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RESEARCH ARTICLE

GRAVITATIONAL CASIMIR EFFECT IN INSPIRALLING NEUTRON STAR BINARY

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Abstract

Currently, the discussions and investigations for the vacuum energy are drawing great both theoretical and experimental attention. The vacuum states of variety of fields, subject to special boundary conditions, may contribute to non-trivial macroscopic vacuum energy, i.e., the Casimir effect, which become an interdisciplinary subject and plays an important role in a variety of fields of physics. We adopt Schwinger's source theory and study the quantization of gravitation contributions to the Casimir effect, i.e., the gravitational Casimir effect, in inspiraling neutron star binaries with wide separation of 10^9 m. By considering gravitoelectromagnetism (GEM) arising from the spiral-in orbital motion and evaluating the gravitoelectromagnetic contributions to the vacuum energy of gravitons radiated during the orbital decay, we demonstrate that, when the radial separation of the system decay a distance L, the GEM results in a small Casimir correction to the gravitational vacuum energy, which contributes to an attractive gravitational Casimir force to the binary, in addition to the gravitational force. We also discuss the possible detections for such gravitational Casimir effect from both the future space-based gravitational wave observatories and cosmological observations.

Introduction:-

The Casimir effect [1,2]manifests the nontrivial properties of quantum fluctuations of the vacuum state, which in its simplest form is the interaction of a pair of neutral, parallel conducting plates due to the disturbance of the vacuum of the electromagnetic field. It was originally calculated [3] the shift in the vacuum energy density of electromagnetic field caused by two parallel conducting plates and was readily applied [4] to show that the classical interactions of two neutral polarizable atoms at large distances are modified by retardation effects. This was later extended by Lifshitz[5] to forces between dielectric macroscopic bodies usually characterized by a dielectric constant. The microscopic approach to the theory of both van der Waals and Casimir forces were formulated in a unified way, non-relativistically in second order perturbation theory from the dipole-dipole interaction energy [6]. It was found that the correlation of the quantized electromagnetic field in the vacuum state is not equal to zero if we increase the distance between the two macroscopic bodies to be so large that the virtual photon emitted by an atom of one body cannot reach the second body during its lifetime. The nonzero correlated oscillations of the induced atomic dipole moments result in a Casimir force [7]. The long-range interactions between polarizable systems also have been widely investigated both theoretically and experimentally [5,8-11], which demonstrate that even though the quantum in nature, an important feature of the Casimir effect is that it predicts the Casimir force and gives rise to nontrivial influence between macroscopic bodies. In addition, Schwinger used the proper-time formalism for the

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effective action and recalculated the Casimir effect in source theoretical methods [12-14], when there is no reference of quantum oscillators and zero point energy in electromagnetism.

In large-scale systems subject to the long-range gravitational interactions, the Casimir effect arises in spacetime with nontrivial topology [15,16], represent by boundaries that can be viewed as external fields. The appearance of an external field always accompanies with the effects of vacuum polarization, characterized by some nonzero vacuum energy and special boundary conditions. In this sense, the Casimir effect can be viewed as a polarization of vacuum by boundary conditions, arising in the case of external gravitational field. Such effect has been investigated in various cases of boundary geometries and different types of field[17,18]. The Casimir effect, associated with the gravitational field, has also been widely investigated theoretically and tested accurately by several experiments in flat spacetime (see [7,19,20] for example). In curved spacetime [21], some interesting results can be obtained in special circumstances, in which the quantization procedure are performed in quite a straightforward way [22,23]. By confining inside a Casimir cavity, the possible influence of the gravitational field on the vacuum energy of a quantum field [24-27] faces the open issue concerning the limits of validity of general relativity at small distances [28]. The gravitational interaction causes a small reduction in the Casimir energy of a massless scalar field confined in a rigid Casimir cavity in a slightly curved, static spacetime background [29]. Based on Schwinger's effective action method [30], it was found that the Casimir effect in a small cavity at rest in the weak gravitational field of a massive, non-rotating source suffers from a same small correction. While there is no first-order gravitomagnetic effect in the vacuum energy shift confined in the Casimir cavity, lying between two parallel massive walls moving in the opposite directions to each other [31].

Anyway, the macroscopic objects with given boundary conditions give rise to nontrivial gravitational Casimir effect (GCE). Such GCE [32] modifies the long-range gravitational interactions by an interaction potential varying with the distance as r^{-7}, which is suppressed by the square of the Planck length, in a system that a massive test point particle interacts with a fluctuating mass distribution. A Lifshitz-type formula for the GCE [33] at zero temperature in real bodies system gives a gravitational Casimir energy depending on the frequency of gravitons, which allows us to quantize the gravitational contribution to the Casimir effect. The inspiraling neutron star (NS) binaries, which behave as the tensor sources, are subject to the gravitational interactions and accompany with spin-2 gravitons, even there is no reference for the zero point energy of quantum oscillators. So far, the Einstein's general relativity has been widely accepted as a sound theory to describe the dynamics of wide NS binaries, with separation of about 10^9 m, which move closer and closer in a spiral-in way and may coalesce and merge in the Hubble time. It is the gravitational force to drive the orbital decay, losing orbital binding energy and emitting gravitational waves (GWs). In this work, we employ Schwinger's source theory [13,14] and investigate the GCE in the inspiraling NS binary systemswith wide separations of about 10^9 m, which may merge during the cosmological time. We use Schwinger's approach and evaluate the shift of gravitational Casimir energy when the separation of two stars decays a small distance $L(L \ll R)$ (Ris the separation of the binary) in radial direction. Firstly, we briefly introduce the preliminary for the scenarios of our calculations, in a weak-field-limit gravitational source. We also decompose the spin-2 gravitational field into two massless spin-1 scalar fields, by considering the periodicity of the orbital motion of the binary and adopting the Dewitt's approach [34], in our calculations. The gravitoelectric and gravitomagnetic contributions to the Casimir energy are calculated in the following two sections, respectively. Finally, we discuss the possible detections for the GCE from direct GW detectors and cosmological observations.

Throughout the paper, we use the natural units $c = 1, G = 1, \hbar = 1$in the calculations and just write out in the results. The metric signature is defined to be diag(-1,1,1,1). Greek indicesμ, ν take values from 0 to 3, while Latin ones i, j take values from 1 to 3.

Preliminary:-

In wide inspiraling NS binaries that display weak gravitational fields and non-relativistic rotation, it naturally allows us to work in the linearized gravity and assume weak-field approximation to general relativity. In the weak-field limit, the gravitational field of a given NS binary is described by a slight metric perturbation deviating from the flat Minkowskian one, namely,

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \quad |h_{\mu\nu}| \ll 1. \quad (1)$$

The solutions of Einstein's field equation read,

$$ds^2 = -\left(1 + \frac{1}{2}\bar{h}^{00}\right) dt^2 + 2\bar{h}_{0i} dt dx^i + \left(1 - \frac{1}{2}\bar{h}^{00}\right) \delta_{ij} dx^i dx^j, \quad (2)$$

where we work in the harmonic gauge, and $\bar{h}_{\mu\nu} \equiv h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}$. The resulting equations resemble those of Maxwell's equations for electromagnetism, referred to as the gravitoelectromagnetism (GEM) [35-38], which is based on the close formal analogy between Newton's law of gravitation and Coulomb's law of electricity. The Newtonian solution of gravitational field can be alternatively interpreted as a gravitoelectric field, and a rotating mass-current gives rise to a gravitomagnetic field. Accordingly, the Newtonian potential of NS binary can be analogously written as a gravitoelectric scalar potential, while the orbital rotation of two massive stars causes a gravitomagnetic vector potential. As consequence, we can write down the gravitoelectric scalar potential and the gravitomagnetic vector potential, respectively,

$$\bar{h}^{00} = 4\Phi_E, \qquad \bar{h}^{0i} = -2\Phi_M^i, \quad (3)$$

with $\Phi_E = -\frac{M}{R}$(M is the total mass of the binary) and $\Phi_M^i \propto \frac{M}{R^2}$. In GEM, analog of mass current to the electric current, the gravitoelectromagnetic fields also satisfy the continuity equation, which is the reduced Lorentz gauge condition in terms of gravitoelectric and gravitomagnetic potentials,

$$\frac{\partial \Phi_E}{\partial t} + \frac{1}{2}\nabla_i \Phi_j^M \delta_{ij} = 0. \quad (4)$$

So, the total potential of the binary system can be written as

$$\Phi(r) = -\frac{M}{R} + \frac{M}{R^2} r + \mathcal{O}\left(\frac{M}{R}\right)^3. \quad (5)$$

Here, r lies in the range of $0 < r < L$ and $\frac{r}{R} \sim \mathcal{O}(\frac{M}{R})$, because of the wide separation R of the binary that leads to the orbital decay in the radial direction $r \ll M$, during an observable duration of $m\mathcal{T}$ ($\mathcal{T}$ denotes the orbital period of the NS binary).

According to the Maxwell-like formulation of the linearized Einstein field equations, i.e., the gravitoelectromagnetic field equations [37], describing the spiral-in dynamics of NS binary, both the gravitoelectric and gravitomagnetic fields should have the form of plane waves,

$$h_{ij}^E = \mathcal{E}_{ij} e^{i(\vec{k}\cdot\vec{r} - \omega t)}, \quad h_{ij}^M = \mathcal{M}_{ij} e^{i(\vec{k}\cdot\vec{r} - \omega t)}, \qquad (6)$$

which are transverse waves with two independent polarizations, "plus" and "cross", each with allowed modes of $(\omega_+, \omega_\times)$, respectively. According to the smoothness principle [39], only the traceless part of the tangential components of the GWs can be smooth across the star surfaces, which gives rise to the Casimir-type boundary conditions,

$$h_{rr}^{E,M} = 0, \qquad h_{\varphi z}^{E,M} = 0, \qquad (7)$$

$$\partial_r h_{\varphi r}^{E,M} = \partial_r h_{zr}^{E,M} = 0. \qquad (8)$$

Here, we use the polar coordinates. The radial component h_{rr} and the cross of angular and perpendicular components $h_{\varphi z}$ for both fields satisfy Dirichlet boundary conditions, while $h_{\varphi z}^{E,M}$ and $h_{zr}^{E,M}$ satisfy Neumann ones.

Following the spirit of DeWitt's approach [34], the dynamics of physical gravitons in linearized gravity is equivalent to that of two free massless scalar fields [40], one with Dirichlet and one with Neumann boundary conditions. Owing to the periodically spiral-in orbital motion of two stars in the binary, the massless scalar fields meet the periodic boundary conditions, yet on a twice separation of the original geometry. Therefore, we in turn combine the two massless scalar fields into a single massless scalar field on a shell with width of 2L after a duration of $m\mathcal{T}$. Consequently, we are allowed to make gravitational analogue of the electromagnetic results and decompose the gravitoelectric and gravitomagetic fields into a scalar part and the polarization-dependent part, respectively,

$$h^{E+} = \sum_k h_E^{TT+}\psi_k, h^{E\times} = \sum_k h_E^{TT\times}\psi_k, h^{M+} = \sum_k h_M^{TT+}\psi_k, h^{M\times} = \sum_k h_M^{TT\times}\psi_k. \quad (9)$$

k is the wave number of the plane waves. Consequently, the scalar part just depends on the allowed modes of the gravitons and is independent of the polarization components, which contributes to the gravitational Casimir energy. So, we turn to evaluate the gravitational Casimir vacuum energy for such a combined massless scalar field when the binary decay an L distance in the radial separation.

According to Schwinger's source theory [13], the shift of gravitational vacuum Casimir energy density of an inspiraling NS binary, after m periods $\mathcal{T}$, should be extracted from

$$\langle 0_+ | 0_- \rangle = e^{iW(G)}, \qquad (10)$$

when the separation shrinks from R to R − L. Therefore, we adopt the Schwinger's formalism [14,41] and evaluate the effective action W(G) for the gravitatioelectromagnetic field of the inspiraling binary, in order to calculate the shift of gravitational-induced Casimir energy when the orbital separation shrinks a distance L in the radial direction. The effective action, arising from the gravitoelectromagnetic sources, contains both the gravitoelectric and gravitomagnetic contributions,

$$\mathrm{W(G)} = \mathrm{W_E(G)} + \mathrm{W_M(G)}. \quad (11)$$

The gravitoelectric part$\mathrm{W_E(G)}$ comes from the Newtonian gravitational interaction, and we refer as the static effect, although it, actually, isn't stationary. While the gravitomagnetic contribution $\mathrm{W_M(G)}$ is called the dynamical effect, which is the representation for the perturbation induced by the gravitomagnetic field.

Static Effect:-

In this section, we calculate the contribution from the Newtonian gravitational potential, i.e., the gravitoelectric effect, which yields a diagonal spacetime metric,

$$ds^2 = -(1 + 2\Phi_E)dt^2 + (1 - 2\Phi_E)\delta_{ij}\, dx^i dx^j, \quad (12)$$

We employ the Schwinger's approach [14] and evaluate the effective action of gravitoelectric part,

$$W_E(G) = \lim_{\upsilon\to 0} W_E(\upsilon) = -\frac{i}{2}\int_{s_0}^{\infty}\frac{ds}{s}s^{\upsilon}Tre^{-isH}, \quad (13)$$

where the Hamiltonian reads $H = \partial_t^2 - \nabla^2 = -\omega^2 + \vec{p}^2$ and$\omega = \omega_+, \omega_\times$ denotes the summation of all allowed modes. We will take the limit $\upsilon \to 0$ and $s_0 \to 0$at the end of our calculations. The trace in (13) is evaluated all over the spacetime degrees of freedom,

$$Tre^{-isH} = \sum_n \int dt d\vec{r} d\vec{p}_\perp d\omega |\langle r|\psi\rangle|^2 e^{-is({\vec{p}_\perp}^2 - \omega^2 + (\frac{n\pi}{2L})^2)}, \quad (14)$$

where $\vec{p}_\perp$denote the angular component $\vec{p}_\varphi$ and the perpendicular component $\vec{p}_z$ of the momentum. The mode solutions of the combined massless scalar field satisfy the Dirichlet conditions in the shrinked Casimir shell,

$$\psi(t,\varphi,z,r = R) = \psi(t,\varphi,z,r = R) = 0. \quad (15)$$

The normalized field modes read,

$$\langle r \mid \psi\rangle^* = \langle\psi|r\rangle = \sqrt{\frac{1}{(1 - 4\Phi_E)2\pi^3 L}}\, e^{i\omega_{E,n}t} e^{-i\vec{p}_\perp\cdot\vec{r}_\perp} \sin\frac{n\pi r}{2L}, \quad (16)$$

where $\vec{r}_\perp$ denotes the angular component $\varphi$and the perpendicular component z of the coordinates, and the allowed gravitoelectric modes inside the shrinked Casimir shell are$\omega_{E,n}^2 = (1 + 4\Phi_E)\left({\vec{p}_\perp}^2 + \left(\frac{n\pi}{2L}\right)^2\right) = (1 + 4\Phi_E)({\vec{p}_\varphi}^2 + {\vec{p}_z}^2 + \left(\frac{n\pi}{2L}\right)^2)$.Integrating over the spacetime degrees of freedom $dtd\vec{r}d\vec{p}_\perp d\omega$ with the duration of m periods$\mathcal{T}$, we obtain the trace in eq. (13),

$$Tre^{-isH} = (1 + 4\Phi_E)\frac{\pi R^2 m\mathcal{T}}{\sqrt{i(4\pi s)^3}}\sum_n e^{-is\left(\frac{n\pi}{2L}\right)^2}, \quad (17)$$

when the separation of NS binary system has decayed L in the radial direction in the duration of $m\mathcal{T}$.

Inserting the trace (17) into eq. (13), we can write down the gravitoelectric effective action,

$$W_E(\upsilon) = -\frac{i}{2}\frac{\pi R^2 m\mathcal{T}}{\sqrt{i(4\pi)^3}}(1 + 4\Phi_E)\int_0^{\infty} ds s^{\upsilon - \frac{3}{2} - 1}\sum_n e^{-is\left(\frac{n\pi}{2L}\right)^2}. \quad (18)$$

Performing the Wick rotation [42] and using both the Euler Γ-function $\Gamma(z) = \int_0^{\infty} ds s^{z-1}e^{-s}$ and the Riemann ς-function $\varsigma(z) = \sum_n \frac{1}{n^z}$, we recast the gravitoelectric effective action as,

$$W_E(G) = -\frac{1}{2}\frac{\pi R^2 m\mathcal{T}}{\sqrt{(4\pi)^3}}(-i)^{\upsilon-2}\left(\frac{2L}{\pi}\right)^{2\upsilon-3}\Gamma(2\upsilon - 3)\varsigma(2\upsilon - 3)(1 + 4\Phi_E). \quad (19)$$

Taking the limit $\upsilon \to 0$and recalling that $\Gamma\left(-\frac{3}{2}\right) = \frac{4\sqrt{\pi}}{3}$, $\varsigma(-3) = \frac{1}{120}$, we finally have the gravitoelectric contribution to the effective action,

$$W_E(G) = \frac{\pi R^2 m\mathcal{T}\pi^2}{1440(2L)^3}(1 + 4\Phi_E). \quad (20)$$

With the identification of the energy shift provided by

$$E_{Cas}^{E} = W_E(G)t, \quad (21)$$

in the duration time t, we get the gravitoelectric-induced shift of gravitational Casimir energy in the gravitational Casimir shell, when the NS binary inspirals$t = m\mathcal{T}$ and the orbital separation shrinks L,

$$E_{Cas}^{E} = \frac{W_E(G)}{m\mathcal{T}} = -\frac{\pi R^2 \hbar c \pi^2}{1440(2L)^3}(1 + 4\Phi_E), \quad (22)$$

The corresponding gravitoelectric Casimir energy density is then given by

$$\mathcal{E}_{Cas}^{E} = \frac{W_E(G)}{m\mathcal{T}} = -\frac{\hbar c \pi^2}{720(2L)^4}(1 + 4\Phi_E), \quad (23)$$

Dynamical Effect:-

We now move to adopt the above technique to the evaluation of the gravitomagnetic correction to the gravitational Casimir energy density, during the spiral-in decay. By taking the gravitomagnetic contributions into account, the spacetime metric then reads

$$ds^2 = -(1 + 2\Phi_E + 2\Phi_M r)dt^2 + (1 - 2\Phi_E - 2\Phi_M r)\delta_{ij}\, dx^i dx^j, \quad (24)$$

and the equation of motion can be written down,

$$-(1 - 4\Phi_E - 4\Phi_M r)\partial_t{}^2\psi + \delta_{ij}\,\partial x^i \partial x^j \psi = 0. (25)$$

We are interested in the mode solutions describing the combined single massless scalar field inside the shrinked Casimir shell with twice width $2L$,

$$\psi = \frac{1}{\sqrt{\left(1 - 4\Phi_E - 4\Phi_M(2L)\right)(2\pi)^3 \omega_{M,n}(2L)}} e^{i\omega_{M,n}t} e^{-i\vec{p}_\perp \cdot \vec{r}_\perp} \sin\frac{n\pi r}{2L}, \quad (26)$$

where the GW frequencies of allowed gravitoelectromagnetic modes inside the decayed Casimir shell are

$$\omega_{M,n}^2 = \pi(1 + 4\Phi_E + 4\Phi_M(2L))\left(\vec{p}_\perp{}^2 + \left(\frac{n\pi}{2L}\right)^2\right) = (1 + 4\Phi_E + 4\Phi_M(2L))(\vec{p}_\varphi{}^2 + \vec{p}_z{}^2 + \left(\frac{n\pi}{2L}\right)^2).$$

Then we are allowed to calculate the gravitomagnetic effective action,

$$W_M(G) = \lim_{\upsilon\to 0} W_M(\upsilon) = -\frac{i}{2}\int_0^\infty ds s^{\upsilon-1} Tr[4i\Phi_M s(r + is\partial_r)\nabla^2 e^{-isH}]. \quad (27)$$

Thereinto, the trace can be got by integrating over the spacetime during the duration of $m\mathcal{T}$ inspiral,

$$\begin{aligned} &Tr[4i\Phi_M s(r + is\partial_r)\nabla^2 e^{-isH}] \\ &= 4i\Phi_M s \int d\vec{r}dt \sum_n \int d\vec{p}_A d\omega |\langle r|\psi\rangle|^2 (r + is\partial_r)(\vec{p}_A{}^2 - (\frac{n\pi}{2L})^2)\sin^2\frac{n\pi r}{2L} e^{-is(\vec{p}_A{}^2 - \omega^2 + (\frac{n\pi}{2L})^2)} \\ &= \frac{4i\Phi_M s}{(2\pi)^{\frac{3}{2}}(2L)} \sum_n \int d\vec{r}dtd\vec{p}_A d\omega \sin^2\frac{n\pi}{2L}(r + is\partial_r)(\vec{p}_A{}^2 - (\frac{n\pi}{2L})^2)\sin^2\frac{n\pi r}{2L} e^{-is(\vec{p}_A{}^2 - \omega^2 + (\frac{n\pi}{2L})^2)} \\ &= \frac{8i\Phi_M \pi R^2 m\mathcal{T}\sqrt{\pi}}{(2\pi)^2 L\sqrt{-i}} \int_0^{2L} dr \sum_n \sin^2\frac{n\pi r}{2L} e^{-is\left(\frac{n\pi}{2L}\right)^2}(-\frac{\pi r}{s^2} + \frac{\pi r}{is}\left(\frac{n\pi}{2L}\right)^2 + \frac{\pi}{s}\frac{n\pi}{2L} - \frac{\pi}{i}\frac{n^3\pi^3}{8L^3}). \quad (28) \end{aligned}$$

Substituting the above result into eq. (27), we evaluate the gravitomagnetic contribution to the effective action,

$$\begin{aligned} W_M(\upsilon) = \frac{4\Phi_M \pi R^2 m\mathcal{T}\sqrt{\pi}}{(2\pi)^3 L\sqrt{-i}} \sum_n \int_0^\infty ds e^{-is\left(\frac{n\pi}{2L}\right)^2}\left[-\pi L^2\left(s^{\upsilon-\frac{3}{2}-1} + i\left(\frac{n\pi}{2L}\right)^2 s^{\upsilon-\frac{1}{2}-1}\right)\right] \\ +\pi L\frac{n\pi}{2L}(s^{\upsilon-\frac{1}{2}-1} + i\left(\frac{n\pi}{2L}\right)^2 s^{\upsilon+\frac{1}{2}-1}). \quad (29) \end{aligned}$$

The contributions all come from the terms in first line of eq. (29), while the terms in the second line of eq. (29), proportional to $\varsigma(-2) = 0$, give any contribution. By taking the limit $\upsilon \to 0$, the gravitomagnetic corrected effective action is written as

$$W_M(G) = \frac{4\Phi_M \pi R^2 m\mathcal{T}\sqrt{\pi}}{(2\pi)^3 L\sqrt{-i}} \frac{\pi^4}{L}\left[-(-i)^{-\frac{3}{2}}\Gamma\left(-\frac{3}{2}\right)\varsigma(-3) + (-i)^{-\frac{1}{2}}\Gamma\left(-\frac{1}{2}\right)\varsigma(-3)\right]. \quad (30)$$

Considering that $\Gamma\left(-\frac{3}{2}\right) = \frac{4\sqrt{\pi}}{3}, \Gamma\left(-\frac{1}{2}\right) = -2\sqrt{\pi},\ \varsigma(-3) = \frac{1}{120}$, we finally obtain the effective action of the gravitomagnetic part,

$$W_M(G) = \frac{\pi R^2 m\mathcal{T}\pi^2}{1440(2L)^3}\Phi_M(2L), \qquad (31)$$

Combined eq. (20) and eq. (31), we can write down the full gravitoelectromagnetic effective action in inspiraling NS binaries,

$$W(G) = W_E(G) + W_M(G) = \frac{\pi R^2 m\mathcal{T}\pi^2}{1440(2L)^3}(1 + 4\Phi_E) + \frac{\pi R^2 m\mathcal{T}\pi^2}{1440(2L)^3}\Phi_M(2L)$$
$$= \frac{\pi R^2 m\mathcal{T}\pi^2}{1440(2L)^3}(1 + 4\Phi_E + 2\Phi_M L). \quad (32)$$

The corresponding gravitoelectromagnetic Casimir energy density, in the gravitational Casimir shell with width of L after inspiraling m periods $\mathcal{T}$, is then

$$\mathcal{E}_{Cas} = -\frac{\hbar c\pi^2}{720(2L)^4}(1 + 4\Phi_E + 2\Phi_M L), \quad (33)$$

Summary And Discussions:-

In an inspiraling NS binary, two stars in the system gravitationally interact and orbit with each other in a spiral-in way, losing orbital energy and releasing GWs. Based upon the Schwinger's source theory, we study the shift of gravitational Casimir energy for the massless spin-2 gravitons, released from the inspiraling NS binaries, in a decayed shell when the orbital separation shrinks L after m orbital periods $\mathcal{T}$. The orbital rotation of two massive stars in the NS binary gives rise to a gravitoelectromagnetic field, which allows us to make analogy to the Casimir effect of electromagnetic field. We consider the Casimir shell forming by two massive stars moving closer and closer in the opposite radial direction to each other and compute the gravitoelectromagnetic corrections experienced by the gravitons in such shell, in the weak-field-limit approximation. It is found that a net GCE due to the GEM appears if the binary decays a nontrivial orbital separation after $m\mathcal{T}$.

Subject to the boundary conditions (8), the GCE behaves as a manifestation of the quantization of released GWs, or gravitons, when the orbital separation shrinks a nontrivial distance L in a duration of m periods $\mathcal{T}$. Associated with the gravitational Casimir energy, a force in the geometrical configuration separated by a width L also acts on the system and may contribute to the orbital motion. In the light of the gravitoelectromagnetic Casimir energy in such a gravitational Casimir shell, we immediately get a consequence that it gives rise to an attractive force. The force density can be evaluated as

$$F_{Cas} = -\frac{\partial\mathcal{E}_{Cas}}{\partial L} = -\frac{\pi^2\hbar c}{240(2L)^4}\left(1 + 4\Phi_E + \frac{4}{3}\Phi_M L\right). \quad (34)$$

That is to say, the inspiraling NS binary is subject to an attractive gravitational Casimir force, in addition to the gravitational interaction. Even though the quantum nature of such an attractive force, it arises from the GEM correction to the general gravitational fields, due to the relativistic massive NS current. If the orbital separation decays 1 m, the binary system may suffer from a gravitational Casimir force of an order of 10^{-21}N. The gravitoelectric correction leads to a contribution of an order of $\frac{M}{R}$ (the total mass of the system is about $M\sim$ several solar mass), which results in a nontrivial correction to the additional attractive force. While the gravitomagnetic part, with the order of $\frac{M}{R^2}$, brings about a relatively insignificant contribution, which is consistent with the result by the previous work about the influence of a gravitomagnetic field on the vacuum energy of a scalar field in a rigid Casimir cavity [31].

By considering the order of 10^{-21} N, it turns out to be only compatible with the extremely sensitive force detectors in order to detect such a GEM-induced attractive gravitational Casimir force, although in a considerable observing time. Even the frustrating result in the sense of measuring gravitoelectromagnetic changes in the Casimir effect because of its extremely smallness but the non-trivialness, we expect the GCE in inspiraling GWs sources can be detected if the modulation signal is higher than the sensitivity of the detector. In the inspiraling NS binaries, the characteristic frequency of GWs is 10^{-4}-1 Hz. The quantum correction to the signals is estimated as of the order of $\sim 10^{-24}$, corresponding to a force of magnitude $\sim 10^{-20}$ N for a frequency of 10^{-3} Hz. While for the GW signals of order of $\sim 10^{-25}$ in coalescing phase, the corresponding to forces of magnitude is about $\sim 10^{-17}$ N at LIGO/VIRGO frequency bands of few tens of Hz. Consequently, the gravitational Casimir force and the gravitoelectric correction in the wide inspiraling NS binaries are very promising to be detected with improved sensitivity of space-based GW observatoriesin the near future. So, we expect that the LISA/eLISA and Taiji GW detectors with sensitivity improvements [25] would give the powerful tool to detect such effectsin near future. The GCE from high-frequency GWs sources, such as the merging black-hole/NS binaries during coalescing phase, also can be expected to be detected by LIGO/VIRGO with improved sensitivity. Apart from the possible direct GW detections, we also suggest the cosmological observations, combining with pulsar timing array, for the GCE, by considering the effects of GEM on large scales [26-28].

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