n}{\partial x_1}(\zeta_1,..., \zeta_n) + ... + y_n\frac{\partial X_n}{\partial x_n}(\zeta_1,..., \zeta_n)$$

Since

$$a_{11} = \frac{\partial X_1}{\partial x_1}(\zeta_1,..., \zeta_n),..., a_{1n} = \frac{\partial X_1}{\partial x_n}(\zeta_1,..., \zeta_n)$$

.

.(22)

$$a_{n1} = \frac{\partial X_n}{\partial x_1}(\zeta_1,..., \zeta_n),..., a_{nn} = \frac{\partial X_n}{\partial x_n}(\zeta_1,..., \zeta_n)$$

Then a dynamical system is to be(linear) such that:

$$\dot{y}_1 = a_{11}y_1 + ... + a_{1n}y_n$$

.

. (23)

.

$$\dot{y}_n = a_{n1}y_1 + ... + a_{nn}y_n$$

Or, equation (23) is written as the form

$$\dot{Y} = A.\bar{Y}, \text{ where } A = \begin{bmatrix} a_{11} & \dots & a_{1n} \\ . & & \\ . & . & . \\ . & & \\ a_{n1} & \dots & a_{nn} \end{bmatrix} = \begin{bmatrix} \frac{\partial X_1}{\partial x_1} & \dots & \frac{\partial X_1}{\partial x_n} \\ . & & \\ . & . & . \\ . & & \\ \frac{\partial X_n}{\partial x_1} & \dots & \frac{\partial X_n}{\partial x_n} \end{bmatrix}_{(\zeta_1,\dots\zeta_n)} \quad (24)$$

Examples

In this paragraph, we gave two examples to expound how to obtain the "fixed points" for the (non-linear) suggest "dynamical system"by using the linearization technique in order to get(linear) system andfoundits stability.

Example(1):

The (non-linear) suggest dynamical system in three dimension as below

$$\dot{X}_1 = X_1(x_1, x_2, x_3) = e^{x_1+x_2} - x_2$$

$$\dot{X}_2 = X_2(x_1, x_2, x_3) = -x_1 + x_1 x_2$$

$$\dot{X}_3 = X_3(x_1, x_2, x_3) = x_3 - x_2$$

Then, by solving above non-linear dynamicalsystem, we get that only one fixed point that is $(\zeta, \eta, \gamma) = (-1,1,1)$.

For finding the linearized for the(non-linear) system nearlya fixed point, we need aJacobian matrixnear fixed point $(\zeta, \eta, \gamma) = (-1,1,1)$ that is:

$$J(x_1, x_2, x_3) = \begin{bmatrix} e^{x_1+x_2} & e^{x_1+x_2} - 1 & 0 \\ x_2 - 1 & x_1 & 0 \\ 0 & -1 & 1 \end{bmatrix}_{(-1,1,1)} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & -1 & 1 \end{bmatrix} = A, |A| = -1 \neq 0$$

Therefore, the linearization near fixed point $(-1,1,1)$ is that: $\dot{Y} = A.\bar{Y} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$, Or, we have that a (linear) system such as:

$$\dot{y}_1 = y_1$$

$$\dot{y}_2 = -y_2$$

$$\dot{y}_3 = -y_2 + y_3$$

For the stability,the eigenvalues $\lambda_1 = 1$, $\lambda_2 = 1$, $\lambda_3 = -1$, of matrix A.

Then the fixed point $(-1,1,1)$ of thesuggested(non-linear) system is unstable by using the (corollary). We also used matlab program (program (1)) for this example to achieve the numerical result.

Example(2):

Considerthe suggestdynamical system that is non-linear in three dimension such that:

$$\dot{X}_1 = X_1(x_1, x_2, x_3) = -x_1^2 + x_3$$

$$\overset{\bullet}{X}_2 = X_2(x_1, x_2, x_3) = -x_2^2 + x_3$$

$$\overset{\bullet}{X}_3 = X_3(x_1, x_2, x_3) = x_1 - x_3^3$$

Then, by solving above system, we get that two real fixed points that are $(\zeta_1, \eta_1, \gamma_1) = (1,1,1)$, $(\zeta_2, \eta_2, \gamma_2) = (1,-1,1)$.

For finding the linearized of thesuggesteddynamical system near fixed point, we need the "Jacobian matrix"near fixed points:

$$J(x_1, x_2, x_3) = \begin{bmatrix} -2x_1 & 0 & 1 \\ 0 & -2x_2 & 1 \\ 1 & 0 & -3x_3^2 \end{bmatrix}$$

The Jacobian matrix near the fixed point $(\zeta_1, \eta_1, \gamma_1) = (1,1,1)$ is that:

$$J(x_1, x_2, x_3) = \begin{bmatrix} -2x_1 & 0 & 1 \\ 0 & -2x_2 & 1 \\ 1 & 0 & -3x_3^2 \end{bmatrix}_{(1,1,1)} = \begin{bmatrix} -2 & 0 & 1 \\ 0 & -2 & 1 \\ 1 & 0 & -3 \end{bmatrix} = A \text{, and}$$

$|A| = \begin{vmatrix} -2 & 0 & 1 \\ 0 & -2 & 1 \\ 1 & 0 & -3 \end{vmatrix} = -10 \neq 0$, Therefore the linearization near $(\zeta_1, \eta_1, \gamma_1) = (1,1,1)$ is that:

$\overset{\bullet}{Y} = A.\overline{Y} = \begin{bmatrix} -2 & 0 & 1 \\ 0 & -2 & 1 \\ 1 & 0 & -3 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$, Or we have that a (linear) system such as:

$$\overset{\bullet}{y}_1 = -2y_1 + y_3$$

$$\overset{\bullet}{y}_2 = -2y_2 + y_3$$

$$\overset{\bullet}{y}_3 = y_1 - 3y_3$$

For the stability all the eigenvalues for the above matrix A are that $\lambda_1 =$ -2.0000 , $\lambda_2 =$ -1.3820 , $\lambda_3 =$ -3.6180.

Then, by the (corollary) the fixed point $(\zeta_1, \eta_1, \gamma_1) = (1,1,1)$ of the system is to beasymptotically stable.

The Jacobian matrixnear fixed point $(\zeta_2, \eta_2, \gamma_2) = (1,-1,1)$ is that:

$J(x_1, x_2, x_3) = \begin{bmatrix} -2x_1 & 0 & 1 \\ 0 & -2x_2 & 1 \\ 1 & 0 & -3x_3^2 \end{bmatrix}_{(1,-1,1)} = \begin{bmatrix} -2 & 0 & 1 \\ 0 & 2 & 1 \\ 1 & 0 & -3 \end{bmatrix} = A$, and we have $|A| = \begin{vmatrix} -2 & 0 & 1 \\ 0 & 2 & 1 \\ 1 & 0 & -3 \end{vmatrix} = 10 \neq 0$,

Therefore, the linearization near fixed point $(\zeta_2, \eta_2, \gamma_2) = (1,-1,1)$ is that: $\overset{\bullet}{Y} = A.\overline{Y} = \begin{bmatrix} -2 & 0 & 1 \\ 0 & 2 & 1 \\ 1 & 0 & -3 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$,

Or, we have that a (linear)
system such as:

$$\dot{y}_1 = -2y_1 + y_3$$

$$\dot{y}_2 = 2y_2 + y_3$$

$$\dot{y}_3 = y_1 - 3y_3$$

For stability the eigenvalues $\lambda_1 = 2.0000$, $\lambda_2 = -1.3820$, $\lambda_3 = -3.6180$formatrix A.

Therefore, $(\zeta_2, \eta_2, \gamma_2) = (1,-1,1)$ is unstable fixed pointby the (corollary).
We used matlab program (program (2)) for this example to achieve the numerical result.

Conclusion:-

1- We can find the stability for(non-linear)dynamical system of three and n dimensions by using the linearization technique.
2- The stability conditions of (linear)system that getting from the (non-linear) dynamical system depends on the (theorem) or the (corollary), (the all values of the eigenvalues for the matrix A).
3-We apply the technique of linearization to find the stability for(non-linear)dynamical system in three dimensionin two examples.

References:-

[1] Arrowsmith, D.K., Place, C.M.,“Ordinary Differential Equations” , Chapman and Hall mathematics series, London, Great Britain (1982).
[2] Boelkins, M.R., Goldberg, J.L., Potter, M.C., “Differential Equation with Linear Algebra”, Oxford University Press, Inc., 198 Madison Avenue, New York, New York 10016, U.S.A. (2009).
[3] Boyce, W. E., Diprima, R.C.,“Elementary differential equations and boundary value problems”, ninth edition. John Wiley and Sons Int., U.S.A. (2009).
[4] Edwards, C.H., Penney, D.E., Calvis, D., “Elementary Differential Equation”, Person Education, Int., New Jersey, U.S.A. (2008).
[5] Jeffrey, A., “Advanced Engineering Mathematics”, Harcourt Academic Press, A Harcourt Science and Technology Company,200 Wheeler Road, Burlington, Massachusetts 01803 ,U.S.A. (2002).
[6]Jordan D.W., Smith P. ,“Nonlinear Ordinary Differential Equations: Problems and Solutions A Sourcebook for Scientists and Engineers”, fourth edition. Oxford University Press Inc., New York, U.S.A. (2007).
[7] Ladas G., Finizio, N., “An Introduction to Differential Equations with Difference Equations, Fourier Series and Partial Differential Equations”, University of Rhode island, Wadsworth, Inc., Publishing Company Belmont, California, U.S.A. (1982).
[8] Matousek, R., Svarc,, I., Pivoňka, P., Osmera, P., Seda, M.,“Simple Methods for Stability Analysis of Nonlinear Control Systems”,Proceedings of the World Congress on Engineering and Computer Science, Vol II,WCECS, October 20-22, San Francisco, U.S.A. (2009).
[9] Nagle, R.K., Saff, E.B., Snider, A.D.,“Fundamentals Differential Equation”, fifth editionAddison Wesley Longman ,U.S.A. (2000).
[10] Schiesser, W.E., Lee, H.J.,“Ordinary and Partial DifferentialEquation Routines in C, C++,Fortran, Java®, Maple®, and MATLAB®”, Chapman & Hall/CRC, New York, U.S.A. (2004).