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RESEARCH ARTICLE

E-SOFT - A VERY SIMPLE AND INNOVATIVE METHOD FOR SOLVING UNBALANCED ASSIGNMENT PROBLEMS

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Abstract

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In this paper, we have proposed a noveland innovative method entitled „E-SOFT‟ fordetermining the optimal assignment plans to the unbalanced assignment problems (UAPs) and viewed its performance with the existing Hungarian method and the Mantra technique. The ESOFT algorithm has beeninheritedfrom the existing SOFTMIN algorithm developed for solving transportation problems and also extended for solving the UAPs.The "Extended SOFTMIN" method is in short termed as „E-SOFT‟ method. The performance of the proposed E-SOFT method over the Hungarian and Mantra methods has been tested on a set of 20 identified UAPs. Experimental results validate that the E-SOFT is an alternative simple method to solve UAPs.

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Introduction:-

The readers of this article know the basics of the assignment problems (APs) and the popular Hungarian method of solving them. The Hungarian method was developed byKuhnH.W. [3]in 1955 and it has been the efficient method for solving the balanced and unbalanced APs.Howsoever, in the recent years various methods,based on zeros assignment approach as well as based on ones assignment approach,have been developed to solve the balanced and unbalanced TPs separately.

In 2021, we have cited all the papers regarding solving the unbalanced APs published during the period 2006-2019 and briefly explained in the paper titled, MANTRA – A Simplified Hungarian Method for finding Optimal Solution of Unbalanced Assignment Problems [4]. MANTRA techniqueis more efficient and very simple and has produced optimal solution to a given unbalanced AP in a less amount of time compared to the Hungarian method.

In the same year 2021, we introduced a new, innovative and efficient zeros allocation method named SOFTMIN (**S**um **O**f**F**irst **T**hree **Min**imum elements) [2] to solve transportation problems. In 2022, weproved through some challenging transportation problems that the SOFTMIN method performs much better than the IASM method (Improved ASM method) [8]projected by us in 2020. As an assignment problem is a special case of the transportation problem, the methods developed to solve the transportation problems can also be extended to solve the assignment problems. Accordingly, we have developed this paper titled E-SOFT (Extended SOFTMIN) by inheriting the salient features of the existing SOFTMIN method.

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In transportation problems, methods such as MODI (**Mo**dified **Di**stribution Method) and SS (**S**tepping **S**tone Method) [9] are available to test the optimality of an initial basic feasible solution obtained through any available method and it can also be improved towards optimal, if it is not optimal. Butsuch an optimality testing and improving method is not available in the case of assignment problems. Really, such an optimality testingand improving method was not necessitated because the Hungarian method has produced optimal solution to a given AP. Howsoever, in 2022 we introduced a new method named CASSI (**C**ustomized **Assi**gnment) [6] which tests the optimality of an assignment solution obtained by an assignment method and improves it towards optimal, if it is not optimal. This was necessitated because certain published methods, such as TERM [5], in APs have not produced the optimal solutions directly to certain problems. In the literature of assignment problems, the CASSI method is of its first kind, which can be used for optimality testing and optimizing a solution obtained through any method based on the zeros assignment approach. Similarly, an equivalent method named MASS (**M**odified **Ass**ignment) [1], which can be used for optimality testing and optimizing a solution obtained by means of any method based on ones assignment approach. The researchers in this field can realize that MASS, MANTRA and CASSI are thethree distinguished methods in the literature of assignment problems.

Organization of this article is as follows: Section 1 – Briefs an introduction. Section 2 – Presents the algorithms of the existing Hungarian and Mantra methods. Section 3 – Presents the algorithm of the proposed E-SOFT method. Section 4 – Solves an unbalanced AP using the said three methods. Section 5 – Lists a set of 20 identified unbalanced APs. Section 6 – Lists the experimental results produced by the said three methods followed by a discussion. Section 7 – Ends with a conclusion.

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2. Algorithms Of The Existing "Hungarian Method" And The "Mantra Technique"

In this section, the algorithms of the existing Hungarian method and the existingMANTRAtechnique have been presented. Before presenting the algorithms, we give the key terminologies related to them.

1.Balanced and Unbalanced AP

An AP is said to be balanced (BAP) if its matrix with the assignment costs has equal numbers of rows and columns; otherwise, it is said to be unbalanced (UAP). That means, an AP is unbalanced if $(m<n)$ or $(n <m)$, where $m \times n$ denote the size of the assignment cost matrix.

As the existing focused two methods generate the solution to an AP based on the reduced cost matrix, first we explain its derivation from the given assignment cost matrix.

2. Row Minimum Subtraction (RMS) operation.

Select the minimum element from each row and subtract it from each element in the corresponding row so that each row will contain at least one 0-entry.

3. Column Minimum Subtraction (CMS) operation.

Select the minimum element from each column and subtract it from each element in the corresponding column so that each column will contain at least one 0-entry.

4. Reduced Cost Matrix (RCM).

The matrix derived by applying the RMS and or CMS operations on the assignment cost matrix of the given AP is called the reduced cost matrix (RCM). It is obvious that there will be at least one 0-entry in each row and in each column of an RCM. In an RCM, the cells with only 0-entries are called 0-entry cells.

Also, for the existing MANTRA technique as the individual assignments on the RCM are done based on the ME Rules, they are explained next.

5.'Me Rules'to Cover All The 0-Entries with Minimum Number of Lines

The word ME is coined from the first letter of the names of the designed authors **M**urugesan and **E**sakkiammal.The advantage of the 'ME Rules' is that it ensures an assignment before drawing a horizontal or vertical line to cover certain zeros. That is, first make an assignment at a 0-entry cell, followed by draw a horizontal or a vertical line through that 0-entry cell.

Rule #1: To draw Minimum number of Lines to cover all zeros
a) Row-wise assignment

1. Look at the rows successively from first to last until a row with exactly one 0-entry is found.
2. Make an assignment to this single 0-entry by marking with a star symbol or by creating a circle or square around it.
3. Draw a vertical line passing through that 0-entry.
4. Continue in this way until all the rows have been scrutinized.
5. After scrutinizing the last row, check whether all the 0s are covered with the drawn lines. If yes, go to Rule (2); otherwise, do column-wise assignment.

b) Column-wise assignment

1. Look at the columns successively from first to last until a column with exactly one unassigned 0-entry is found.
2. Make an assignment to this single 0-entry by marking with a star symbol or by creating a circle or a square around it.
3. Draw a horizontal line passing through that 0-entry.
4. Continue in this way until all the columns have been scrutinized.
5. After scrutinizing the last column, check whether all the 0s are covered with the overall drawn lines. If yes, go to Rule (2); otherwise, do row-wise and column-wise assignments, if possible. Then go to Rule (2)

//* 6.Complete assignment plan

By a complete assignment plan (or a feasible solution or simply a solution) for an unbalanced assignment cost matrix of size m × n, we mean an assignment plan or program or pattern or policy containing exactly k (where k = Min{m, n}) assigned independent 0s, one in each of the k rows and one in each of the k columns.

7.Conditions to be testedto attain a complete assignment plan

A complete assignment plan for an UAP is said to be achieved if the following two conditions are satisfied:

1. Minimum number of lines drawn to cover all the 0-entries must be exactly equal to k.
2. Each of the row (k rows among m rows) and each of the column (k columns among n columns) of the assignment matrix must have a unique 0s assignment. ***//**

Rule #2: To test the conditions for a complete assignment

Test whether the conditions for a complete assignment is achieved. If yes, write the complete assignment plan and compute the corresponding overall assignment cost; otherwise, select the smallest element (say d_{ij}) out of those which do not lie on any of the lines drawn in the RCM. Then subtract by d_{ij} from each element of the uncovered row only or column only in which d_{ij} lies on it. If tie occurs in selecting d_{ij}, then select the one corresponding to the least original assignment cost for obtaining better assignment plan. If tie occurs still, then select the cell at random. This operation creates some new 0s to this row or column. Thus, we obtain a further reduced cost matrix, called Revised RCM. Then, go to Rule #1.

If the conditions for a complete assignment are not satisfied through the above said two rues, then apply Rule #3.

Rule #3:

After performing the row-wise assignment and column-wise assignment completely as far as possible, if more than one 0-entries are present in certain rows and columns, then

1. Select any one 0-entry arbitrarily and make an assignment to that 0-entry by marking with a star symbol or by creating a circle or square around it.
2. Draw a horizontal line through the row of the assigned 0-entry and put an X mark on all the remaining 0s on the column of that assigned 0-entry. (Or) Draw a vertical line through the column of the assigned 0-entry and put an X mark on all the remaining 0s on the row of that assigned 0-entry.
3. Repeat (i) and (ii) until the conditions for a complete assignment are satisfied.

Such a situation creates an alternative assignment plan to the given UAP.

2.1 Algorithm of the existing "Hungarian for UAP"

Hungarian method,developed by KuhnH.W. [6]in 1955, is an efficient method for solving balanced and unbalanced APs. This method is based on the principle that "if a constant is added to, or subtracted from, every element of a row and/or a column of the given cost matrix of an AP, the resulting AP has the same optimal solution as the original AP. For deriving the solution to a givenunbalanced AP, the following steps are used.

Step 1: Conversion into Minimization UAP.
If the given UAP with size m × n is of maximization type, then convert it into a minimization one.

Step 2: Conversion into Balanced AP.
Add required numbers of dummy row(s) if m < n or add required numbers of dummy column(s) if n < m. The assignment cost in each of the cells in the dummy row(s) or dummy column(s) is assumed to be zero. Let N be the size of the resulting balanced AP.

Step 3: Derivation of an RCM.
On the balanced AP with size N perform the RMS operation only, if (m < n) or perform the CMS operation only if(n < m) to obtain a reduced cost matrix (RCM)

Step 4: Cover all the 0-entries.
Cover all the 0-entries in the obtained balanced RCM by using onlyminimum numbers of horizontal and vertical lines.

Step 5: Test for Optimality
If the minimum number of covering lines is N, an optimal assignment plan is possible and go to step 7 for making the individual assignments.Else, if lines are lesser than N, an optimal assignment plan is not found and must proceed to step 6.

Step 6: Determine the smallest entry not covered by any line. Subtract this entry from each uncovered entries, and then addit to each of the entries at the intersection of a horizontal and a vertical line.Keep the remaining entries as it is. Return to step 4.

Step 7: Examine the row successively until a row-wise exactly single zero is found, encircle this zero to make the assignment and mark cross (×) over all zeros in that column. Continue in this manner until all the rows have been examined. Repeat the same procedure for columns also.

Step 8: Repeat the Step 7 successively until one of the following situations arise: If no un-circledand uncrossed zero is left, then the assignment process is over; or if there lie more than one un-circled zeros in any column or row, then encircle anyone of the un-circled zeros arbitrarily and cross over all other zeros lying in that row or column. Repeat the process until no un-circled and uncrossed zero is left in the reduced cost matrix.

Step 9: Thus, in each row and in each column exactly one encircled zero is obtained. The sum of original assignment costs corresponding to these encircled zeros will give an overall optimal assignment cost.

Important Note:
In Step 4 of the Hungarian algorithm, it is given just that "cover all the 0-entries using minimum numbers of horizontal and vertical lines". The technique of how to draw the horizontal and vertical lines to cover all the 0entries is not provided. Because of this, most of the teachers (including myself!), students and scholars find it difficult to cover all the 0-entries present in an RCM and thereby they say that "the Hungarian algorithm fails to produce an optimal solution to certain APs". To overcomethisdifficulty, we have designed a set of three simple rules, named "ME Rules", to cover all the 0-entries using minimum numbers of horizontal and vertical lines. Consequently, we are able to get the optimal solution to a given AP using the Hungarian algorithm. Really, Hungarian algorithm is an efficient one to solve all kinds of APs.

2.2 Algorithm of the existing "Mantra for UAP"
"Mantra" is a simplified form of the existing Hungarian method of solving balanced and unbalanced assignment problems, which successfully derives the optimal solution to a given unbalanced assignment problem (UAP). The term "Mantra" consist of two words, "man" or mind and "tra" the technique that quiets the mind and brings consciousness in bliss for those who are practicing it. Likewise, the simplified technique "Mantra" brings optimal solution to UAPs, for those who are applying it.

The salient features of the „Mantra" technique is as follows: In the Hungarian method of solving, it is required to convert the given UAP into a balanced one by introducing required numbers of dummy row(s) or column(s) having zero effectiveness in each of the dummy cells. But, the "Mantra" technique can be applied directly on the given UAP without converting it into a balanced one. Also, the "Mantra" technique consumes less amount of time to solve an UAP compared to the Hungarian method. The following are the three simple steps involved in "Mantra for UAP"

Step 1: Conversion into Minimization UAP.
If the given UAP is of maximization type, then convert it into a minimization one.

Step 2: Find the "minimum of" number of rows and columns of the assignment cost matrix and decide the operation.
If m × n is the size of the given assignment cost matrix, find the minimum of m and n, in brief Min{m, n}. Let Min{m, n} = k

a) If m < n, then perform the RMS operation to obtain an RCM.
b) If n < m, then perform the CMS operation to obtain an RCM.

Step 3: Build the assignments one by one on the RCM by applying the ME Rules.
Apply the ME rules on the obtained RCMin order to place theindividual assignments.

3. Algorithm for the Proposed "E-SOFTfor UAP"
This algorithm is quite different from the existing algorithms of Hungarian and Mantra.It inherits certainsalient features fromthe SOFTMIN method developed for solving transportation problems and extends it for solving unbalanced APs. The "Extended SOFTMIN" method is simply named as "E-SOFT" method.

In Hungarian and Mantra, individual assignments are made at the appropriate 0-entry cells based on covering all the 0-entries in an RCM by drawing minimum numbers of horizontal and vertical lines. But, in the proposed E-SOFT method every individual allocation is placed at a 0-entry cell on the basis of the **S**um **O**f**F**irst **T**hree **min**imum (SOFTMIN) elements computed for every row and every column of the RCM derived from the given UAP. The following are the sequence of four steps involved in it:

Step 1: Conversion into Minimization UAP.
If the given UAP with size m × n is of maximization type, then convert it into a minimization one.

Step 2: Conversion into Balanced AP.
Add required numbers of dummy row(s) only if m < n or add required numbers of dummy column(s) only if n < m. The assignment cost in each of the cells in the dummy row(s) or dummy column(s) is assumed to be zero. Let N be the size of the balanced AP.

Step 3: Derivation of an RCM.
On the balanced AP with size N, perform the RMS operation only if (m < n) orperform the CMS operation only if (n < m) to obtain an RCM.

Step 4:Build the Assignmentsone by one in the RCM by computing "Soft Min" elements

1. For each row, find the **s**um **o**f **f**irst **t**hree (soft) **min**imum (min) elements. Write the resulting sum under the Soft Minelements column by enclosing it in parentheses against the respective row. Similarly, do the same computation for each column.
2. Mark by *, the maximum among the Soft Min elements computed for rows and columns, along the corresponding row(s) and/ column(s).
3. Making the assignments

a. Select the row or column which is marked by * and assign at the cell having unique0-entry in that row or column.
b. If tie occurs among certain 0-enty cells in that selected row or column, then select the 0-entry cell which has the least original assignment cost figure for assignment.
c. If tie occurs among the least original assignment cost figure, then consider each such 0-entry cell for assignment as a separate case and finally choose the best assignment plan among them. Such a situation may produce an alternative assignment plan to the given UAP.

Priority Rules

1. In Step 4(iii-b or c), while selecting the 0-entry cell for assignment corresponding to the least original assignment cost figure, give priority to the dummy cell when all the non-dummy cells have NONZERO assignment costs.
2. While selecting a 0-entry cell for assignment corresponding to the least original assignment cost figure, give priority to the non-dummy cell when at least one non-dummy cell has ZERO assignment cost.

Specific Cases

When an RCM is of size 2 × 2, then there can be three possibilities (two 0-entries, three 0-entries, four 0-entries) of 0-entry cells in it.

1. If the RCM has only two 0-entry cells, which are at diagonally opposite positions, then select both the cells for two individual assignments.
2. If the RCM has three 0-entry cells, then select only the two cells which are at diagonally opposite positions for two individual assignments.
3. If the RCM has all four 0-entry cells, then selectany two cells which are at diagonally opposite positions for two individual assignments. Such a situation creates alternative assignment plans

4. Numerical Illustration

Suitable illustrative explanation helps the readers to understand the difference in the process of assignments by the E-SOFT, Hungarian and Mantra algorithms. Keeping in mind, one benchmark unbalanced AP from the literature has been illustrated.

Example: Consider the following profit maximization type unbalanced AP with six jobs and four machines, as given in Table 1.

Table 1:- The given maximization UAP.

Job	**Machine**			
	1	**2**	**3**	**4**
1	3	6	2	6
2	7	1	4	4
3	3	8	5	8
4	6	4	3	7
5	5	2	4	4
6	5	7	6	2

4.1 Solution by The Proposed E-SOFT Method

Conversion to Minimization UAP

As the given UAP is of maximization category, first we convert it into a minimization one. This is done by subtracting all the profit values from the highest profit value of 8. The equivalent minimization UAP is shown in Table 2.

Table 2:- The equivalent minimization UAP.

Job	**Machine**			
	1	**2**	**3**	**4**
1	5	2	6	2
2	1	7	4	4
3	5	0	3	0
4	2	4	5	1
5	3	6	4	4
6	3	1	2	6

Conversion to balanced AP

As the given UAP has 6 rows and 4 columns, we add two dummy columns say Machine 5 and Machine 6 with 0 assignment cost in the cells. The resulting balanced AP is shown in Table 3.

Table 3:- The resulting balanced AP.

Job	Machine					
	1	**2**	**3**	**4**	**5**	**6**
1	5	2	6	2	0	0
2	1	7	4	4	0	0
3	5	0	3	0	0	0
4	2	4	5	1	0	0
5	3	6	4	4	0	0
6	3	1	2	6	0	0

Perform the CMS operation

In each column, identify the smallest cost and subtract the same from every cost of that column. The resulting reduced cost matrix (RCM #1) is shown in Table 4.

Table 4:- The RCM #1.

Job	Machine					
	1	**2**	**3**	**4**	**5**	**6**
1	4	2	4	2	0	0
2	0	7	2	4	0	0
3	4	0	1	0	0	0
4	1	4	3	1	0	0
5	2	6	2	4	0	0
6	2	1	0	6	0	0

Make the 1st allocation

By applying the Step 4(i) – (ii) of the proposed E-SOFT algorithm, one can get the Soft Min elements distinguished under the "Soft Min elements" column and "Soft Min elements" row as shown in Table 5. The maximum amongst the overall Soft Min elements (I) is 3 (and is marked with *), which corresponds to the first four columns (machines). By applying Step 4(iii) in thesefour columns, we see the tie breaking situations as shown in Table 6.

Among the four 0-entry cells, the cells(3, 2) and (3, 4) are with the least original assignment cost figure zero. Therefore, each of these two cells is considered for assignment as a separate case and finally the best assignment plan is taken. We consider the cell (3, 2) as the first case and the cell (3, 4) as the second case.

Case 1: Assign at the cell (3, 2)

We place the 1st assignment at the cell (3, 2), marked with **†,**as shown in Table 5.

Table 5:- The RCM #1.

Job	Machine						Soft Min elements-I
	1	**2**	**3**	**4**	**5**	**6**	
1	4	2	4	2	0	0	(2)
2	0	7	2	4	0	0	(0)
3	4	**0†**	1	0	0	0	(0)
4	1	4	3	1	0	0	(1)
5	2	6	2	4	0	0	(2)
6	2	1	0	6	0	0	(0)
Soft Min elements-I	**(3)** *	**(3)** *	**(3)** *	**(3)** *	(0)	(0)	

Table 6:- Tie breaking situation during the 1st assignment.

Column #	0-entry cell	Original assignment cost	Nature of cell
1	(2, 1)	1	Non-dummy
2	**(3, 2)**	**0**	Non-dummy

3	(6, 3)	2	Non-dummy
4	(3, 4)	**0**	Non-dummy

Make the 2nd assignment

Now, delete the 3rd row and the 2nd column from RCM #1. After the deletions, observe that the resultant matrix is not an RCM because of the 4th column. By applying the CMS operation, we can get further reduced cost matrix, say RCM #2. In RCM #2, the computed Soft Min elements (II) for the rows and columns are shown in Table 7. The maximum amongst the overall Soft Min elements (II) is 4 (and is marked with *), which corresponds to the 3rd and 4th columns (machine 3 and 4). As tie occurs among these columns, by applying Step 4(iii), in these columns we locate the 0-entry cell for which the original assignment cost figure is the least. The located 0-entry cells with theirassignment cost figures are shown in Table 7.

Table 6:- The RCM #2.

Job		**Machine**					Soft Min elements		
		1	**3**	**4**	**5**	**6**	II	III	IV
1		4	4	1	**0†**	0	(1)	**(4)***	
2		**0†**	2	3	0	0	(0)	(0)	(2)
4		1	3	**0†**	0	0	(0)		
5		2	2	3	0	**0†**	(2)	(2)	**(4)***
6		2	**0†**	5	0	0	(0)	(0)	(2)
Soft Minelements	II	(3)	**(4)** *	**(4)** *	(0)	(0)			
	III	**(4)** *	**(4)** *		(0)	(0)			
	IV	**(4)** *	**(4)** *			(0)			

Table 7:- Tie breaking situation during the 2nd assignment.

Column #	0-entry cell	Original assignment cost	Nature of cell
1	(6, 3)	2	Non-dummy
4	**(4, 4)**	**1**	Non-dummy

In Table 7, among the two 0-entry cells, the cell (4, 4) has the least assignment cost. Therefore, the cell (4, 4) is selected for the 2nd assignment and is made (marked with †) as shown in Table 6.

Make the 3rd assignment

Now, delete the 4th row and the 4th column from RCM #2. After the deletions, observe that the resultant matrix is an RCM only. In that RCM, the computed Soft Min elements (III) for the rows and columns are shown in Table 6. The maximum amongst the overall Soft Min elements (III) is 4 (and is marked with *), which corresponds to the 1st and 3rd columns and the 1st row. As tie occurs among these columns and row, by applying Step 4(iii), in these columns and row we identify the 0-entry cell for which the original assignment cost figure is the least. The identified 0-entry cells with their original assignment cost figures are shown in Table 8.

Table 8:- Tie breaking situation during the 3rd assignment.

Column #/ Row #	0-entry cell	Original assignment cost	Nature of cell
1	(2, 1)	2	Non-dummy
3	(6, 3)	1	Non-dummy
1	(1, 5)	0	Dummy
	(1, 6)	0	Dummy

As in Table 8, among the four 0-entry cells, the cells (1, 5) and (1, 6) are dummy in nature have zero assignment costs and the other two non-dummy cells have non-zero assignment costs. Therefore,by Priority Rule #1 we select a dummy cell for the assignment. We select the dummy cell (1, 5) arbitrarily for the 3rd assignment. This is shown in Table 6.

Make the 4th assignment

At present, delete the 1st row and the 5thcolumn. Due to the deletions the resultant matrix remains an RCM. As usual, the tie breaking situation is shown in Table 9.

Table 9:- Tie breaking situation during the 4th assignment.

Column #/ Row #	0-entry cell	Original assignment cost	Nature of cell
1	(2, 1)	1	Non-dummy
3	(6, 3)	2	Non-dummy
5	(5, 6)	0	Dummy

Among the three 0-entry cells, the cell (5, 6) is dummy in nature have zero assignment cost and the other two non-dummy cells have non-zero assignment cost. Therefore, by Priority Rule #1 we select the dummy cell (5, 6) for the 4thassignment. This is shown in Table 6.

Make the 5th and 6thallocations

At present, delete the 5th row and the 6th column. Due to the deletions the resultant matrix remains an RCM but, with size 2 × 2. Therefore, the remaining assignments are very obvious as specified by the Specific cases. Thereby, the 5th and 6th assignments are made at the diagonally opposite cells (2, 1) and (6, 3) respectively as shown in Table 6. Now, the assignment process is over.

Write the assignment plan

The obtained assignment plan by applying the proposedE-SOFT method is shown in Table 10. As a result, the maximumoverall profit computed is $28. Interestingly, for verification one can test the optimality of this solution by the Phase-II of the existing CASSI method [6].

Table 10:- Assignment plan with the maximumoverall profit by the E-SOFT method.

Job	Machine	Original Profit
1	5 (or 6)	--
2	1	7
3	2	8
4	4	7
5	6 (or 5)	--
6	3	6
Overall profit due to assignment		**28**

Case 2: Assign at the cell (3, 4)

If we make the 1stassignment at the cell (3, 4), then it will lead to the assignment plan 1 – 5 or 6, 2 – 1, 3 – 4, 4 – 6 or 5, 5 – 3, 6 – 2 with the overall profit of $26 only.

Decision

Comparing the two assignment plans, we decide that the assignment plan, shown in Table 6, is the best (optimal)assignment plan with the maximumoverall profit of $28 for the given maximization UAP.

Solution by the Hungarian Method

By applying the steps of the Hungarian method, we obtain the same identical optimal assignment plan, as shown in Table 10, to the given maximization UAP.

Solution by the Mantra Technique

By applying the steps of the Mantratechnique, we obtain the same identical optimal assignment plan, (without the dummy machines 5 and 6) as shown in Table 10, to the given maximization UAP.

Decision

For the given maximization UAP, the existing Hungarian and Mantra methods as well as the proposed E-SOFT method have produced the same identical optimal assignment plan with the maximum verall profit of $28.

Numerical Examples

To justify the efficiency of the proposed „E-SOFT method, we have solved a set of 20 numbers of benchmark APs of unbalanced category in different sizes, from various literature and textbooks, which are listed in Table 11.

Table 11:- List of benchmark unbalanced APs for testing.

Problem #	Problem #
Problem 1 $[C_{ij}]$ 3×4= [18 24 28 32; 8 13 17 19; 10 15 19 22]	**Problem** 11 $[C_{ij}]$ 7×10= [21 11 16 9 15 10 12 32 26 16; 14 15 20 10 16 3 6 9 21 14; 9 17 11 31 21 16 7 9 10 11; 16 23 8 15 10 3 6 3 20 23; 12 40 14 36 9 21 14 19 4 13; 8 18 9 42 8 11 19 9 32 20; 21 9 12 9 32 10 19 25 116 10]
Problem 2 $[C_{ij}]$ 4 ×3= [21 14 7; 15 10 5; 15 10 5; 12 8 4]	**Problem** 12 $[C_{ij}]$ 8×4= [53 62 42 89; 18 35 39 55; 93 80 91 83; 79 23 96 56; 43 16 12 20; 87 70 87 31; 35 79 25 59; 27 16 1220]
Problem 3 $[C_{ij}]$ 5×4= [9 14 19 15; 7 17 20 19; 9 18 21 18; 10 12 18 19; 10 15 21 16]	**Problem** 13 $[C_{ij}]$ 8×5= [300 290 280 290 210; 250 310 290 300 200; 180 190 300 190 180; 320 180 190 240 170; 270 210 190 250 160; 190 200 220 190 140; 220 300 230 180 160; 260 190 260 210 180]
Problem 4 $[C_{ij}]$ 5×6= [10 8 13 20 16 6; 8 16 23 13 14 10; 9 8 1 6 3 7; 4 12 8 11 11 10; 6 10 9 5 11 8]	**Problem** 14 $[C_{ij}]$ 10×4= [11 8 9 8; 4 5 29 33; 10 5 29 33; 1 18 25 31; 23 22 33 30; 3 9 13 19; 6 8 27 32; 32 30 39 38; 36 35 31 21; 15 11 10 28]
Problem 5 $[C_{ij}]$ 5×6= [80 140 80 100 56 98; 48 64 94 126 170 100; 56 80 120 100 70 64; 99 100 100 104 80 90; 64 90 90 60 60 70]	**Problem** 15 $[P_{ij}]$ 5×4= [13 15 12 14; 12 14 10 12; 16 18 14 14; 15 15 13 13; 16 15 14 12]
Problem 6 $[C_{ij}]$ 5×8= [300 250 180 320 270 190 220; 290 310 190 180 210 200 300; 280 290 300 190 190 220 230; 290 300 190 240 250 190 180; 210 200 180 170 160 140 160]	**Problem** 16 $[C_{ij}]$ 5×6= [12 3 6 -- 5 9; 4 11 -- 5 -- 8 ; 8 2 10 9 7 5; -- 7 8 6 12 10; 5 8 9 4 6 1]
Problem 7 $[C_{ij}]$ 6×4= [6 5 1 6; 2 5 3 7; 3 7 2 8; 7 7 5 9; 12 8 8 6; 6 9 5 10]	**Problem** 17 $[P_{ij}]$ 4×3= [11 8 8; 4 33 5; 10 33 5; 1 25 10]
Problem 8 $[C_{ij}]$ 6×5= [6 2 5 2 6; 2 5 8 7 7; 7 8 6 9 8; 6 2 3 4 5; 9 8 9 7; 9 7 4 6 8]	**Problem** 18 $[P_{ij}]$ 4×5= [62 78 50 101 82; 71 84 61 73 59; 87 92 111 71 81; 48 64 87 77 80]

Problem 9 $[C_{ij}]$ 6×10= [10 2 14 9 6 7 21 32 18 11; 7 12 9 3 5 6 9 16 54 12; 4 8 6 12 21 9 21 14 45 13; 21 9 12 9 32 10 19 25 16 10; 10 12 30 15 12 17 30 12 12 9; 15 7 34 17 7 16 14 17 9 5]	**Problem** 19 $[P_{ij}]$ 5×4= [13 15 12 14; 12 14 10 12; 16 18 14 14; 15 15 13 13; 16 15 14 12]
Problem 10 $[C_{ij}]$ 7×6= [126 207 254 245 214 243; 229 238 242 228 213 285; 118 253 306 218 245 216; 172 247 218 248 217 243; 309 207 105 136 194 139; 99 168 220 140 215 116; 95 174 168 145 249 98]	**Problem** 20 $[P_{ij}]$ 6×4= [3 6 2 6; 7 1 4 4; 3 8 5 8; 6 4 3 7; 5 2 4 4; 5 7 6 2]

Note: Problems numbered with 1-16 are UAP of minimization case, and 17-20 are UAP of maximization case. 16 is of restricted assignments.

Evaluationand Analysis

To measure the effectiveness of the proposed E-SOFT assignment technique, 20 benchmark instances, listed in Table 11, have been tested and the results are compared with the optimal results obtained by Hungarian and Mantra methods. The comparison of results is shown in Table 12.

Table 12:- Comparison of solutions by E-SOFT with optimal solutions.

Prob. #	**E-SOFT**	**Hungarian/**	**Mantra/ Optimal**	**Prob. #**	**E-SOFT**	**Hungarian/**	**Mantra/ Optimal**
1.	50		50	**11.**	43		43
2.	27		27	**12.**	73		73
3.	54		54	**13.**	870		870
4.	30*		28	**14.**	24		24
5.	328*		326	**15.**	61		61
6.	870		870	**16.**	18		18
7.	16*		15	**17.**	54		54
8.	16		16	**18.**	376		376
9.	28		28	**19.**	61		61
10.	881		881	**20.**	28		28

Analysis As of Table 12, we conclude that out of 20 unbalanced TPs tested the" Hungarian" method as well as the "Mantra" technique have produced optimal assignment plans directly to all 20 problems. But, the proposed E-SOFT method has produced optimal assignment plans directly to 17 problems only and assignment plans very close to the optimal assignment plans to 3 problems (problems numbered with 4, 5 and 7).

Decision

Therefore, we conclude that for a given unbalanced AP the proposed E-SOFT method produces either an optimal assignment plan or an assignment plan very close to the optimal assignment plan.

Remedy for the Near-Optimal Solution

Howsoever, by applying the Phase #2 (Optimality testing and optimizing the obtained solution) of the existing CASSI method [6] one can improve thenon-optimal assignment plans (of the problems numbered with 4, 5 and 7)towards the optimal assignment plans.

Important Observation:-

It is observed that the problem numbered with 13 is the transpose of the problem numbered with 6. From the results of Table 12, we see that both the problems have the overall optimal assignment cost of $870. Hence, we conclude that a given assignment problem and its transpose produce the optimal assignment plans with the same identical optimaloverall assignment cost. This is due to the „strong duality theorem" found in the „relations between primal and dual".

Conclusion:-

In this research article, we have proposed a very simple and innovative method named "E-SOFT" to find optimal assignment plans to the unbalanced assignment problems.The E-SOFT algorithm has been inborn from the SOFTMIN algorithm developed for solving transportation problems and extended for solving the unbalanced assignment problems. The "Extended SOFTMIN" method is simply entitled as "E-SOFT" method. The performance of the proposed E-SOFT method over the existing Hungarian and Mantra methods has been experienced on a set of 20 identified unbalanced APs. Experimental outcomes authenticate that the E-SOFT is an alternative simple method to solve the unbalanced APs. Also, the proposed E-SOFT method can be used to solve balanced APs.

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