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RESEARCH ARTICLE

STUDY OF SELF-SUPERPOSABLE LIQUID IN OBLATE SPHEROIDAL SHAPE

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Abstract

The paper studiesthe self-superposable motion of a liquid of a fluid which is incompressible in nature in oblate spheroidal shape. An incompressible fluid is defined as the fluid whose volume or density does not change with pressure. Thus, the main aim of this paper is to solve the basic equations of fluid dynamics in oblate spheroidal coordinates considering self-superposable nature of the fluid. The paper includes the study of nature of vorticity and irrotationality and has not considered the boundary conditions in theanalysis. Lastly, the paper determines the pressure distribution and the solutions contain a set of constants.

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Introduction:-

If a liquid flow with velocity $\overline{q}$ and it satisfy the condition

$$\text{Curl}\left(\vec{q}\times\text{curl }\vec{q}\right)=0 \qquad ....(1)$$

Then the liquid is said to be self-superposable. This condition was given by Ram Ballabh(1940). In this paper liquid velocities are considered for which $\left(\overrightarrow{q}\times\text{curl}\overrightarrow{q}\right)=\overline{p}$ (say) and these can be represented by the gradient of a scalar quantity i.e. $\overline{p}$ can be represented as the gradient of a scalar quantity θ (say). Some velocities of some incompressible liquids are found which satisfy the above condition in oblate spheroidal system of coordinates. These solutions show the self-superposable nature of fluid.

Every solution has a set of constants. If $\overline{q_1}$ and $\overline{q_2}$ are velocities of self-superposable fluids then $\overline{q_1}\pm\overline{q_2}$ will also be self-superposable then $\overline{q_1}$ and $\overline{q_2}$ are mutually superposable.By using this property, we along with others [Mittal P K, (1981), Mittal P K, (1992)] tried to find some more self-superposable flows. Some work isdone in the field of fluid dynamics by several researchers viz.Bhattacharya T K, et al (1997), Kumar P, (2004), Shruti R, et al (2015), Nicolas F, et al(2018), Joao V N D, (2020).Natures of vorticity and irrotationality of the flows are also studied and analyzed in this paper.

Formulation of Problem:

Let $\overline{q}\times\text{curl }\overline{q}=\overline{p}.....(2)$

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For incompressible fluids, we have div $\overline{q} = 0.....(3)$

Now if a solution of equation (3) be found in such a way that P can be represented as

$\overline{p} = \text{grad}\,\theta...........(4)$

It will give a self-superposable flow. It has been shown [Agarwal G.K. (1984)] that such solutions will also satisfy the equation of motion for a steady flow. For determining a flow of liquid let us consider the flow in oblate spheroidal co-ordinates (u, v, w). If q_u, q_v, q_w be the components of $\overline{q}$ at any point (u, v, w) in oblate spheroidal co-ordinates [Spiegel M.R., (1968)]then in order to make equation(3)intergrable we may consider the following cases:

Case I: Let $q_u = 0$. In this case equation (3) will be satisfied by a solution

$$\left.\begin{aligned} q_u &= 0 \\ q_v &= \frac{AU(u)W(w)}{\cosh u \cos v\sqrt{\cosh^2 u + \cos^2 v}} \\ q_w &= \frac{BU_1(u)V(v)}{\left(\cosh^2 u + \cos^2 v\right)} \end{aligned}\right\} \qquad ....(5)$$

Where $U(u)$, $U_1(u)$ are integral functions of u, $V(v)$ and $W(w)$, the integral functions of v and w respectively and A & B are the constants. For this fluid velocity, it can be shown that P can be represented by the gradient of a scalar quantity θ given by

$$q = \frac{A^2W^2}{a\cos^2 v}\int \frac{UU'du}{\cosh^2 u\left(\cosh^2 u + \cos^2 v\right)^{3/2}} \frac{A^2W^2}{a\cos^2 v}\int \frac{U^2 \operatorname{sech} u \tanh u\, du}{\left(\cosh^2 u + \cos^2 v\right)^{3/2}}$$

$$\frac{B^2U_1^2}{a\cosh u}\int \frac{VV^1dv}{\cos v\left(\cosh^2 u + \cos^2 v\right)^2} + \frac{B^2U_1^2}{a\cosh u}\int \frac{V^2\left(\cosh^2 u - \cos^2 v\right)\sin v}{\left(\cosh^2 u + \cos^2 v\right)^2 \cos^2 v}dv$$

$$+\frac{ABUU_1}{a\left(\cosh^2 u + \cos^2 v\right)}\left[\left(V' + V\tan v\right)\left(\cosh^2 u + \cos^2 v\right) + V\sin 2v\right]\int W(w)\,dw$$

$$-\frac{A^2U^2}{a\left(\cosh^2 u + \cos^2 v\right)^2 \cosh^2 u \cos^2 v}\int WW'dw \qquad ......(6)$$

Here $U(u), U_1(u), V(v), W(w)$ are represented by U, U_1, V & W respectively and $U', U'_1 V'$ & W' represent their differentials.

By choosing different suitable sets of values of U, U_1, V and W we may get a number of self-superposable fluid velocities. One of such velocities can be obtained by taking

$$\left.\begin{aligned} &U_1 = U = a\cosh u, \\ &V = a\cos v, \\ &W = a\sin w \\ &and\;\; B = A \end{aligned}\right\} \qquad ....(7)$$

The fluid velocity becomes,

$$\left.\begin{aligned} q_u &= 0 \\ q_v &= \frac{A\sin W}{\cos v\sqrt{\cosh^2 u+\cos^2 v}} \\ q_w &= \frac{A\cosh u\cos v}{\left(\cosh^2 u+\cos^2 v\right)} \end{aligned}\right\} \qquad ....(8)$$

And

$$q=\frac{A^2}{a}[\frac{\sin^2 W}{\cos^4 V}\{\frac{1}{2\cos v}\log(\frac{\sqrt{\cosh^2 u+\cos^2 v}-\cos v}{\sqrt{\cosh^2 u+\cos^2 v}+\cos v})-\frac{(1+\cosh u)}{\sqrt{\cosh^2 u+\cos^2 v}}\}$$
$$+2\sinh\{\frac{\sqrt{\cosh^2 u+\cos^2 v}-\cosh^2 u}{\sqrt{\cosh^2 u+cos^2 v}}\}^{1/2}+\frac{\cosh^2 u\sin v\cos w\left(2\cosh^2 u+3\cos^2 v\right)}{\left(\cosh^2 u+\cos^2 v\right)}$$
$$-\frac{\cos^2 w}{2\left(\cosh^2 u+\cos^2 v\right)}] \qquad ....(9)$$

If, U, U_1, V, W are constants, then

$$\left.\begin{aligned} q_u &= 0 \\ q_v &= \frac{C_1}{\cosh u\cos v\sqrt{\cosh^2 u+\cos^2 v}} \\ q_w &= \frac{D_1}{\left(\cosh^2 u+\cos^2 v\right)} \end{aligned}\right\} \qquad ....(10)$$

And

$$\theta=\frac{D_1^2}{a\cos^6 v}[\frac{3}{2\cos v}\sin^{-1}(\frac{\cos v}{\sqrt{\cosh^2 u+\cos^2 v}})-\frac{\cosh u}{2\left(\cosh^2 u+\cos^2 v\right)}-\frac{1}{\cosh u}]$$

$$-\frac{2D_1^2\cosh u}{a\cos^3 v\sqrt{\cosh^2 u+\cos^2 v}}+\frac{D_1^2}{a\cosh^6 u}[\frac{3}{2\cosh u}\sin^{-1}(\frac{\cosh u}{\sqrt{\cosh^2 u+\cos^2 v}})$$
$$-\frac{\cos v}{2\left(\cosh^2 u+\cos^2 v\right)}-\frac{1}{\cos v}]-\frac{2D_1^2\cos v}{a\cosh^3 u\sqrt{\cosh^2 u+\cos^2 v}}\ \backslash$$
$$+\frac{C_1D_1}{a}[\frac{\cosh u\tan v\left(\cos^2 v-\cosh^2 u\right)}{\left(\cosh^2 u+\cos^2 v\right)^3}]w \qquad ....(11)$$

Case II: where $q_v = 0,$ in this case, the self-superposable flows may be

$$(i)\quad \left.\begin{aligned} q_u &= \frac{A_1 v_1 w_1}{\cosh u \cos v\sqrt{\cosh^2 u+\cos^2 v}} \\ q_v &= 0 \\ q_w &= \frac{B_1 U_2 V_2}{\left(\cosh^2 u+\cos^2 v\right)} \end{aligned}\right\} \qquad ....(12)$$

And

$$\theta = -\frac{R_1 B_1 V_1 V_2 W'_1}{a\cos^3 v}\int \frac{U_2 du}{\cosh^2 u\left(\cosh^2 u+\cos^2 v\right)} + \frac{B_1^2 V_2^2}{a\cos v}\int \frac{U_2^2\left(\cos^2 v-\cos^2 u\right)\sinh u du}{\cosh^2 u\left(\cosh^2 u+\cos^2 v\right)}$$

$$+\frac{B_1^2 V_2^2}{a\cos v}\int \frac{U_2 U'_2\, du}{\cosh u\left(\cosh^2 u+\cos^2 v\right)^2} + \frac{B_1^2 U_2^2}{a\cosh u}\int \frac{V_2^2 \sin v\left(\cosh^2 u-\cos^2 v\right)}{\cos^2 v\left(\cosh^2 u+\cos^2 v\right)^3} dv$$

$$+\frac{B_1^2 U_2^2}{a\cosh u}\int \frac{V_2 V'_2\, dv}{\cos v\left(\cosh^2 u+\cos^2 v\right)^2} + \frac{A_1^2 W_1^2}{a\cosh^2 u}\int \frac{\left(V_1 V_1^{'} \sec v - V_1^2 \sec v \tan v\right) dv}{\cos v\left(\cosh^2 u+\cos^2 v\right)}$$

$$+\frac{A_1^2 V_1^2}{a\cosh^2 u\cos^2 v\left(\cosh^2 u+\cos^2 v\right)^2}\int W_1 W_1^{'} dw$$

$$-\frac{A_1 B_1 V_1 V_2}{a\cosh u\left(\cosh^2 u+\cos^2 v\right)^2}\left[\frac{U_2 \sinh u\left(\cos^2 v-\cosh^2 u\right)}{\left(\cosh^2 u+\cos^2 v\right)^2} + \frac{U'_2 \cosh u}{\left(\cosh^2 u+\cos^2 v\right)}\right]\int W_1 dw \qquad ....(13)$$

$$(ii)\quad \left.\begin{aligned} q_u &= \frac{A_1 \sin w}{\cosh u\sqrt{\cosh^2 u+\cos^2 v}} \\ q_v &= 0 \\ q_w &= \frac{A_1 \cosh u \cos v}{\left(\cosh^2 u+\cos^2 v\right)} \end{aligned}\right\} \qquad ....(14)$$

And

$$\theta = -\frac{A_1^2 \cos w}{a\cos^3 v}\left[\tanh u - \frac{2}{\cos v}\tan^{-1}\left(\frac{\coth u}{\cot v}\right)\right]$$

$$+\frac{A_1^2 \cos v}{a}\left[\frac{2\cosh u}{\cosh^2 u+\cos^2 v} - \frac{\left(2\cos^2 v+1\right)}{\cos v}.\sin^{-1}\left(\frac{\cos v}{\sqrt{\cosh^2 u+\cos^2 v}}\right)\right.$$

$$\left.-\frac{1}{4\cos^3 v}.\sin\left(\frac{2\cos v}{\sqrt{\cosh^2 u+\cos^2 v}}\right)\right] + \frac{A_1^2}{a}\sin v \log\left(\frac{\cosh^2 u}{\sqrt{\cosh^2 u+\cos^2 v}}\right)$$

$$+\frac{A_1^2}{a}\cosh u\left[\frac{2\cos v}{\left(\cosh^2 u+\cos^2 v\right)} - \frac{2\left(\cosh^2 u+1\right)}{\cosh^2 u}.\sin^{-1}\left(\frac{u}{\sqrt{\cosh^2 u+\cos^2 v}}\right)\right.$$

$$\left.-\frac{1}{4\cosh^3 u}.\sin\left(\frac{2\cosh u}{\sqrt{\cosh^2 u+\cos^2 v}}\right)\right] + \frac{A_1^2}{2a}\sinh^2 u\left[\frac{\cosh u \cos v}{\left(\cosh^2 u+\cos^2 v\right)}\right.$$

$$-\sin^{-1}\left(\frac{\cosh u}{\sqrt{\cosh^2 u+\cos^2 v}}\right)]-\frac{\alpha_1^2}{2a}\frac{\cos^2 w}{\cosh^2 u\left(\cosh^2 u+\cos^2 v\right)^2}$$

$$-\frac{2\alpha_1^2}{a}\frac{\cos w\cos^4 v\tanh u}{\cosh^2 u\left(\cosh^2 u+\cos^2 v\right)^2} \qquad ....(15)$$

$$(iii)\quad \left.\begin{aligned} q_u &= \frac{c_2}{\cosh u\cos v\sqrt{\cosh^2 u+\cos^2 v}} \\ q_v &= 0 \\ q_w &= \frac{D_2}{\left(\cosh^2 u+\cos^2 v\right)} \end{aligned}\right\} \qquad ....(16)$$

And

$$\theta=\frac{D_2^2}{a\cos v}[\sin^7 v\left\{\cos ec\, v.\operatorname{sech} u-\frac{1}{32}\sin(4\sin^{-1}.(\frac{\cos v}{\sqrt{\cosh^2 u+\cos^2 v}})\right\}$$

$$-\frac{\cos v\cosh u}{\left(\cosh^2 u+\cos^2 v\right)}-\left(\frac{9+8\cos^4 v}{8}\right)\sin^{-1}\left(\frac{\cos v}{\sqrt{\cosh^2 u+\cos^2 v}}\right)\}$$

$$-\frac{\cosh u}{\cos^2 v\left(\cosh^2 u+\cos^2 v\right)}]+\frac{D_2^2}{a\cosh u}[\sin^7 v\{\cosh u\sin v-\frac{1}{32}\sin(4\sin^{-1}(\frac{\cosh u}{\sqrt{\cosh^2 u+\cos^2 v}}))$$

a

$$-\frac{\cos v\cosh u}{\left(\cosh^2 u+\cos^2 v\right)}-\left(\frac{9+8\cosh^2 u}{8}\right).\sin^{-1}\left(\frac{\cosh u}{\sqrt{\cosh^2 u+\cos^2 v}}\right)\}$$

$$-\frac{\cos v}{\cosh^2 u\left(\cosh^2 u+\cos^2 v\right)}+\frac{C_2D_2}{a}.\frac{\cos v\tanh u\left(\cos^2 v-\cosh^2 u\right)}{\left(\cosh^2 u+\cos^2 v\right)}.w \qquad ......(17)$$

Case III: when $q_v=0.$some self-superposable flow may be

$$\left.\begin{aligned} q_u &= \frac{A_2V_3W_2}{\cosh u\cos v\sqrt{\cosh^2 u+\cos^2 v}} \\ q_v &= \frac{B_2U_3W_3}{\cosh u\cos v\sqrt{\cosh^2 u+\cos^2 v}} \\ q_w &= 0 \end{aligned}\right\} \qquad ....(18)$$

And

$$q=\frac{B_2^2W_3^2}{a\cos^2 v}\int\frac{\left(U'_3U_3\cosh v-V_3^2\sinh u\right)}{\cosh^3 u\left(\cos^2 u+\cos^2 v\right)^{3/2}}du$$

$$-\frac{A_2B_2W_3W_2}{a\cos^3 v}\{V'_3\cos v-V_3\sin v\}\int\frac{U_3\sinh^2 u\,du}{\left(\cosh^2 u+\cos^2 v\right)^{3/2}}$$

$$-\frac{A_2B_2W_2W_3}{a\cosh^3 u}\{U'_3\cosh u-U_3\sinh u\}\int\frac{V_3\sec^2 v\,dv}{\left(\cosh^2 u+\cos^2 v\right)^{3/2}}$$

$$+\frac{A_2^2W_2^2}{a\cosh^2 u}\int\frac{V_3'V_3\cos v-V_3^2\sin v}{\cos^3 v\left(\cosh^2 u+\cos^2 v\right)^{3/2}}dv+\frac{A_2^2V_3^2}{a\cosh^2 u\cos^2 v\left(\cosh^2 u+\cos^2 v\right)}\int W_2W_2'dw$$

$$+\frac{B_2^2U_3^2}{a\cosh^2 u\cos^2 v\left(\cosh^2 u+\cos^2 v\right)}\int W_3W_3'dw \qquad ....(19)$$

$$\left.\begin{aligned} q_u &= \frac{A_2\sin w}{\cosh u\sqrt{\cosh^2 u+\cosh^2 v}} \\ q_v &= \frac{A_2\sin w}{\cos v\sqrt{\cosh^2 u+\cos^2 v}} \\ q_w &= 0 \end{aligned}\right\} \qquad ....(20)$$

And

$$\theta=\frac{A_2^2}{2a}\cosh^2 u\cos^2 v\cos^2 w \qquad ....(21)$$

(iv)
$$\left.\begin{aligned} q_u &= \frac{C_3}{\cosh u\sqrt{\cosh^2 u+\cos^2 v}} \\ q_v &= \frac{D_3}{\cos v\sqrt{\cosh^2 u+\cos^2 v}} \\ q_w &= 0 \end{aligned}\right\} \qquad ....(22)$$

And $\theta=\text{constant}$ ....(23)

Case IV:when $q_u=q_v=0,$ a possible solution of equation (3) is given by

$$\left.\begin{aligned} q_u &= 0 \\ q_v &= 0 \\ q_w &= \frac{A_3U_4V_4}{\left(\cosh^2 u+\cos^2 v\right)} \end{aligned}\right\} \qquad ....(24)$$

Case V:when $q_v=0, q_w=0$ the self-superposable flow is given by

$$\left.\begin{aligned} q_u &= \frac{A_4V_5W_4}{\cosh u\cos v\sqrt{\cosh^2 u+\cos^2 v}} \\ q_v &= 0 \\ q_w &= 0 \end{aligned}\right\} \qquad ....(25)$$

Case VI: when $q_w=0, q_u=0,$ the flow is

$$\left.\begin{array}{l} q_u = 0 \\ q_v = \dfrac{A_5 V_5 W_5}{\cosh u \cos v \sqrt{\cosh^2 u + \cos^2 v}} \\ q_w = 0 \end{array}\right\} \qquad ....(26)$$

In all the above cases $U_n, V_n, W_n \ (n = 1, 2, 3, 4, ...)$ areintegral functions of u, v and w Respectively and $A_n, B_n, C_n, D_n \ (n = 1, 2, 3, ...)$ are constants which may be determined by boundary conditions.

Superposable fluid Motion:

It has already been shown that the hydrodynamic flows given by equations (5) and (25) are self-superposable. It can also easily be shown that if $\overline{q_1}$ and $\overline{q_2}$ be the two flows given by equations (5) and (25) then p for $\overline{q_1} \pm \overline{q_2}$ can also be represented by the gradient of a scalar quantity. Thus $\overline{q_1}$ and $\overline{q_2}$ will be mutually superposable and a flow

$$\left.\begin{array}{l} q_u = \dfrac{AV(v)W(w)}{\cosh u \cos v \sqrt{\cosh^2 u + \cos^2 v}} \\ q_v = \dfrac{BU(u)W(w)}{\cosh u \cos v \sqrt{\cosh^2 u + \cos^2 v}} \\ q_w = \dfrac{CU(u)V(v)}{\left(\cosh^2 u + \cos^2 v\right)} \end{array}\right\} \qquad ....(27)$$

is possible.The same flow can be determined by mutually superposing the flows (12) and (26), (18) and (24)

Pressure Distribution:

It is interesting to note that θ is nothing but Bernoulli function given by

$$\theta = (\frac{q^2}{2g}) + h + \frac{P}{g} \qquad ....(28)$$

Where q, g, h and P denote velocity, acceleration due to gravity, height above some horizontal plane of reference and the pressure head.

It is a well-known fact that for an incompressible fluid the pressure head P is given by

$$P = \frac{þ}{\rho_0} + \text{Constant}$$

Where, p is the pressure distribution.

Also if the motion of the fluid be steady and slow then the value of h can be taken without much loss of generality as u, for the flow $(5), (8)$ and (10) v for the flows $(12), (14)$ & (16) and w for the flows $(18), (20)$ and (22). Thus for the flow (20), the pressure distribution,

$$p = K_1 + K_2.\sin W + K_3 \frac{\left(\operatorname{sech}^2 u + \sec^2 v\right)}{\left(\cosh^2 u + \cos^2 v\right)} \qquad ....(29)$$

Similarly for the flow (22) taking $C_3 = D_3$ we have

þ = $K_4 + K_5 \sin w + K_6 \operatorname{sech}^2 u \sec^2 v \qquad ....(30)$

Where, $K_1, K_2, K_3, K_4, K_5, K_6$ are constant.

Similarly the pressure distribution for the other flows can be determined.

Vorticity of the flow:

It was shown [Ballabh R, (1942)] that for a self-superposable flow, vorticity is constant along its stream lines. If T is a unit tangent along a stream line, then

$$\bar{T} \times \bar{q} \qquad ....(31)$$

By equation (16) and (31) it can readily be shown that

$$T = [\frac{\cosh^2 u + \cos^2 v}{\cos^2 v\left(\cosh^4 u + 2\cosh^2 u\right) + \cosh^4 u + \cos^4 v}, 0,$$

$$\frac{\cosh^2 u \cos v}{\{\cos^2 v\left(\cosh^4 u + 2\cosh^2 u\right) + \cosh^4 u + \cos^4 v\}^{1/2}}] \qquad ....(32)$$

Hence, the vorticity of the flow (16) is constant along the curve represented by equation (32). Similarly, the curves of constant vorticity can also be found for other flows.

Irrotationality:

Vorticity ζ for the flow[equation (10)] can be calculated as

$$\zeta = \frac{D_1}{a\cos v\left(\cosh^2 u + \cos^2 v\right)^{1/2}}\{\frac{(\cosh^2 u - \cos^2 v)\sin v}{(\cosh^2 u + \cos^2 v)^2}\}e_1$$

Vorticity

$$-\frac{D_1}{a\cosh u\left(\cosh^2 u + \cos^2 v\right)^{1/2}} \times \{\frac{(\cos^2 v - \cosh^2 u)\sinh u}{(\cosh^2 u + \cos^2 v)^2}\}e_2 - c_1\frac{\operatorname{sech} u \tan u \sec v}{a(\cosh^2 u + \cos^2 v)} \qquad ......(33)$$

It is clear from equation (33) that flow (10) is not irrotational. For the flow (22)

$$\bar{\zeta}=0$$

Hence, the flow (22) is irrotational throughout. Similar conclusions can be drawn for the flows discussed earlier.

Conclusion:-

To sum up the study of the equations of fluid dynamics in oblate spheroidal coordinates considering self-superposable nature of the fluid, the authors can ably deduce that:

1. Pressure Distribution is $P = \frac{p}{\rho_0}$ + Constant
2. The vorticity of the flow is constant along the curve/shape.
3. The flow is irrotational throughout.

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