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RESEARCH ARTICLE

TWINS - AN ALTERNATIVE OPTIMAL TOUR PLANS GENERATION METHOD TO TRAVELING SALESMAN PROBLEMS

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Abstract

A traveling salesman problem (TSP) is an assignment problem (AP) with extra conditions, in which the aim is to find the best possible way of visiting all the planned cities only once and returning back to the opening point that minimize the travel distance (or time or cost) on the whole. In the modern world, TSP occupies a major role. The eminent method used to solve the TSPs is the Hungarian method. In this research article, we have identified and established a list of TSPs which are having alternative optimal tour plans (OTPs). For the identified TSPs, the alternative OTPs are generated by applying the proposed TWINS method. The main aim of developing alternative OTPs by the management is to have the best possible decision out of the available alternative solutions. Solution alternatives also can make it easier to make use of these options in the future during disaster situations. The alternatives identified become a base for subsequent analysis by the management and eventually the decision itself.

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Introduction:-

The TSP is a problem in combinatorial optimization deliberated in operations research. A set of cities is given to a salesman and he has to start from a city, visit all the cities only once and return back to the start city to complete a round tour such that the length of the tour is the shortest among all possible tour plans. Because the TSP is a particular class of Assignment Problem (AP) with extra conditions, a solution to the TSP, in general, is found using the methods existing for solving APs. In the recent years significant number of zeros assignment methods and ones assignment methods have been published by quite a few researchers for solving APs. In this section, we briefly bring in the important zeros and ones assignment methods published so far recently.

During May 2014, Hadi Basirzadeh [2] presented an approach namely, Ones Assignment Method for solving the TSP, by a little modification in the procedure given in [1] to obtain a tour of the TSP. In this method, priority rule plays a vital role in order to assign the 1-entries.

In June 2016, Mohammed Ahmed Shihab Alkailany [5] presented a New Revised Ones Assignment Method to solve TSP and the author claims that the results of the tests show that this method is better than the Ones Assignment Method. But, the solutions generated by this method for the TSPs shown in Example 1 and Example 3 of this paper have cycles and hence not feasible to the given TSPs. This method has not provided any rule to make a tour plan when the solution consisting cycles.

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In June 2019, Janusz Czapik [4] offered a new polynomial time algorithm which uses the Hungarian algorithm for AP to solve TSP. The running time of this algorithm is as the algorithm of AP. This algorithm removes the cycles generated by the Hungarian algorithm, but does not guaranteed to yield an optimal solution to a TSP. By this algorithm, Example 3 shown in this paper has got a tour plan with length $Z = 34$ units instead the optimal tour plan generated by our algorithm [6, 8] with the minimum length $Z = 26$ units.

In the same year 2019, Susanta Kumar Mohanta [7] projected a direct approach to discover an optimal solution to the TSP in a single shot with a rational amount of time from the network of a complete graph, complete digraph or connected graph.

R. Murugesan and T. Esakkiammal [6], in February 2021, presented a new two phase method named ESA-ESAN for generating the optimal tour plans (OTPs) of TSPs based on ones assignment approach.

T. Esakkiammal and R. Murugesan [8], in April 2021, introduced another new two phase method named ESAN-ESA for producing the OTPs of TSPs based on zeros assignment approach.

In this article, we have identified and established a set of TSPs which are having alternative OTPs. The generation of the alternative OTPs is done by applying the ESAN-ESA and ESA-ESAN methods.

This article is well-designed as follows: In Section 1, the basic information about the TSP has been recited. In Section 2, the algorithm of the new proposed TWINS method for generating the alternative OTPs to TSPs is presented. In Section 3, one benchmark problem, having alternative OTPs, from the literature has been illustrated. A set of 10 benchmark TSPs (having alternative OTPs) identified from the literature has been tabulated in Section 4. The OTPs generated by the proposed TWINS method on the 10 benchmark TSPs and consequently the alternative OTPs identified are tabulated in Section 5. The article ends with the drawn conclusion in Section 6.

Length of a Tour For a complete round tour or simply a tour to an n city TSP, the salesperson travels exactly n arcs (or n edges). Addition of the values (distances or costs) in every arc of a tour yields the length of the tour. The length of a tour may be in terms of time or distance or currency units.

Optimal Tour A tour with the minimum length is called a least length tour or an optimal tour. The length of an optimal tour is denoted by the symbol Z . It is clear that, $Z \leq$ Length of a tour.

Lower Bound (LB) Addition of the minimum values in every row of a TSP provides a lower bound (LB) for Z .

Tighter Lower Bound (TLB) It is well-known that the TSP is a restricted version (having added constraints) of the AP. Therefore, we relax it temporarily by removing the added constraints and solve it as an AP. If the optimal assignment plan to the AP is feasible to the TSP, then it would have been the optimal tour plan to the TSP. If it is infeasible to the TSP, then the overall total distance (or cost) corresponding to the optimal assignment plan is a lower bound to the optimal value Z . This lower bound is, in general, tighter than the existing LB for Z and therefore, it is called as tighter lower bound (TLB). Note that, $LB \leq TLB \leq Z$.

Upper Bound (UB) Sum of the average values in every row of a TSP gives an upper bound (UB) for Z . Note down that, always $Z \leq$ Length of a tour \leq UB.

Relation between the Bounds and the Length of a Tour

The relation between the various bounds and the length of a tour is given by

$$LB \leq TLB \leq Z \leq \text{Length of a tour} \leq UB.$$

However, the bounding condition $TLB \leq Z$ is not true for very few instances.

Algorithm of the proposed TWINS method

The name TWINS is crown to the proposed method as it contains two matching methods namely ESAN-ESA and ESA-ESAN. This section presents the algorithms of the ESAN-ESA method due to T. Esakkiammal and R. Murugesan [8] and the ESA-ESAN method due to R. Murugesan and T. Esakkiammal [6], to solve the TSPs.

Algorithm of the ESAN-ESA Method

The term ESAN-ESA is coined from the names of the authors Murugesan and Esakkiammal. The ESAN-ESA method consists of two phases. In the first phase, a set of optimal subtours is generated using the 'zeros assignment approach' and in the second phase linking of all the subtours together to form a complete round tour is carried out based on the available 0-entry or 1-entry or the next higher entry among the upcoming unassigned cells. The algorithm is as follows:

Phase-I (Generating subtours)

Step 1: Construct the Reduced Cost Matrix (RCM).

- a. **Perform the Row Minimum Subtraction (RMS) Operation.** Subtract from each of the costs of every row of the TSP by its minimum cost. This will result in a reduced cost matrix, which will have at least one zero in each row. If each row and each column of the reduced cost matrix has at least one 0s, go to Step 2 to cover all the 0s entry; otherwise, go to Step 1(b).
- b. **Perform the Column Minimum Subtraction (CMS) Operation.** Subtract from each of the entries of every column of the resultant cost matrix obtained in Step 1(a) by its minimum entry. These operations create at least one zero in each column. Go to Step 2 to cover all the 0s entry.

Note: The resultant matrix obtained in Step 1(b) is known as the reduced cost matrix (RCM). It is noted that there will be at least one 0-entry in each row and in each column of an RCM. The cells having only 0-entry in an RCM are called 0-entry cells.

Step 2: Cover all the 0s with minimum number of lines using 'ME rules' to obtain the complete assignments

The readers may refer Appendix-A for the details of ME rules, by assuming '0-entry' in the place of '1-entry'.

Phase-II

(Linking of all the subtours together aptly to form a complete round tour)

Now we will link all the subtours together rightly to form a complete round tour. The linking operation is done by following the below steps:

Step 1: Consider the 'subtour of shortest length'. If more than one subtour of shortest length occurs, then consider the subtour arbitrarily.

Step 2: In the shortest length subtour, start the tour plan from the first city and travel ahead and identify the finally visited city (not the start city) and let it be i . In the i^{th} city (i^{th} row) of the assignment table, look for the city (row), not visited already, with the existing next 0-entry (or 1-entry or next higher entry). Let it be j .

Step 3: Identify the subtour in which the city j lies and continue the journey from city j in the sequence given in the subtour in which j lies.

Step 4: If all the cities are visited only once, then return back to the start city; otherwise, spot the finally visited city in the subtour in which j lies. Let it be city k . In the k^{th} city (k^{th} row) of the assignment table, look for the city (row), not visited already, with the existing next 0-entry (or 1-entry or next higher entry). Let it be l .

Step 5: Repeat Steps (3) and (4) until a complete round tour with size n (a feasible solution) has been obtained to the given TSP. Compute the length of the tour.

Step 6: In the shortest length subtour, start the tour plan from the second city and travel ahead and identify the finally visited city (not the start city) and let it be p . In the p^{th} city (p^{th} row) of the assignment table, look for the city (row), not visited already, with the next existing 0-entry (or 1-entry or next higher entry). Let it be q . Repeat Steps (4) and (5) until a complete round tour with size n (a feasible solution) has been obtained to the given TSP. Compute the length of the tour.

Step 7: Repeat Steps (3) to (6) until a complete round tour with size n (a feasible solution) has been obtained to the given TSP starting from the last but one city in the shortest length subtour. Among the generated complete round tours, identify the tour with minimum length and is the optimal tour plan to the given TSP.

Algorithm of the ESA-ESAN Method

The phrase ESA-ESAN has been coined from the names of the authors Esakkiammal and Murugesan. The ESA-ESAN method consists of two phases. In the first phase, a set of optimal subtours is generated using the 'ones assignment approach' and in the second phase connecting of all the subtours together to form a complete round optimal tour is carried out based on the least entry, just greater than 1, among the upcoming unassigned cells. The algorithm is as follows:

Phase-I**(Generating subtours)****Step 1: Construct the Ratio of Distances Matrix (RDM).****a) Perform the Row Minimum Division (RMD) Operation.**

Divide each of the distances of every row of the TSP by its smallest distance. This will result in a ratio matrix, which will have at least one 1s in each row. If each row and each column of the ratio matrix has at least one 1s, go to Step 2 to cover all the 1s entry; otherwise, go to Step 1(b).

b) Perform the Column Minimum Division (CMD) Operation.

Divide each of the entries of every column of the ratio matrix obtained in Step 1(a) by its least entry. These operations create at least one 1s in each column. Go to Step 2 to cover all the 1s entry.

Note: The ensuing matrix obtained in Step 1(b) is known as the ratio of distances matrix (RDM). It is well-known that there will be at least one 1-entry in each row and in each column of an RDM. The cells having only 1-entry in an RDM are called 1-entry cells.

Step 2: Cover all the 1s with minimum number of lines using ‘ME Rules’ to obtain the complete assignments

The readers may refer Appendix-A for the details of ‘ME rules’.

Phase-II**(Connecting of all the subtours properly to form a complete round tour)**

Now we will link all the subtours together properly to form a complete round tour. The linking operation is completed by following the below steps:

Step 1: Consider the ‘subtour of shortest length’. If more than one subtour of shortest length occurs, then consider the subtour arbitrarily.

Step 2: In the shortest length subtour, start the tour plan from the first city and travel ahead and spot the finally visited city (not the start city) and let it be i . In the i^{th} city (i^{th} row) of the assignment table, look for the city (row), not visited already, with least entry 1 or least entry immediately after 1. Let it be j .

Step 3: Identify the subtour in which the city j lies and go on the journey from city j in the sequence given in the subtour in which j lies.

Step 4: If all the cities are visited only once, then return back to the start city; otherwise, spot the finally visited city in the subtour in which j lies. Let it be city k . In the k^{th} city (k^{th} row) of the assignment table, look for the city (row), not visited already, with least entry immediately after 1. Let it be l .

Step 5: Repeat Steps (3) and (4) until a complete round tour with size n (a feasible solution) has been obtained to the given TSP. Calculate the length of the tour.

Step 6: In the shortest length subtour, start the tour plan from the second city and travel ahead and recognize the finally visited city (not the start city) and let it be p . In the p^{th} city (p^{th} row) of the assignment table, look for the city (row), not visited already, with least entry immediately after 1. Let it be q . Repeat Steps (4) and (5) until a complete round tour with size n (a feasible solution) has been obtained to the given TSP. Calculate the length of the tour.

Step 7: Repeat Steps (3) to (6) until a complete round tour with size n (a feasible solution) has been obtained to the given TSP starting from the last but one city in the shortest length subtour. Among the generated complete round tours, identify the tour with least length and is the optimal tour plan to the given TSP.

Note 1: In Steps (2) and (4) of Phase-II, if there is tie in bringing the least entry cell into the assignment, then consider each such least entry cell one by one separately for making assignment and choose the best tour obtained.

Note 2: If the consideration of ‘subtour with shortest length’ at first does not produce optimal tour plan, then consider another subtour whose length is equal to or greater than the length of the already considered subtour and follow the steps of Phase-II.

Remark: In Section 2.1 and 2.2, the steps involved in Phase-II seem to identical and repeats. But, there are differences in Step 2 and Step 4 of each.

Illustrative TSP

Exact illustrative explanation makes the readers to be familiar with the proposed TWINS method systematically. Thereby, the objective of this article – finding alternative optimal tour plans (OTPs) – is achieved. Keeping in mind, one TSP from the literature has been illustrated.

Example: Consider the following 5-city symmetric TSP, whose distance (in miles) matrix is shown in Table 1.

Table 1:- The given 5-city TSP.

| City | City | | | | |
|------|------|----|----|----|----|
| | 1 | 2 | 3 | 4 | 5 |
| 1 | -- | 10 | 8 | 9 | 7 |
| 2 | 10 | -- | 10 | 5 | 6 |
| 3 | 8 | 10 | -- | 8 | 9 |
| 4 | 9 | 5 | 8 | -- | 6 |
| 5 | 7 | 6 | 9 | 6 | -- |

Solution by the Twins Method

As the proposed TWINS method consisting of two matching methods namely ESAN-ESA and ESA-ESAN, we solve the given 5-city TSP by applying them one by one.

Solution by the ESAN-ESA method

By applying the steps of Phase-I, one can get the subtours $1 \rightarrow 3 \rightarrow 1$; $2 \rightarrow 4 \rightarrow 5 \rightarrow 2$ (OR) the tour plan $1 \rightarrow 3 \rightarrow 4 \rightarrow 5 \rightarrow 2 \rightarrow 1$ with length 38 miles. The generated subtours along with their length are shown in Table 2.

Table 2:- Subtours and their length.

| Subtour | Length of Subtour (in miles) |
|---|------------------------------|
| $1 \rightarrow 3 \rightarrow 1$ | $8 + 8 = 16$ |
| $2 \rightarrow 5 \rightarrow 4 \rightarrow 2$ | $6 + 6 + 5 = 17$ |
| Tighter Lower Bound (TLB) | 33 |

The least length subtour is $1 \rightarrow 3 \rightarrow 1$, which is considered first in Phase-II.

By applying the steps of Phase-II, one can have the following tours along with their length, which are shown in Table 3:

Table 3:- Generated tours and their length.

| Subtour | Start city | Generated Tour | Length of Tour (in miles) |
|---------------------------------|------------|---|---------------------------|
| $1 \rightarrow 3 \rightarrow 1$ | 1 | $1 \rightarrow 3 \rightarrow 4 \rightarrow 5 \rightarrow 2 \rightarrow 1$ | $8 + 8 + 6 + 6 + 10 = 36$ |
| | 3 | $3 \rightarrow 1 \rightarrow 5 \rightarrow 2 \rightarrow 4 \rightarrow 3$ | $8 + 7 + 6 + 5 + 8 = 34$ |

Writing the OTP: As of Table 3, we see that, the tour with least length is 34. Consequently, the OTP is $3 \rightarrow 1 \rightarrow 5 \rightarrow 2 \rightarrow 4 \rightarrow 3$ (equivalently, $1 \rightarrow 5 \rightarrow 2 \rightarrow 4 \rightarrow 3 \rightarrow 1$) with $Z = 34$.

Solution by the ESA-ESAN method

By applying the steps of Phase-I, the produced subtours are:

$1 \rightarrow 3 \rightarrow 1$; $2 \rightarrow 5 \rightarrow 4 \rightarrow 2$ (OR) the tour plan $1 \rightarrow 3 \rightarrow 4 \rightarrow 5 \rightarrow 2 \rightarrow 1$ with length 38 miles. The generated subtours along with their lengths are shown in Table 4.

Table 4:- Subtours and their length.

| Subtour | Length of Subtour (in miles) |
|---|------------------------------|
| $1 \rightarrow 3 \rightarrow 1$ | $8 + 8 = 16$ |
| $2 \rightarrow 5 \rightarrow 4 \rightarrow 2$ | $6 + 6 + 5 = 17$ |
| Tighter Lower Bound (TLB) | 33 |

The minimum length subtour is $1 \rightarrow 3 \rightarrow 1$, which is considered first in Phase-II.

By applying the steps of Phase-II, one will have the following tours along with their length, which are shown in Table 5:

Table 5:- Generated tours and their length.

| Subtour | Start city | Generated Tour | Length of Tour (in miles) |
|---------------------------------|------------|---|---------------------------|
| $1 \rightarrow 3 \rightarrow 1$ | 1 | $1 \rightarrow 3 \rightarrow 4 \rightarrow 2 \rightarrow 5 \rightarrow 1$ | $8 + 8 + 5 + 6 + 7 = 34$ |
| | 3 | $3 \rightarrow 1 \rightarrow 4 \rightarrow 2 \rightarrow 5 \rightarrow 3$ | $8 + 9 + 5 + 6 + 9 = 37$ |

Writing the OTP: From Table 5, we see that, the tour with shortest length is 34 miles. Therefore, the OTP is 1 → 3 → 4 → 2 → 5 → 1 with Z= 34.

Decision:-

As of the proposed TWINS method, the OTPs generated on the whole are: 1 → 5 → 2 → 4 → 3 → 1; 1 → 3 → 4 → 2 → 5 → 1. Hence, the given TSP has two separate OTPs. In other words, the given TSP has alternative OTPs.

Benchmark TSPs

In order to validate the objective of this article, we have identified and solved ten (nine classical and one non-classical) benchmark TSPs, in different relatively small sizes, having alternative OTPs, from a range of literature and textbooks, which are shown in Table 6. A TSP is said to be ‘classical’, if each pair of given cities is connected by a desired mode of travel.

Table 6:- List of benchmark TSPs having alternative optimal tour plans.

| TSP | TSP |
|--|---|
| Problem 1 [C _{ij}] _{5×5} = [-- 16 4 12 --; 16 -- 6 -- 8; 4 6 – 5 6; 12 -- 5 – 20; -- 8 6 20 --] | Problem 6 [C _{ij}] _{5×5} = [-- 17 16 18 14; 17 – 18 15 16; 16 18 – 19 17; 18 15 19 – 18; 14 16 17 18 --] |
| Problem 2 [C _{ij}] _{5×5} = [-- 3 6 2 3; 3 -- 5 2 3; 6 5 – 6 4; 2 2 6 – 6; 3 3 4 6 --] | Problem 7 [C _{ij}] _{6×6} = [-- 13 2 15 15 15; 13 -- 14 1 12 12; 2 14 – 16 14 14; 15 1 16 – 10 10; 15 12 14 10 – 4; 15 12 14 10 4 --] |
| Problem 3 [C _{ij}] _{5×5} = [-- 10 8 9 7; 10 -- 10 5 6; 8 10 – 8 9; 9 5 8 – 6; 7 6 9 6 --] | Problem 8 [C _{ij}] _{6×6} = [-- 1 5 4 -- --; 1 -- 2 2 -- 2; -- 2 – 1 4 1; 4 2 1 – 2 2; -- -- 4 2 – 3; -- 2 1 2 3 --] |
| Problem 4 [C _{ij}] _{5×5} = [-- 10 12 14 8; 10 -- 13 8 9; 12 13 – 12 8; 14 8 12 – 11; 8 9 8 11 --] | Problem 9 [C _{ij}] _{6×6} = [-- 12 29 22 13 24; 12 -- 19 3 25 6; 29 19 – 21 23 28; 22 3 21 – 4 5; 13 25 23 4 – 16; 24 6 28 5 16 --] |
| Problem 5 [C _{ij}] _{5×5} = [-- 8 4 9 9; 8 -- 6 7 10; 4 6 – 5 6; 9 7 5 – 4; 9 10 6 4 --] | Problem10 [C _{ij}] _{7×7} = [-- 86 49 57 31 69 50; 86 -- 68 79 93 24 5; 49 68 – 16 7 72 67; 57 79 16 – 90 69 1; 31 93 7 90 – 86 59; 69 24 72 69 86 – 81; 50 5 67 1 59 81 --] |

Outcomes Analysis

On behalf of every one of the ten numbers of TSPs, listed in Table 6, the subtours generated by Phase-I along with their lengths and the OTPs generated by Phase-II along with their least lengths as a result of ESAN-ESA method are shown in Table 7 and that of by the ESA-ESAN method are shown in Table 8.

Table 7:- Subtours and the resultant OTPs produced by ESAN-ESA method.

| Prob. No. # | Subtours generated by Phase-I | Length of Subtours | OTPs generated by Phase-II | Length of Tour (Z) |
|-------------|--|------------------------------------|--|--------------------|
| 1. | 1 → 3 → 4 → 1 2 → 5 → 2 | 21 16 | 2 → 5 → 3 → 4 → 1 → 2 | 47 |
| 2. | 1 → 4 → 2 → 1 3 → 5 → 3 | 07 08 | 3 → 5 → 1 → 4 → 2 → 3 | 16 |
| 3. | 1 → 3 → 1 2 → 4 → 5 → 2 (Alternative subtours) 1 → 3 → 1 2 → 5 → 4 → 2 | 16 17 16 17 | 3 → 1 → 5 → 2 → 4 → 3 1 → 3 → 4 → 2 → 5 → 1 | 34 34 |
| 4. | 1 → 5 → 3 → 1 2 → 4 → 2 | 28 16 | 4 → 2 → 1 → 5 → 3 → 4 | 46 |
| 5. | 1 → 2 → 3 → 1 4 → 5 → 4 | 18 08 | 4 → 5 → 3 → 1 → 2 → 4 | 29 |

| | | | | |
|-----|---------------|-----------|-------------------------------|-----|
| 6. | 1 → 3 → 5 → 1 | 47 | 4 → 2 → 5 → 1 → 3 → 4 | 80 |
| | 2 → 4 → 2 | 30 | | |
| | 1 → 5 → 3 → 1 | 47 | 2 → 4 → 3 → 1 → 5 → 2 | 80 |
| | 2 → 4 → 2 | 30 | | |
| 7. | 1 → 3 → 1 | 04 | 2 → 4 → 6 → 5 → 3 → 1 → 2 | 44 |
| | 2 → 4 → 2 | 02 | | |
| | 5 → 6 → 5 | 08 | | |
| 8. | 1 → 2 → 1 | 02 | 2 → 1 → 4 → 5 → 6 → 3 → 2 | 13 |
| | 3 → 6 → 3 | 02 | | |
| | 4 → 5 → 4 | 04 | | |
| 9. | 1 → 5 → 1 | 26 | 4 → 6 → 2 → 3 → 1 → 5 → 4 | 76 |
| | 2 → 3 → 2 | 38 | | |
| | 4 → 6 → 4 | 10 | | |
| 10. | 1 → 3 → 5 → 1 | 87 | 7 → 4 → 3 → 5 → 1 → 6 → 2 → 7 | 153 |
| | 2 → 6 → 2 | 48 | | |
| | 4 → 7 → 4 | 02 | | |

Table 8:- Subtours and the resultant OTPs generated by ESA-ESAN method.

| Prob. No. # | Subtours generated by Phase-I | Length of Subtours | OTPs generated by Phase-II | Length of Tour (Z) |
|-------------|-------------------------------|--------------------|-------------------------------|--------------------|
| 1. | 1 → 4 → 3 → 1 | 21 | 5 → 2 → 1 → 4 → 3 → 5 | 47 |
| | 2 → 5 → 2 | 16 | | |
| 2. | 1 → 4 → 1 | 04 | 4 → 1 → 5 → 3 → 2 → 4 | 16 |
| | 2 → 5 → 3 → 2 (OR) | 12 | | |
| | 1 → 5 → 3 → 2 → 4 → 1 | 16 | | |
| 3. | 1 → 3 → 1 | 16 | 1 → 3 → 4 → 2 → 5 → 1 | 34 |
| | 2 → 5 → 4 → 2 | 17 | | |
| 4. | 1 → 3 → 5 → 1 | 28 | 2 → 4 → 3 → 5 → 1 → 2 | 46 |
| | 2 → 4 → 2 | 16 | | |
| 5. | 1 → 3 → 2 → 1 | 16 | 5 → 4 → 2 → 1 → 3 → 5 | 29 |
| | 4 → 5 → 4 | 08 | | |
| 6. | 1 → 5 → 3 → 1 | 47 | 2 → 4 → 3 → 1 → 5 → 2 | 80 |
| | 2 → 4 → 2 | 30 | | |
| 7. | 1 → 3 → 1 | 04 | 2 → 4 → 5 → 6 → 3 → 1 → 2 | 44 |
| | 2 → 4 → 2 | 02 | | |
| | 5 → 6 → 5 | 08 | | |
| 8. | 1 → 2 → 1 | 02 | 1 → 2 → 3 → 6 → 5 → 4 → 1 | 13 |
| | 3 → 6 → 3 | 02 | | |
| | 4 → 5 → 4 | 04 | | |
| 9. | 1 → 3 → 1 | 58 | 4 → 5 → 1 → 3 → 2 → 6 → 4 | 76 |
| | 2 → 6 → 2 | 12 | | |
| | 4 → 5 → 4 | 08 | | |
| 10. | 1 → 5 → 3 → 1 | 87 | 4 → 7 → 2 → 6 → 1 → 5 → 3 → 4 | 153 |
| | 2 → 6 → 2 | 48 | | |
| | 4 → 7 → 4 | 02 | | |

Table 9 displays the OTPs produced by the proposed TWINS method for the identified ten benchmark TSPs shown in Table 6. Note that, each TSP has two distinct OTPs.

Table 9:- Alternative OTPs for the TSPs listed in Table 6.

| Prob. No. # | OTPs generated by the focused two algorithms | Length of Tour (Z) | Number of OTPs |
|-------------|--|--------------------|----------------|
| 1. | 2 → 5 → 3 → 4 → 1 → 2 | 47 | 2 |
| | 5 → 2 → 1 → 4 → 3 → 5 | 47 | |

| | | | |
|-----|--|------------|---|
| 2. | 4 → 1 → 5 → 3 → 2 → 4 3 → 5 → 1 → 4 → 2 → 3 | 16 16 | 2 |
| 3. | 3 → 1 → 5 → 2 → 4 → 3 1 → 3 → 4 → 2 → 5 → 1 | 34 34 | 2 |
| 4. | 4 → 2 → 1 → 5 → 3 → 4 2 → 4 → 3 → 5 → 1 → 2 | 46 46 | 2 |
| 5. | 4 → 5 → 3 → 1 → 2 → 4 5 → 4 → 2 → 1 → 3 → 5 | 29 29 | 2 |
| 6. | 4 → 2 → 5 → 1 → 3 → 4 2 → 4 → 3 → 1 → 5 → 2 | 80 80 | 2 |
| 7. | 2 → 4 → 6 → 5 → 3 → 1 → 2 2 → 4 → 5 → 6 → 3 → 1 → 2 | 44 44 | 2 |
| 8. | 2 → 1 → 4 → 5 → 6 → 3 → 2 1 → 2 → 3 → 6 → 5 → 4 → 1 | 13 13 | 2 |
| 9. | 4 → 6 → 2 → 3 → 1 → 5 → 4 4 → 5 → 1 → 3 → 2 → 6 → 4 | 76 76 | 2 |
| 10. | 7 → 4 → 3 → 5 → 1 → 6 → 2 → 7 4 → 7 → 2 → 6 → 1 → 5 → 3 → 4 | 153 153 | 2 |

Reasons for choosing the ESAN-ESA and ESA-ESAN methods for generating tour plans of TSPs

By means of our research articles [6, 8] in TSP, we have proved through 30 benchmark TSPs of different smaller sizes (size 4 to size 10) from the literature and textbooks that ESAN-ESA and ESA-ESAN methods are the most efficient ones which produce optimal tour plans directly to all 30TSPs. Therefore, in the proposed TWINS method, the two matching methods ESAN-ESA and ESA-ESAN are chosen for generating tour plans for a TSP.

Novelty in terms of methodology in the proposed TWINS method

The generated set of subtours by Phase-I of each of the matching methods are connected together exactly in Phase-II to form OTPs by considering the 'subtour of shortest length' first. By starting the tour plan from the first (or next) city of this subtour and travel ahead along the sequence given in this subtour and connecting of all the remaining subtours together to form a tour plan is carried out based on the available 0-entry or 1-entry or the next available immediate higher entry among the upcoming unassigned cells in the associated reduced cost matrix. In this way, the subtours are connected rightly to form a tour plan by removing the cycles (subtours) easily.

How is the proposed TWINS method different from the existing methods in TSP?

In the proposed TWINS method, we have seen that how the cycles (subtours) are removed to form a single cycle (or a tour plan) rightly and easily. Thereby, the proposed method is entirely different from the existing methods and the difference found in each is given below:

1. In Hungarian Method: The given TSP considered as an AP is solved by the Hungarian method of assignment [3], thereby an optimal solution is obtained. If this optimal solution violates the condition that salesman can visit each city only once, then one looks for the 'next best' solution by bringing the next (non-zero) cost element 1 along with the zero elements into the solution. If more than one cost element 1 occurs, then each such 1 is considered separately until a feasible solution to the TSP is obtained.
2. In Ones Assignment Method: In Ones Assignment Method [2], priority rule plays an important role to make a tour.
3. In the New Revised Ones Assignment Method: In the New Revised Ones Assignment Method [5], no clear cut rule is given to make a tour from the solution consisting cycles.
4. In the Direct Approach: In the Direct Approach [7], assignments are made based on choosing a suitable assignment preference table consisting of absolute favorable costs of from cities or to cities and their frequencies.
5. In the Application of Hungarian Algorithm to solve TSP [4]: The given TSP considered as an AP is solved by the Hungarian method of assignment, by this means an optimal solution is obtained. If this optimal solution contains more cycles, then it is removed by constructing a Modifying Distance (Cost or Time) Matrix and then solving the same by Hungarian Algorithm in order to obtain an optimal solution with only one cycle. This algorithm removes the cycles generated by the Hungarian algorithm, but does not guaranteed to yield an optimal solution to a TSP.

Important real life applications of TSP

The foremost objective of any management is to 'minimize the overall cost' and to 'maximize the overall profit' to its organization. In this direction the whole subject Operations Research is very much helpful in achieving the objectives. As the areas of study such as Transportation Problems, Assignment Problems and Traveling Salesman Problems are the main branches of Operations Research, they are very much useful and helpful to attain the objective of the management. The following are the important real life direct applications of the TSP reported and established through various researchers: Printing press scheduling problem, School bus routing problem, Crew scheduling problem, Interview scheduling problem, Hot rolling scheduling problem, Mission planning problem, Design of global navigation satellite system surveying networks, Drilling of printed circuit boards, Overhauling gas turbine engines, X-ray crystallography, Computer wiring, The order picking problem in warehouses, Vehicle routing, and Mask plotting in PCP production.

Conclusion:-

In this research paper, we have identified a set of ten benchmark TSPs and confirmed that each of the TSPs is having alternative optimal tour plans. The optimal tour plans and their alternatives have been generated through the proposed method named TWINS. Hence, it is definite that by applying the TWINS method one can generate (optimal tour plans and) alternative optimal tour plans to a given TSP, provided it exists. For the management, the alternatives identified become the basis for subsequent analysis and finally the decision itself.

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APPENDIX-A

'ME Rules' for covering all 1s by Lines

The word ME is coined from the first letter of the names of the authors Murugesan and Esakkiammal. In this section, we present a new set of rules, called 'ME rules', of how to draw the minimum number of horizontal and vertical lines to cover all the 1s of a reduced cost matrix and also the conditions for a complete assignment.

Rule 1: To draw Minimum number of Lines to cover all 1s

- a) Row-wise assignment
 - (i) Look at the rows successively from first to last until a row with exactly one 1 entry is found.
 - (ii) Make an assignment to this single 1 entry by creating a circle or square around it.
 - (iii) Draw a vertical line passing through that 1 entry.
 - (iv) Continue in this way until all the rows have been scrutinized.
 - (v) After scrutinizing the last row, check whether all the 1s are covered with the drawn lines. If yes, go to Rule (2); otherwise, do column-wise assignment.
- b) Column-wise assignment

- (i) Look at the columns successively from first to last until a column with exactly one unassigned 1 entry is found.
- (ii) Make an assignment to this single 1 entry by creating a circle or a square around it.
- (iii) Draw a horizontal line passing through that 1 entry.
- (iv) Continue in this way until all the columns have been scrutinized.
- (v) After scrutinizing the last column, check whether all the 1s are covered with the overall drawn lines. If yes, go to Rule (2); otherwise, do row-wise and column-wise assignments, if possible. Then go to Rule (2)

/* By a 'complete assignment' for a square cost matrix of order n , we mean an assignment plan or program containing exactly n assigned independent 1s, one in each row and one in each column. A complete assignment plan for an AP is said to be 'achieved' if the following two conditions are satisfied:

- (a) Minimum number of lines drawn must be exactly equal to n , the order of the assignment matrix.
- (b) Each row and each column of the assignment matrix must have a unique ones assignment. */

Rule 2: To test the conditions for a complete assignment

Test whether the conditions for complete assignment is achieved. If yes, write the 'complete assignment plan' and compute the corresponding 'overall cost (or profit)'; otherwise, select the smallest (largest) element (say d_{ij}) out of those which do not lie on any of the lines in the above matrix. Then divide by d_{ij} each element of the uncovered row(s) or column(s), which d_{ij} lies on it. This operation creates some new ones to this row or column and hence a revised RCM is obtained. Then, go to Rule 1.

If the conditions for a complete assignment are not satisfied through the above said two rules, then apply rule 3.

Rule 3:

After performing the row-wise assignment and column-wise assignment completely as far as possible in the revised RCM, if more than one 1s are present in certain rows and columns, then

1. Select any one 1 entry arbitrarily and make an assignment to that 1 entry by creating a circle or square around it.
2. Draw a horizontal line through the row of the assigned 1 entry and put an X mark on all the remaining 1s on the column of that assigned 1 entry. (Or) Draw a vertical line through the column of the assigned 1 entry and put an X mark on all the remaining 1s on the row of that assigned 1 entry.
3. Repeat (i) and (ii) until the conditions for a complete assignment are satisfied.

The situation of applying Rule 3 creates an alternative assignment plan to the given AP.