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### RESEARCH ARTICLE

#### EXPLORATION WITH RETURN OF HIGHLY DYNAMIC NETWORKS

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#### Abstract

In this paper, we study the necessary and sufficient time to explore with return constantly connected dynamic networks modelled by a dynamic graphs. Exploration with return consists, for an agent operating in a dynamic graph, of visiting all the vertices of the graph and returning to the starting vertex. We show that for constantly connected dynamic graphs based on a ring of size  $n, 3n-4$  time units are necessary and sufficient to explore it. Assuming that the agent knows the dynamics of the graph.

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#### Introduction:-

This paper studies the problem of exploration with return of constantly connected dynamic graphs based on a ring assuming that the agent knows the dynamics of the graph. The problem of exploration is fundamental in distributed computing by mobile agents. It has been extensively studied in static graphs since the seminal paper by Claude Shannon [13]. Since a decade or so, researchers began modeling distributed computing systems by dynamic graphs. This is motivated by the fact that they become more and more dynamic partly due to the very important increase of the number of communicating objects. Many more or less equivalent models of dynamic graphs have been developed taking into account the extreme dynamism of some communication networks. Among these models, we have the constantly connected dynamic graphs model. A dynamic graph  $\mathcal{G} = (G_1, G_2, \dots, G_n)$  is called **constantly connected** if each graph  $G_i$  which composes it is connected. This class of graphs was used in [12] to study the problem of information dissemination. In 2010, Kuhn, Lynch and Oshman [11] generalize this class of dynamic graphs by introducing the notion of  $T$ -interval-connectivity. Roughly speaking, given an integer  $T \geq 1$ , a dynamic graph is  $T$ -interval-connected if for any window of  $T$  time units, there is a connected spanning subgraph that is stable throughout the period. (The notion of constant connectivity is equivalent to the notion of  $1$ -interval-connectivity.) This new concept, which captures the connection stability over time, allows to derive interesting results: the  $T$ -interval-connectivity allows a savings of a factor about  $\theta(T)$  on the number of messages necessary and sufficient to achieve a complete exchange of information between all vertices [6, 11].

During these last few years, several studies consider constantly connected dynamic graphs where the underlying graph of the dynamic graph is a ring of  $n$  vertices. The problem of exploration with termination by a mobile agent is considered in [4, 8, 9]. If the dynamics of the graph is known, [9] shows that a single agent can solve the problem, and  $2n - 3$  time units are necessary and sufficient. If the dynamics is not known in advance, [4] shows that two agents knowing an upper bound  $N$  on the number of vertices can solve the problem, and  $3N - 6$  time units are sufficient if all agents are active at each time step, and  $O(N^2)$  moves are sufficient if a subset of the agents might be active at each time step. The case when the agent has partial information about network graphs is considered in [8].

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More precisely, the authors study the exploration time for a single agent which knows the dynamics of the graph for the next  $S$  steps in its  $H$ -hop neighborhood, for given parameters  $S$  and  $H$ .

The problem of perpetual exploration is considered in [2, 7]. In [2], the authors consider that all agents are active at each time step and show that to solve the problem, one agent is sufficient in the rings of size two, two agents are sufficient in the rings of size three, and three agents are sufficient for all other rings. The authors define a ring of size two as a two-node path if the graph is simple, or as two nodes linked by two bidirectional edges otherwise. In [7] the authors consider time varying graphs whose topology is arbitrary and unknown to the agents and investigate the number of agents that are necessary and sufficient to explore such graphs. In addition to the problem of exploration, the problem of dispersion of a team of agents [1], gathering [5] and patrolling by a team of agents [3] are studied, considering constantly connected dynamic graphs based on the ring.

### Our results.

We consider the problem of exploration with return of constantly connected dynamic graphs the case where the underlying graph is a ring of  $n$  vertices. We show that to explore that dynamic graphs,  $3n - 4$  time units are necessary and sufficient, where  $n$  is the size of the ring.

### Definitions and model

This section provides precise definitions of the concepts and models informally mentioned above. We also give some previous results from the literature on the problem studied. The proofs of the theorems mentioned in this section are given in [14].

#### Definition 1. (Dynamic graph)

A dynamic graph is a pair  $\mathcal{G} = (V, \mathcal{E})$ , where  $V$  is a set of  $n$  static vertices, and  $\mathcal{E}$  is a function which maps every integer  $i \geq 1$  to a set  $\mathcal{E}(i)$  of undirected edges on  $V$ .

#### Definition 2. (Underlying graph)

Given a dynamic graph  $\mathcal{G} = (V, \mathcal{E})$ , the static graph  $G = (V, \cup \mathcal{E}(i))$  is called the underlying graph of  $\mathcal{G}$ . Conversely, the dynamic graph  $\mathcal{G}$  is said to be based on the static graph  $G$ .

#### Definition 3. (Constant connectivity)

A dynamic graph  $\mathcal{G}$  is said to be constantly connected if, for any integer  $i$ , the static graph  $G_i = (V, \mathcal{E}(i))$  is connected.

We give in the following definitions the different types of exploration of graphs by a mobile agent that exists, namely perpetual exploration, periodic exploration, exploration with stop and exploration with return.

#### Definition 4. (Perpetual exploration)

The agent must visit each vertex of the graph at least once. It does not have to stop after visiting all the vertices of the graph (it may not know if the graph is anonymous). The exploration time is the number of steps between the start of the exploration and the first moment when all the vertices have been visited.

#### Definition 5. (Periodic exploration)

In a periodic exploration, the agent must visit all the vertices of the graph periodically. In this type of exploration too, the agent does not have to stop after visiting all the vertices of the graph.

#### Definition 6. (Exploration with stop)

In this type of exploration, the agent must visit each vertex of the graph at least once. It must then stop once it has finished visiting all the vertices (not necessarily immediately after the last unknown vertex has been visited).

#### Definition 7. (Exploration with return)

The exploration with return is an exploration with stop with an additional constraint: the mobile entity (agent) must stop at its starting position.

In this paper, we study the time complexity of the exploration with return of constantly connected dynamic graphs based on a ring of size  $n$ . A mobile entity, called agent, operates on these dynamic graphs. The agent can traverse at most one edge per time unit. We say that an agent explores with return the dynamic graph if and only if he visits all its vertices and returns to his starting vertex. We also assume that the agent knows the dynamics of the graph, that is to say, the times of appearance and disappearance of the edges of the dynamic graph.

In this article, we will use the following results from the literature.

#### Theorem 1. (From [12])

For any integer  $n \geq 3$  and for any constantly connected dynamic graph based on a ring with  $n$  vertices, there exists an agent (algorithm), **Explore-ring**, exploring this dynamic graph in time at most  $2n - 3$  (the agent knows the dynamics of the graph).

**Sketch of proof.**

Consider  $n$  virtual agents placed on the  $n$  vertices (one agent on each vertex). Make all agents move in the clockwise direction for  $n - 1$  time units from time  $t$ . Since at most one edge is removed at a time, it holds that, at each time, at most one such virtual agent is blocked at this time without having been blocked before. Thus, one of the  $n$  virtual agents is never blocked during then  $- 1$ time units, and the starting vertex of this agent is the vertex  $v(t)$  we are looking for. This can be done in at most  $n - 1$ time units.  $\diamond$

**Theorem 2. [15]**

For any constantly connected dynamic graph on  $n$  vertices, at most  $n - 1$ time units are sufficient for an agent to go from any vertex to any other vertex in the graph, when the agent knows the dynamics of the graph.

**Sketch of proof.**

Let  $u$  be some arbitrary vertex of the dynamic graph. For any integer  $i \geq 0$ , let  $V_i$  be the set of vertices reachable from  $u$  in at most  $i$  time units. We have that  $V_i \subsetneq V_{i+1}$  until  $V_i$  contains all the vertices. Indeed, before all vertices are reachable, there exists a vertex not in  $V_i$  which is neighbor of a vertex in  $V_i$ , because the dynamic graph is constantly connected.  $\diamond$

**Upper bound**

This section gives an upperbound on the exploration time of constantly connected dynamic graphs based on a ring of size  $n$ . The algorithm executed by the agent named **Explore-ring-with-return** to explore with return any constantly connected dynamic graph based on a ring of  $n$  vertices is very simple, and we show in the next section that it is optimal. Let's describe it.

From the start vertex, the agent explores the ring using the algorithm **Explore-ring** whose complexity is given by **Theorem 1**. Once the exploration is finished, he returns to his starting vertex.

**Theorem 3.** An agent executing the algorithm **Explore-ring-with-return** requires at most  $3n - 4$  time units to explore with return any constantly connected dynamic graph based on a ring of  $n$  vertices.

**Proof.**

First of all, note that the algorithm is composed of two procedures:

- a) Exploring-the-ring
- b) Returning-to-the-starting-vertex

The complexity of the algorithm is given by the sum of the complexities of this two procedures. The complexity of the procedure Exploring-the-ring is  $2n - 3$  time units (exploration time of a ring with  $n$  vertices **cf. Theorem 1**). And the complexity of the procedure Returning-to-the-starting-vertex is  $sn - 1$  time units **cf. Theorem 2**. The sum of these two complexities gives the claimed bound. Which concludes the proof of the theorem.  $\square$

**Lower bound**

This section gives the lower bound on the exploration time of constantly connected dynamic graphs based on a ring of size  $n$ . We prove that the simple algorithm described in the **section 2** is optimal. For that, we show that for any agent (algorithm), there exists a constantly connected dynamic graph based on a ring of size  $n$  such that the agent must pay at least  $3n - 4$  time units to explore it with return. We have the following Theorem.

**Theorem 4.** For any integer  $n \geq 3$ , there exists a constantly connected dynamic graph based on a ring of size  $n$  such that any agent must pay at least  $3n - 4$  time units to explore with return the ring. This bound remains even if the agent knows the dynamics of the graph.

**Proof.** For any integer  $n \geq 3$ , we define the constantly connected dynamic graph  $\mathcal{G}$  based on the ring  $C_n$ . Let  $v_0, v_1, \dots, v_{n-1}$  be the vertices of  $C_n$  in clockwise order. Without loss of generality, assume that the exploration starts from  $v_0$  at time 0. By construction, in  $\mathcal{G}$ , the edge  $\{v_0, v_1\}$ , respectively  $\{v_1, v_2\}$ , is absent in the time interval  $[0, n - 2) \cup [2n - 3, 3n - 4)$ , respectively  $[n - 2, 2n - 3)$ . **See Figure 1**. Note that the dynamic graph  $\mathcal{G}$  is indeed constantly connected.

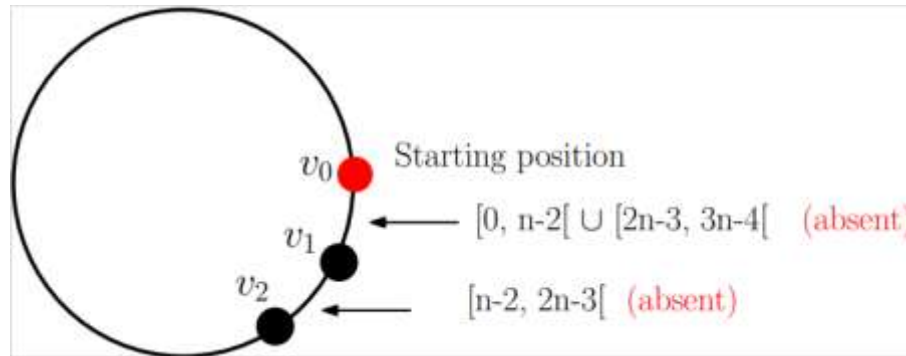


Figure 1. Difficult dynamic Ring

Consider any agent (algorithm). We will now prove that the time the agent uses to explore with return  $\mathcal{G}$  is at least  $3n - 4$  time units. To explore the dynamic graph with return, the agent must visit all vertices of  $\mathcal{G}$  in particular the vertices  $v_1, v_2$  and return at  $v_0$ . We consider the following cases.

### 1. CASE1.

#### The agent explore $v_2$ before $v_1$ .

To visit  $v_2$  without going through  $v_1$ , the agent must go around the ring and pay at least  $n - 2$  time units. After visiting  $v_2$ , to visit  $v_1$ , the agent has two choices. Either he goes around the cycle and pay  $n - 1$  other time units or wait on  $v_2$ ,  $n - 1$  time units, the time that the edge  $\{v_1, v_2\}$  appears. In both cases, the agent pays at least  $2n - 3$  time units to visit  $v_1$  for the first time. After visiting  $v_1$ , the agent must return to  $v_0$  to complete his exploration with return. To do this, two ways are available to him. First, he can return to  $v_0$  without going through  $v_2$  after visiting  $v_1$ . To do this, he must wait on  $v_1$  until the edge  $\{v_0, v_1\}$  reappears. This after  $3n - 4$  time units. The other way is to return to  $v_0$  by going around the ring through  $v_2$ . In this case, he pays  $n - 1$  other time units to return to  $v_0$ . In both cases, the agent pays at least  $3n - 4$  time units to explore with return  $C_n$ .

### 2. CASE2

#### The agent explore $v_1$ before $v_2$ .

First, note that the edge  $\{v_0, v_1\}$  is absent during the time interval  $[0, n - 2) \cup [2n - 3, 3n - 4)$ . So to visit  $v_1$  without going through  $v_2$ , the agent must wait for the appearance of the edge  $\{v_0, v_1\}$ . This after  $n - 2$  time units. And then pays one more time unit to cross the edge  $\{v_0, v_1\}$ . So to visit  $v_1$  without going through  $v_2$ , the agent pays  $n - 1$  time units. After visiting  $v_1$ , to visit  $v_2$  for the first time, the agent has two options. He can go around the ring or wait on  $v_1$  the time the edge  $\{v_1, v_2\}$  absent during the time interval  $[n - 2, 2n - 3)$  reappears. In both cases, the agent pays  $n - 1$  other time units to visit  $v_2$  for the first time. So  $2n - 2$  time units are necessary to visit  $v_2$  for the first time. After visiting  $v_2$ , to return to  $v_0$ , the agent can either go around the ring and pay  $n - 2$  other time units or go through  $v_1$  and wait for the reappearance of the edge  $\{v_0, v_1\}$  absent during the time interval  $[2n - 3, 3n - 4)$ . So the agent will arrive on  $v_0$  and finish his exploration with return after paying at least  $3n - 4$  time units which gives the claimed bound. This concludes the proof of the theorem.

### Conclusion:-

In this paper, we studied the time complexity for exploring with return constantly connected dynamic graphs based on a ring, under the assumption that the agent knows the dynamics of the graph. We gave a simple exploration algorithm for dynamic graphs that we called Explore-ring-with-return, and we have shown that it is optimal. For exploring with return constantly connected dynamic graphs based on a ring of  $n$  vertices  $3n - 4$  time units are necessary and sufficient. This study opens several perspectives. An interesting question to investigate would be if  $T$ -interval connectivity (for  $T \geq 1$ ) allows to save a significant factor in the exploration time. A further perspective is to consider the exploration problem of dynamic graphs using more than one agent, assuming standard models of communication between the agents. The objective would be to study whether dynamic graph exploration can be performed more efficiently by using more than one agent.

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