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RESEARCH ARTICLE

DEPENDENCE OF GRAVITATIONAL CASIMIR EFFECTS ON GW FREQUENCY IN NS BINARY

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Abstract

In this paper, we investigate the dependence of gravitational Casimir effects on the frequencies of gravitational waves (GWs) in inspiraling neutron star (NS) binaries, with wide separation of $R \sim 10^9 m$. We introduce a mapping to describe the gravitational interactions and reflections on quantum scale and the frequency-dependent operator to depict the GWs/gravitons scattering. Accordingly, the gravitational fluctuation fields, i.e., GWs, can be decomposed as the gravitoelectric fields and gravitomagnetic fields, arising from the gravitoelectromagnetism due to the orbital motion of compact NS binaries, which can be unified as the gravitoelectric fields, because of their mutual induction. Because frequencies of GWs depend on both geometry and mass-density of the sources, we divide the calculations into geometric effects and mass effects. It is found that the dependence of gravitational Casimir energy arising from the quadruple moments on the GWs frequencies mainly give expressed to the post-Newtonian expansion coefficient. While the mass variation induced dependence on GW frequencies scales as ω^5 in low-frequency GW sources, which reflects the temperature dependence of gravitational Casimir energy in wide inspiraling NS binaries.

Introduction:-

One of nontrivial properties of the nonzero vacuum energy is the Casimir effect [1,2]. It originates from the corrections to the spectrum of zero point fluctuations of the electromagnetic field, when the plates are brought into close distance, and manifests the attraction between two neutral, parallel metallic plates [3]. Such effect between atoms at asymptotically large distances relates to the polarizability of the atoms [4]. It was later extended to account for the fluctuating fields between dielectric macroscopic bodies [5], which was generalized to include magnetic effects between two spheres [6]. The long-range interactions between polarizable systems [7] demonstrate that, even though the quantum in nature, the Casimir effect gives rise to nontrivial influence between macroscopic bodies. The Casimir effects between macroscopic objects depend on shape and material [8,9], which appears through susceptibility to current fluctuations [10] and relates to their scattering [11].

However, the electromagnetic field is of course not the only field that gives rise to Casimir effect, and the contributions to such quantum effects from any field should be nontrivial [12-14]. The quantum fluctuations arising from mass and mass currents in weak gravitational fields contribute to gravitational Casimir energy in a slightly curved spacetime background [15-18]. The gravitational Casimir effect (GCE) modifies the long-range gravitational

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interactions by an interaction potential varying with the distance as r^{-7} [19], when a mass distribution appears, and results in additional dissipation to the gravitational bound systems, such as wide inspiraling neutron star (NS) binaries [20]. The additional attractive interactions between two compact objects in gravitation induced inspiraling NS binaries [18] may accelerate the orbital decay and subsequently modify the frequencies of gravitational waves (GWs) [20]. In this work, we investigate how the GCE relates to the frequencies in low-frequency GW sources, i.e., inspiraling NS binaries with wide separation of $R \sim 10^9 m$.

Throughout the paper, we use the natural units $c=1, G=1, \hbar=1$ in our calculations and just write them out in the final results. The metric signature is defined to be diag(-1,1,1,1). Greek indices μ, ν take values from 0 to 3, while Latin ones i, j take values from 1 to 3.

Scenario:-

The dynamics of wide inspiraling NS binaries in the weak-field limit can be described by

$$ds^2 = -(1+\frac{1}{2}\bar{h}^{00})dt^2 + 2\bar{h}_{0i}dtdx^i + (1-\frac{1}{2}\bar{h}^{00})dx^i dx^j, \qquad (1)$$

where we work in the harmonic gauge, and $\bar{h}_{\mu\nu} \equiv h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}h$. The fluctuating fields obey the linear Einstein field equation,

$$\Box \bar{h}_{\mu\nu} = -16\pi T_{\mu\nu}, \qquad (2)$$

under Lorentz gauge $\bar{h}^{\mu\nu}_{,\nu} = 0$, where $T_{\mu\nu}$ behaves as the source of $\bar{h}_{\mu\nu}$,

$$\bar{h}_{\mu\nu}(\vec{r}, \vec{r}') = 4\int d\vec{r}' \frac{T_{\mu\nu}(t-|\vec{r}-\vec{r}'|, \vec{r}')}{|\vec{r}-\vec{r}'|}. \qquad (3)$$

In the near zone ($\lambda >> r > R$) of slowly spiral-in NS binaries, the Newtonian gravitational potential can be expanded as [21]

$$\Phi = -(\frac{M}{r} + \frac{d_i x^i}{r^3} + \frac{Q_{ij}x^i x^j}{r^5} + \cdots). \qquad (4)$$

Here, $M = \int T^{00} d^3\vec{r}$ gives the total mass of the source, and the dipole moment $d_i = \int T^{00} x^i d^3\vec{r}$ describes the displacement of the centre of mass from chosen origin. The first two terms $\frac{M}{r} + \frac{d_i x^i}{r^3}$ denote the static part of gravitational field. While the GWs $\bar{h}_{ij} \sim \frac{\ddot{Q}_{ij}}{r}$ arise from the reduced quadruple moment $Q_{ij} = \int T^{00}(x_i x_j - \frac{1}{2}R^2\delta_{ij})d^3\vec{r}$. Based on close formal analogy between Newton's law of gravitation and Coulomb's law of electricity, the linear Einstein field equation (2) is referred to as gravitoelectromagnetism (GEM) [22-24]. The Newtonian solution of the gravitational field can be alternatively interpreted as a gravitoelectric field, and a rotating mass current gives rise to a gravitomagnetic field. Accordingly, the Newtonian potential of an NS binary can be analogously written as a gravitoelectric scalar potential, while the spiral-in orbital rotation of two massive stars causes a gravitomagnetic vector potential. As consequence, we relate the gravitoelectric scalar potential and the gravitomagnetic vector potential to the fluctuating fields,

$$\bar{h}^{00} = 4\Phi_E, \bar{h}^{0i} = -2\Phi_M^i, \qquad (5)$$

with $\Phi_E = -\frac{M}{r}$ and $\Phi_M^i = -\frac{d_i x^i}{r^3}$, respectively. The term involving quadruple moments in the potential (4) associates with $\bar{h}_{ij}$, which can be divides into the gravitoelectric field h_{ij}^E and the gravitomagnetic field h_{ij}^M.

Consequently, we are allowed to describe the gravitational interactions in inspiraling NS binaries as the couplings between gravitoelectric field h_{ij}^{E} and the gravitomagnetic field h_{ij}^{M}. In light of the Schwinger's source theory [25], the fluctuating gravitoelectric charge and gravitomagnetic current densities $\rho(\vec{r}), \vec{J}(\vec{r})$ result in the GCE,

$$[\Phi_M(\vec{r}), \Phi_E(\vec{r}\,')] = \int d^3\vec{r} G_0(\vec{r}, \vec{r}\,')[\vec{J}(\vec{r}\,'), \rho(\vec{r}\,')], \quad (6)$$

where $G_0(\vec{r}, \vec{r}\,') = \langle \vec{r} \,|\, \frac{1}{-\nabla^2 + \omega^2} \,|\, \vec{r}\,' \rangle$, and ω denotes the frequency of GWs.

Unified description for dynamics:-

In compact NS binaries, the two stars ``A" and ``B" gravitationally related by interactions and reflections, and thus the gravitational Casimir interaction between two objects involves mechanical response [20]. Although the two objects in an NS binary orbit with each other on the binary plane and move closer and closer in a spiral way, we can treat the orbital motion as periodic orbits with radial shrink, by considering the very small radial decay on an observable time comparing with the cosmological coalescence time because of the wide separation of $10^9 m$. Because of the periodically orbital motion, the gravitational reflection in the binary system should be symmetrical. In order to express the reflection symmetry of the gravitational bound binary system on quantum scale, we apply a mapping $\hat{H}: A \rightarrow B$ and $\hat{H}(r,\theta,\varphi) = (R-r,\theta,\varphi)$, which is unitary and defined as . Th $\hat{H} h_{ij}^{E,M}(\vec{r}) = h_{ij}^{E,M}[\hat{H}(\vec{r})]$e gravitational connection between stars ``A" and ``B" due to the gravitational interactions and reflections in an inspiraling NS binary, which results in loses of orbital binding energy and release of GWs, can be characterized as frequencies of GWs. Consequently, we introduce a positive and bounded $\hat{\Im}$ operators,

$$\hat{\Im}_{A/B} = \frac{\omega^2}{1 + \omega^2 \chi(\omega) G_{0AB/0BA}} \chi(\omega), \quad (7)$$

to describe the propagations and scattering of GWs/gravitons [26] that depends on their frequencies. $\chi(\omega)$ appearing in the $\hat{\Im}$ operator (7) is the frequency-dependent gravitational susceptibility, which bridges the sources and fluctuating fields via $T_{ij} = \chi(\omega) h_{ij}^{E} = -\chi(\omega)\omega^2 \frac{h_{ij}}{2}$. Therefore, the gravitational dynamics of the NS binaries on quantum scale can be expressed by the mapping $\hat{H}$ and $\hat{\Im}$ operator, $\hat{\Im}_B = \hat{H}\hat{\Im}_A \hat{H}^{+}$. The fluctuating fields $h_{ij}^{E}(\vec{r})$ and $h_{ij}^{M}(\vec{r})$ iteratively induce each other during the whole spiral-in orbital motion, according to

$$h_{B,ij}^{E} = \hat{H} h_{A,ij}^{E}, h_{A,ij}^{M} = \hat{\Im}_B h_{A,ij}^{E} = \hat{H}\hat{\Im}_A \hat{H}^{+} h_{A,ij}^{E}, \ldots \quad (8)$$

At the initial state of the binary, the gravitational potential of the system is the Newtonian one $\Phi_E = -\frac{M}{R}$. So, the initial fluctuating field plays a role of the static gravitoelectric field $h_{ij}^{0}(\vec{r}) = h_{S,ij}^{E}(\vec{r})$. With the spiral-in orbital motion, the mass current produces an gravitomagnetic field, in analogy with that an electrical current induces the magnetic field. Subsequently, an iterative induction between gravitoelectric field and gravitomagnetic field occurs,

$$h_{ij}^{M}(\vec{r}) = \frac{1}{ik} \nabla \times h_{ij}^{E}(\vec{r}), h_{ij}^{E}(\vec{r}) = ik \int d\vec{r}\,' G_0(\vec{r}, \vec{r}\,') \vec{J}(\vec{r}\,'), \quad (9)$$

where k is the wave number of GWs. According to Eq. (9), we can unify the gravitoelectric fields and gravitomagnetic fields and uniformly describe the totally induced fluctuating fields as the gravitoelectric fields,

$$h_{ij}^{E}(\vec{r}) = ik \int d\vec{r}\,' G_0(\vec{r}, \vec{r}\,') \vec{J}(\vec{r}\,'), \ (10)$$

where the gravitomagnteic current density $\vec{J}(\vec{r}\,')$ gives expression to the gravitomagnetic fields h_{ij}^{M} . Therefore, the action, describing the GCE in inspiraling NS binaries arising from the couplings between the gravitoelectric field and the gravitomagnetic current, can be written as,

$$S[\{J\}] = \int d\vec{r}\vec{J}^{*}(\vec{r})\frac{1}{ik}h_{ij}^{E}(\vec{r}) + \int d\vec{r}d\vec{r}\,'\vec{J}^{*}(\vec{r})G_{0}(\vec{r},\vec{r}\,')\vec{J}(\vec{r}\,'). \qquad (11)$$

Dependence on GWfrequencies:-

The characteristic frequencies of GWs radiated from inspiraling compact binaries have been well predicted, which depend on the mean mass density of the sources. Based on the potential (4), the mass of the sources also depends on the geometry due to the multiple moments, which appears variations. In order to investigate the dependence of GCE on GW frequencies, we calculate the contributions to the gravitational Casimir energy from two points of view, i.e., the geometric-dependent contributions and that resulting from mass variations. Because the geometric-dependent contributions come from the quadruple moment, we, enlightened by the spirit of Dewitt's approach [27], decompose the unified fluctuating fields into three parts,

$$h_{ij}^{E}(\vec{r}) = -k\sum_{ij} h^{TT} Q_{ij}\psi_{k}(\vec{r}). \qquad (12)$$

Here, the scalar field $\psi_k(\vec{r})$ is independent of the polarizations and gives expression to the allowed modes of the outgoing solutions of GWs, which represents the combination of $\psi_k^{E}(\vec{r})$ and $\psi_k^{M}(\vec{r})$ on a twice interval [18,27]. h^{TT} is responsible for the two independent polarizations, "plus" and "cross", of allowed modes for the unified fluctuating fields in transverse-traceless (TT) gauge, which have no any contributions to the GCE. Based on the decomposition Eq.(12), we are allowed to compute the geometrically resultant effects, i.e., the effects associating with the quadruple moment Q_{ij} , and the contributions from the outgoing propagations of GWs that depends on the mass variations, respectively.

Geometrical effects -- In order to compute the contributions to gravitational Casimir energy from the quadruple moment, we express currents and field strengthes in Eq.(11) as quadruple moments by transforming the variables from currents and fields into quadruple moment and scalar field, and then we perform quantization. Accordingly, the effective action arising from the gravitational quadruple moment can be extracted,

$$S^{eff}[Q_{ij}] = \frac{i}{k}\sum_{ij,kl}[Q_{ij}^{*}H_{ijkl}Q_{kl} + Q_{ij}^{*}\Im_{ijkl}^{-1}Q_{kl}], \qquad (13)$$

where H_{ijkl} and $\Im_{ijkl}$ are the matrices of operators $\hat{H}$ and $\hat{\Im}$ that describe the gravitational interactions in quantum framework. H_{ijkl} depends on $(\frac{r}{R})^{n}$ according to post-Newtonian expansions of gravitational radiation in compact binaries. According to the observations of wide inspiraling NS binaries and performing dimensional analysis, we obtain the contributions to gravitational Casimir energy arising from the quadruple moment,

$$\begin{aligned} E_{Cas}^{Q_{ij}} &= \frac{1}{2\pi}\int_{0}^{\infty} d\omega \ln\det(1-\hat{H}^{-1}\hat{\Im}\hat{H}^{+}\hat{\Im}) \\ &\sim -\frac{1}{\pi}(\beta(0)\frac{1}{R^{7}} + \beta(\omega)\frac{1}{R^{9}} + \beta(0)\beta(\omega)\frac{1}{R^{11}} + \cdots) \qquad (14) \\ &\sim -\frac{\hbar c}{\pi}\frac{r^{6}}{R^{7}}\sum_{n=0}^{\infty} c_{n}(\frac{r}{R})^{n}, \end{aligned}$$

where c_n is the post-Newtonian expansion coefficient. In the second line, we introduce the susceptibility tensor of gravitational quadruple moment $\beta_{ijkl}(\omega) = i\int_{0}^{\infty} dt e^{i\omega t}\langle[Q_{ij}(t), Q_{kl}(0)]\rangle$, and $\beta(0) \sim M$. For the completeness of the results, we write out $\hbar$ and c obviously in the last line in Eq.(14).

Mass effects -- The contributions to gravitational Casimir energy from the mass density depend on frequencies of released GWs via the scalar field $\psi_k(\vec{r})$ (or ϕ_ω) in Eq. (12). The corresponding effective action can be written as

$$S^{eff}[\phi_\omega] = \frac{1}{2}\int d\vec{r}\int \frac{d\omega}{2\pi}\phi^*_\omega[\nabla^2 + \omega^2\varepsilon(\vec{r},\omega)]\phi_\omega. \quad (15)$$

The gravitational dielectric function $\varepsilon(\vec{r},\omega)$ of GEM sources in action (15) is defined according to the gravitational susceptibility $\chi(\omega)$, i.e., $\varepsilon(\vec{r},\omega) = 1 + \chi(\omega)$. in which the gravitational susceptibility $\chi(\omega)$ depends on the mass density of the GW sources [10]. The frequency-dependent gravitational susceptibility gives rise to changes of gravitational Casimir energy,

$$\begin{aligned} E^E_{Cas} &= E_{\chi(\omega)} - E_{\chi\to 0} \\ &= -\int_0^\infty \frac{d\omega}{2\pi}\ln\det[1 + \omega^2\chi(\omega)(\nabla^2 + \omega^2)^{-1}] \quad (16) \\ &= \int_0^\infty \frac{d\omega}{2\pi}\ln\det[1 + \omega^2\chi(\omega)G_0(\vec{r},\vec{r}')], \end{aligned}$$

where $\chi \to 0$ denotes the physical cutoff at high frequencies.

In order to investigate the interactions on quantum scale in gravitationally bound NS binaries, we quantize the propagator of the gravitational interactions in Eq. (16), by considering $(1 + \omega^2\chi(\omega)G_0)$ as the mapping operator $\hat{H}$. Accordingly, the change of gravitational Casimir energy is written as

$$E^M_{Cas} = \int_0^\infty \frac{d\omega}{2\pi}\ln\det\begin{pmatrix} 1 + \omega^2\chi G_0^{AA} & \hat{\Im}_A G_0^{AB} \\ \hat{\Im}_B G_0^{BA} & 1 + \omega^2\chi G_0^{BB} \end{pmatrix}. \quad (17)$$

It turns out that the diagonal elements contain contributions that don't depend on the relative positions of two star components, i.e. the self-currents induced contributions, which come from the quadruple moments and have been computed in the part of geometrical effects. Therefore, we subtract the diagonal contributions to the determinant, which are not sensitive to the binary separation, which yields

$$E^M_{Cas} = \int_0^\infty \frac{d\omega}{2\pi}\ln\det\begin{pmatrix} 1 & \hat{\Im}_A G_0^{AB} \\ \hat{\Im}_B G_0^{BA} & 1 \end{pmatrix}, \quad (18)$$

where $\hat{\Im}_A G_0^{AB}\hat{\Im}_B G_0^{BA}$ is a trace class operator without any cutoffs, by considering the positive and bounded $\hat{\Im}$ operators and the exponential decay of $G_0^{AB/BA}$. Therefore, the determinant is regularized and rigorously well defined for low-frequency GWs of $\omega \sim 10^{-4} - 1Hz$, which contributes to convergent gravitational Casimir energy.

In inspiraling NS binaries, the orbital separation appears to be durative decay, which makes the systems behave as mixers of GW frequencies. In the mixed space-frequency representation, the propagations of fluctuating fields obey

$$\left\langle (h_{00})^2 \right\rangle = \sum_{n=-\infty}^{\infty} \frac{\omega_n}{2\pi n} D(\vec{r},\vec{r}', i\omega_n), \quad (19)$$

where $\omega_n = 2\pi n k_B \mathrm{T}, (n = 0, \pm1, \pm2, ...)$ are Matsubara frequencies in the fluctuation dissipation theorem, and T is the temperature, $D(\vec{r},\vec{r}', i\omega_n) = \langle \vec{r} | \frac{1}{\nabla\times\nabla\times\omega^2[1+\chi(\omega)]} | \vec{r}' \rangle$ is the temperature Green's function of the gravitational field. Consequently, we have $\int \frac{d\omega}{2\pi} \to \sum_n \omega_n$ in Eq. (18), and the corresponding gravitational Casimir energy depends on GWs frequencies according to the order of

$$E^M_{cas} \sim \sum_n \omega_n^5. \quad (20)$$

Conclusion and Remarks:-

We compute the dependence of gravitational Casimir energy on the GW frequencies in wide inspiraling NS binaries, with separation of $10^9 m$. The mass current, caused by orbital motion of massive stars in compact NS binaries, gives rise to GEM, which allows us to describe the gravitational fluctuation fields as gravitoelectric and gravitomagnetic fields. In order to investigate the gravitational interactions and reflections between two objects on quantum scale, we introduce a mapping $\hat{H}$ and the frequency-dependent scattering operator $\hat{\Im}$. Accordingly, the mutual induction between gravitoelectric fields and gravitomagnetic fields, due to the orbital motion, allows us to unify them and uniformly describe as the gravitoelectric fields. Because the GWs frequencies depend on both geometry and mass-density of the sources, we divide the calculations into geometric effects and mass effects. It is found that the dependence of gravitational Casimir energy arising from the quadruple moments on the GWs frequencies mainly give expressed to the post-Newtonian expansion coefficients. While the mass variation induced gravitational Casimir energy depends on GW frequencies scaling as ω_n^5. The gravitational susceptibility $\chi(\omega)$ caused by the GEM sources depends on the mass density of the objects, which is considerable for low frequencies [10]. The mass density of the compact objects tightly relates to its temperature T, according to $\frac{\Delta M}{M} \sim -(k_B \mathrm{T})^4$ [19]. So, it remarks that the dependence of gravitational Casimir energy arising from the mass variation on the GW frequencies reflects the temperature dependence of GCE, which scales as $E_{Cas}^{E} \sim \mathrm{T}^{20}$.

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Refrences:-

[1] G. Plunien, B. Muller and W. Greiner, Phys. Rept. 134, 87-193 (1986)
[2] V. M. Mostepaneko and N. N. Trunov, The Casimir effffect and its applications (Clarendon press, Oxford,1997)
[3] H. B. G. Casimir, Proc. Kon. Ned. Akad. Wet. 51, 793 (1948)
[4] H. B. G. Casimir and D. Polder, Phys. Rev. 73, 360-372 (1948)
[5] E. M. Lifshitz, Sov. Phys. JETP 2, 73-83 (1956) [Zh. Eksp. Teor. Fiz. 29, 94 (1956)]
[6] G. Feinberg and J. Sucher, Phys. Rev. A 2, 2395-2415 (1970)
[7] J. S. Schwinger, L. L. DeRaad, Jr. and K. A. Milton, Annals Phys. 115, 1-23 (1979)
[8] A. Bulgac, P. Magierski and A. Wirzba, Phys. Rev. D 73, 025007 (2006)
[9] T. Emig, N. Graham, R. L. Jaffffe and M. Kardar, Phys. Rev. Lett. 99, 170403 (2007)
[10] J. Q. Quach, Phys. Rev. Lett. 114, no.8, 081104 (2015) [erratum: Phys. Rev. Lett. 118, no.13, 139901 (2017)]
[11] O. Kenneth and I. Klich, Phys. Rev. Lett. 97, 160401 (2006)
[12] P. Hays, Annals Phys. 121, 32-46 (1979)
[13] J. Ambjorn and S. Wolfram, Annals Phys. 147, 1-32 (1983)
[14] M. Bordag, U. Mohideen and V. M. Mostepanenko, Phys. Rept. 353, 1-205 (2001)
[15] F. Sorge, Class. Quant. Grav. 22, 5109-5119 (2005)
[16] F. Sorge, Class. Quant. Grav. 26, 235002 (2009)
[17] F. Sorge, Class. Quant. Grav. 36, no.23, 235006 (2019)
[18] J. Wang, [arXiv:2202.10772 [gr-qc]].
[19] O. Panella and A. Widom, Phys. Rev. D 49, 917-922 (1994)
[20] J. Wang, Phys. Lett. B 827, 136980 (2022)
[21] S. Bonazzola and J. A. Marck, Ann. Rev. Nucl. Part. Sci. 44, 655-717 (1994)
[22] R. Maartens and B. A. Bassett, Class. Quant. Grav. 15, 705 (1998)
[23] B. Mashhoon, F. Gronwald and H. I. M. Lichtenegger, Lect. Notes Phys. 562, 83-108 (2001)
[24] M. L. Ruggiero and A. Tartaglia, Nuovo Cim. B 117, 743-768 (2002)
[25] J. Schwinger, Lett. Math. Phys., 1, 43-47 (1975)
[26] R. G. Newton, Scattering Theory of Waves and Particles (Springer-Verlag, Berlin, 1982); D. B. Pearson, Quantum Scattering and Spectral Theory (Academic, New York, 1988)
[27] B. S. DeWitt, Phys. Rept. 19, 295-357 (1975).