Journal Homepage: -www.journalijar.com

INTERNATIONAL JOURNAL OF ADVANCED RESEARCH (IJAR)

ISSN NO. 2320-5407

Article DOI:10.21474/IJAR01/14324
DOI URL: http://dx.doi.org/10.21474/IJAR01/14324

RESEARCH ARTICLE

IS GRAVITATIONAL VACUUM ENERGY RENORMALIZEDIN NEUTRON STAR BINARY?

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Manuscript Info

Manuscript History
Received: 21 December 2021
Final Accepted: 24 January 2022
Published: February 2022

Abstract

The vacuum energy of fluctuating quantum fields has been intensively studied by analyzing boundary conditions on the objects. By treating the two star components, making up of a wide neutron star (NS) binary with orbital separation of $R \sim 10^9 m$, as two Dirichlet point particles on the radial line, we calculate the quantum vacuum energy of fluctuating gravitational fields, arising from the Newtonian gravitational scalar potential and a gravitational vector potential that leads to the spiral-in orbital motion of the system. It is found that the stress tensor, which is responsible for the fluctuations of gravitational fields, gives rise to a finite quantum vacuum energy inside the binary system, i.e., in the region of $-R < r < R$. Accordingly, both objects making up of the binary are imposed by an additionally finite and attractive stress of $-\frac{\pi}{96R^2}$. While outside the system, $|r| > R$, the gravitational vacuum energy consists of a divergent term u_{vac}, resulting from the free Green's function without any presence of gravitational sources, and a term of $-\frac{1}{16\pi(r-R)}$ that disappears when the distance is far away from the sources. However, the gravitational Casimir force imposed on NS binary is a finite one, because the fluctuating gravitational fields vanish on the star, on which the stress tensor appears discontinuity.

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Introduction:-

Quantum vacuum energy comes of the oscillations of quanta in their ground state, making up of the corresponding field, which is a consequence of quantum mechanics and involves renormalization in quantum field theory, because of the highly divergences in the ultraviolet due to the summation over infinite oscillating modes imposed by the boundary [1-3]. The calculations for vacuum energy ab initio devolved upon the changes in the zero-point energy of fluctuating quantum field, when a background or a manifold is introduced [4]. Such traditional study idealized the problem by replacing the Casimir interactions into boundary conditions. However, the physical interactions result from the coupling between fluctuating fields and the matter, which give rise to stresses on the manifold. A real body cannot constrain modes of the field with wavelengths much smaller than the typical length scale of its interactions. It has long been known that the vacuum energy of a fluctuating field diverges when a boundary condition is imposed [5,6]. In order to cancel the divergences, several regular boundary conditions were introduced to get formal solutions to the problem [7-12]. Nevertheless, the divergences show dramatic dimensional dependence. For example, the energy with a circular boundary in two dimensions was infinite [13,14]. While for a hyperspherical shell in an odd D

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spatial dimensions, the Casimir energy of a massless scalar field was finite [15]. If the fluctuating field has a mass, the divergences still are unremovable [16-19], except for the case of plane boundaries.

The only physical way to regulate and remove divergences is renormalization in quantum field theory without boundaries. It was firstly developed that the idealized boundary conditions were replaced by the interactions of fluctuating field with an external static background field, and the coupling strength produces the desired boundary conditions on specific surfaces. By using analytic properties of scattering data to compute Green's functions in time-independent background fields, the conventional Casimir sum over modes can be transformed into a sum over bound states and scattering states in terms of phase shifts. Consequently, the divergent terms are replaced by the perturbative calculations for Feynman diagrams [20]. In this way, the renormalized one-loop quantum energies of interfaces in arbitrary dimensions were calculated [21] (see [22] for a review), which found that the Casimir energy is finite functionals of smooth background potentials in one-dimension. However, the Casimir energy for massless fields diverges on a circular boundary in two space dimensions [23], which indicates that the involved physical quantities depend on the properties of the material in detail so that they provide the physical ultraviolet cutoff [24]. An infinite Casimir energy for massive fields on curved boundaries also was reinforced by calculating the leading Feynman diagrams in D dimensions [25]. Even in three dimensions imposed by a Dirichlet boundary condition in a particular limit, the Casimir energy also depends on the detailed interactions between the fluctuating fields and the background, which are still not be captured by the idealized Casimir problem [26,27] andsimilar to the context of dispersive media [28]. While Neumann boundary conditions on a zero width surface, by coupling the field to an interaction term, can control the ultraviolet divergences, with intrinsic regularization [29].

In this work, we calculate the quantum vacuum energy of fluctuating gravitational fields, arising from the spiral-in orbital motion of wide neutron star (NS) binaries, with separation of $R \sim 10^9 m$. By comparing the typical radius of $10^4 m$ for an NS with the binary separation of $R \sim 10^9 m$, we are allowed to treat the two NS components, making up of system, as two massive Dirichlet point particles. The massive NS binary imposes a static Newtonian potential on the flat background space-time $\eta_{\mu\nu}$, which scales down $\sim -\frac{1}{r}$ and goes to zero at a distance far away from the system. In addition, two massive objects, subject to the gravitational interactions, orbit with each other and give rise to mass current, which modifies the static Newtonian potential by the higher order terms of relativistic expansions. Consequently, the flat space-time background is fluctuated by a small perturbation $h_{\mu\nu} << 1$. We compute Green's functions in time-independent background and study the quantum vacuum energy of fluctuating gravitational field. The paper is organized as follows. Firstly, we give the preliminary for our calculations. The spiral-in motion of the massive NS binaries give rise to the fluctuations of stress tensor, whose solution results in two kinds of transverse plane waves. Both of transverse plane waves display "cross" and "plus" polarizations. According to DeWitt's formulation, we ascribe the quantum effects of physical gravitons to a massless scalar field, while the other 2×2 spacial parts are responsible for the polarizations. By computing the scattering Green's function in radial direction, we investigate the gravitational vacuum energy both outside and inside the NS binaries consisting of two Dirichlet point stars in section III. Finally, we give a brief summary and come to the conclusion.

Preliminary:-

In wide NS binaries with separation of $R \sim 10^9 m$, the two massive star components bind together gravitationally and produce a static Newtonian potential. Furthermore, they are orbiting with each other under the control of gravitation and moving closer and closer in a spiral way. By considering the cosmological time for coalescence, the radial decay of the periodically orbital motion during an observable time is very small. As a result, the spiral-in orbital motion of NS binaries can be handled with periodically orbital motion with radial fluctuations. The spiral-in orbital motion disturbs the static gravitational background field and gives rise to fluctuations of stress tensor, following the Einstein's field equation,

$$\Re_{\mu\nu} - \frac{1}{2} g_{\mu\nu} \Re = \frac{8\pi G}{c^4} T_{\mu\nu}. \qquad (1)$$

The fluctuated metric comes from a slight perturbation $h_{\mu\nu}$ deviating from the flat Minkowskian one $\eta_{\mu\nu}$,

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, (|h_{\mu\nu}| << 1) \qquad (2)$$

The solution of stress tensor $T_{\mu\nu}$ reads,

$$ds^2 = (1+\frac{1}{2}\bar{h}^{00})dt^2 + 2\bar{h}_{0i}dtdx^i + (1-\frac{1}{2}\bar{h}^{00})\delta_{ij}dx^i dx^j, \tag{3}$$

where $\bar{h}_{\mu\nu} \equiv h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}h$. As consequence, the static Newtonian gravitational potential $-G\frac{M}{R}$ (M is the total mass of the binary system) is perturbed by the radial decay, and the totally gravitational potential of the NS binary is modified as,

$$\Phi(r) = -G\frac{M}{R} + G\frac{M}{R^2}r + \mathrm{O}(\frac{M}{R})^3. \tag{4}$$

The first term gives the static Newtonian gravitational scalar potential imposed by the initial system, which corresponds to the 00 components of the fluctuating fields, i.e $\bar{h}^{00} \sim G\frac{M}{R}$. While the second term, $G\frac{M}{R^2}$, represents components $\bar{h}_{0i}$ and is the gravitational vector potential, which results from the mass current, due to the orbital motion of massive objects, and contributes to dynamical effects. Consequently, the fluctuating gravitational fields, arising from the spiral-in dynamics of NS binary, can be divided into two kinds of transverse waves [30] with plane wave form,

$$h_{ij}^N = \pi_{ij}^N e^{i(\vec{k}\cdot\vec{r}-\omega t)}, h_{ij}^D = \pi_{ij}^D e^{i(\vec{k}\cdot\vec{r}-\omega t)}. \tag{5}$$

Each displays two independent polarizations, “plus” and “cross”, with allowed physical modes of $(\omega_+, \omega_\times)$, respectively. In the light of DeWitt's approach [31], the dynamics of physical gravitons in linearized gravity are equivalent to that of two free massless scalar fields, which can be combined into a single massless scalar field, yet on a twice scale of the original geometry, when meeting periodic boundary conditions. By considering the periodically orbital motion of the binary, we are allowed to decompose both kinds of transverse plane waves into a massless scalar field and the polarization-dependent part, respectively,

$$\begin{gathered} h_N^+ = \sum_k h_N^{TT+}\psi_k, h_N^\times = \sum_k h_N^{TT\times}\psi_k \\ h_D^+ = \sum_k h_D^{TT+}\psi_k, h_D^\times = \sum_k h_D^{TT\times}\psi_k \end{gathered} \tag{6}$$

Here, h^N and h^D are caused by the static Newtonian potential and the dynamical part of $G\frac{M}{R^2}r$, respectively. k denotes the wave number of the plane waves. Consequently, only the oscillations of massless scalar field $\psi_k(r,t)$ contributes to the quantum vacuum energy, while the other parts form a 2×2 space and are responsible for the polarizations. When calculating the quantum effects, the contributions of two kinds of transverse waves can be considered as $\psi_k(r,t)$ on a twice of binary separation $2R$. According to the smoothness principle [32], only the traceless part of the tangential components can be smooth across the star surfaces. Therefore, the massless scalar field is subject to the Dirichlet boundary condition on the star points,

$$\psi_k(0) = \psi_k(R) = 0. \tag{7}$$

The propagations of ψ_k all over the space-time obeys the Klein-Gordon equation,

$$-\partial^2\psi_k = 0. \tag{8}$$

ψ_k also satisfies the equal-time commutation relation,

$$[\psi(r,t), \psi(r',t')] = i\delta(r-r')\delta(t-t').$$

Gravitational Casimir effects for Dirichlet NS binaries:-

Considering the periodically planar orbital motion and the radial decay of NS binaries, we concentrate on $1+1$ dimensions and relate the matrix elements of the energy density operator to the scattering Green's function in radial direction $G(r,t;r',t')$ at the star points. Tho two-point Green's function is defined as the time-order product of the fields,

$$G(r,t;r',t') \equiv -i0\langle 0|\hat{\mathrm{T}}\psi(r,t)\psi(r',t')|0\rangle \tag{10}$$

Corresponding to the dynamics of massless scalar field ψ_k Eq. (8), the Green's function satisfies

$$-\partial^2 G(r,t,r',t') = \delta(r-r')\delta(t-t'), \tag{11}$$

which is translationally invariant in time. At the star points, the boundary conditions are

$$G(r,t;r',t')\big|_{|r|=R,0} = 0. \tag{12}$$

In our calculations, we consider the Feynman Green's function and impose the outgoing boundary conditions of $|r| \to \infty$. The physical quantum forces and physical quantum energy can be calculated from the vacuum expectation value of the stress tensor, by applying the differential operator to the Green's function,

$$F = \langle 0|T_{\mu\nu}|0\rangle = \frac{i}{2}\partial_r \partial_{r'} G(r,t,r',t')\big|_{r=r',|r|=R}. \tag{13}$$

For the 00 component, the differential operator is

$$\partial_0 \partial'_0 + \frac{1}{2}\partial^\lambda \partial'_\lambda = \frac{1}{2}\partial_0 \partial'_0 + \frac{1}{2}\partial_r \partial'_r. \tag{14}$$

While the relevant differential operator for the radial component of the stress tensor is

$$\partial_r \partial'_r - \frac{1}{2}\partial^\lambda \partial'_\lambda \to \frac{1}{2}(\omega^2 + \partial_r \partial'_r). \tag{15}$$

In order to obtain the vacuum expectation value of stress tensor $\langle 0|T_{\mu\nu}|0\rangle$, it is necessary to solve the Klein-Gordon equation of Green's function. We reduce the Green's function by taking the time Fourier transformation,

$$G(r,t,r',t') = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} e^{i\omega(t-t')} \Im_\omega(r,r'), \tag{16}$$

where $\omega = 0$ in the integral $\int\limits_{-\infty}^{\infty}$ are automatically removed for Feynman Green's function.The reduced Green's function reads

$$\Im_\omega(r,r') = \int_{-\infty}^{\infty} dt e^{-i\omega(t-t')} G(r,t,r',t'), \tag{17}$$

which obeys the differential equation

$$-(\omega^2 + \frac{d^2}{dr^2})\Im_\omega(r,r') = \delta(r-r'). \tag{18}$$

Because the gravitational waves propagate all over the space-time, it cannot form standing waves even in the region of $r < R$, by taking the point star sources into account. The physical solutions should be

$$-\Im_\omega(r,r') = \frac{1}{\omega}\sin\omega(r-R)e^{i\omega(r-R)}. \tag{19}$$

By using the equation of reduced Green's function (18) and the differential operator (14), we can calculate the energy density, whose 00 components are

$$\begin{aligned} \langle T^{00}\rangle &= \int \frac{d\omega}{2\pi}\frac{i}{2}(\omega^2 + \partial_r \partial'_r)\Im_\omega(r-r')\big|_{r=r'} \\ &= \int \frac{d\omega}{2\pi}[\frac{1}{2i}\omega^2 \Im_\omega(r) + \frac{1}{2i}\partial_r \partial_r \Im_\omega(r). \end{aligned} \tag{20}$$

Substituting the solution (19) into the above and making the complex frequency rotation $\omega \to i\xi$, we obtain

$$\begin{aligned}\left\langle T^{00}\right\rangle &= \int \frac{d\omega}{2\pi}\left[\frac{\omega}{4}e^{2i\omega(r-R)}+\frac{\omega}{4}\right]\\ &= \int -\frac{d\xi}{2\pi}\frac{\xi}{4}e^{2\xi(r-R)} - \int \frac{d\xi}{2\pi}\frac{\xi}{4}\\ &= -\frac{1}{16\pi(r-R)^2}+u_{vac}.\end{aligned} \qquad (21)$$

The first term disappears for a distance far away from the systems. However, the second term of Eq. (21), $u_{vac} = -\int_0^\infty \frac{d\xi}{4\pi}\xi$, appears divergent. Therefore, the gravitational Casimir energy is divergent outside the NS binaries, i.e., in the region of $r > R$. The divergence of u_{vac} reflects exactly the behavior of stress tensor that would be found everywhere, when we consider the free Green's function without presence of objects,

$$\Im_0(r, r', \omega) = \frac{i}{2\omega}e^{i\omega|r-r'|}. \qquad (22)$$

Accordingly, the gravitational Casimir energy shift outside the NS binary is

$$E^{r>R} = -\frac{1}{16\pi(r-R)}. \qquad (23)$$

For $r < R$, we compute in the same way and get a finite result,

$$\left\langle T^{00}\right\rangle = -\frac{\pi}{24(2R)^2}. \qquad (24)$$

The corresponding gravitational Casimir energy is

$$E^{r<R} = -\frac{\pi}{96R}. \qquad (25)$$

Using the radial differential operator (15), we also calculate $\left\langle T^{rr}\right\rangle$ by the same way and get the same results as $\left\langle T^{00}\right\rangle$. Consequently, the gravitational Casimir force on the two components is completely finite, which is given by

$$F = \left\langle T^{rr}\right\rangle\Big|_{r=R} = -\frac{\pi}{24(2R)^2}. \qquad (26)$$

The reason why an infinite quantum energy results a completely finite force is that the discontinuity of rr component of stress tensor across the objects, because of its physical nature of the flux of momentum.

While the off-diagonal terms $\left\langle T^{0r}\right\rangle$, resulting from $\frac{1}{2}(\partial^0\partial'^r + \partial'^r\partial^0)$, disappears. Therefore, we can finally write down the vacuum expectation value of stress tensor in $1+1$ dimensions,

$$\begin{aligned}\left\langle T^{\mu\nu}(r)\right\rangle &= -\frac{\pi}{24(2R)^2}\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}(r<R)),\\ &= u_{vac} - \frac{1}{16\pi(r-R)^2}\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}(r>R).\end{aligned} \qquad (27)$$

Conclusion:-

The quantum vacuum energy of fluctuating fields, subject to boundary conditions, has been studied intensively since Casimir's prediction for a force between grounded metal plates [4]. Such a quantum force arises from interactions between the fluctuating fields and matter. As consequence, it is posed as the response of a fluctuating quantum field to externally imposed boundary conditions, which is referred to the Casimir problem. The Casimir energy diverges in the limit that the fields vanish on a surface. Conventional computation for the Casimir energy and the corresponding forces is to analyze the effects of boundary conditions on the fluctuating quantum field. We consider the wide inspiraling NS binaries, with orbital separation of $\sim 10^9 m$, and treat the two objects consisting of the system as two Dirichlet point particles on the radial line. By calculating the scattering Green's function, we study the vacuum energy of fluctuating gravitational field. We obtain a finite gravitational Casimir energy of $-\frac{\pi}{96R}$, when considering the expectation value of stress tensor inside the binary system, i.e $-R < r < R$, which imposes a finitely attractive gravitational Casimir force on two objects. While outside the system, $|r| > R$, the gravitational vacuum energy resulting from an inspiraling NS binary consists of a divergent term u_{vac}, resulting from the free Green's function without any presence of gravittaional sources, and a term of $-\frac{1}{16\pi(r-R)}$ that disappears when the distance is far away from the system. The appearance of u_{vac} exactly reflects that the stress tensor would be found everywhere. The gravitational Casimir forces imposed on the star components are still finite, i.e., $-\frac{\pi}{96R^2}$, because the fields vanish on the star surface, where the stress tensor, which physically describes the flux of momentum, appears discontinuity.

Acknowledgments:-

This work is supported by the Guangxi Natural Science Foundation Program (Grant no. 007151339018).

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