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RESEARCH ARTICLE

T-SOFT - AN ADDITIONAL OPTIMAL TOUR PLAN FORMATION METHOD TO TRAVELING SALESMAN PROBLEMS

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Abstract

A traveling salesman problem (TSP) is an assignment problem (AP) with additional conditions, in which the intent is to find the best possible way of visiting all the planned cities only once and returning back to the starting city that minimizes the travel distance (or time or cost) on the whole. In the modern world with all the transportation facilities, TSP plays a most important role. The well-known method used to solve the TSPs is the Hungarian method. In this research article, we have identified and established a list of TSPs which are having additional 'optimal tour plans' (OTPs). For the identified TSPs, the OTP and the additional OTP are formed by applying the proposed T-SOFT method. The central aspire of developing additional OTPs by the administration is to have the best possible decision during the critical and disaster situations. Thereby, the additional OTPs become a base for subsequent analysis by the administration and eventually the decision itself.

Introduction:-

The traveling salesman problem (TSP) is a problem in combinatorial optimization deliberated in operations research. A set of cities is given to a salesman and he has to start from a city, visit all the cities only once and return back to the start city to complete a round tour such that the length of the tour is the shortest among all possible tour plans. Because the TSP is a particular class of Assignment Problem (AP) with extra conditions, a solution to the TSP, in general, is found using the methods available for solving APs. In the recent years significant number of zeros assignment methods and ones assignment methods have been published by quite a few researchers for solving APs. In this section, we briefly bring in the important zeros and ones assignment methods published so far recently.

During May 2014, Hadi Basirzadeh [3] presented an approach namely, Ones Assignment Method for solving the TSP, by a little modification in the procedure given in [1] to obtain a tour of the TSP. In this method, priority rule plays a vital role in order to assign the 1-entries.

In June 2016, Mohammed Ahmed Shihab Alkailany [5] presented a New Revised Ones Assignment Method to solve TSP and the author claims that the results of the tests show that this method is better than the Ones Assignment Method. But, the solutions generated by this method for the TSPs shown in Example 1 and Example 3 of this paper have cycles and hence not feasible to the given TSPs. This method has not provided any rule to make a tour plan when the solution consisting cycles.

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In June 2019, Janusz Czopik [4] offered a new polynomial time algorithm which uses the Hungarian algorithm for AP to solve TSP. The running time of this algorithm is as the algorithm of AP. This algorithm removes the cycles generated by the Hungarian algorithm, but does not guaranteed to yield an optimal solution to a TSP. By this algorithm, Example 3 shown in this paper has got a tour plan with length Z = 34 units instead the optimal tour plan generated by our algorithm [1, 6] with the minimum length Z = 26 units.

In the same year 2019, Susanta Kumar Mohanta [10] projected a direct approach to discover an optimal solution to the TSP in a single shot with a rational amount of time from the network of a complete graph, complete digraph or connected graph.

Murugesan R. and Esakkiammal T. [6], in February 2021, presented a new two phase method named ESA-ESAN for generating the 'optimal tour plans' (OTPs) of TSPs based on 'ones assignment approach'.

Esakkiammal T. and Murugesan R. [1], in April 2021, introduced another new two phase method named ESAN-ESA for producing the OTPs of TSPs based on 'zeros assignment approach'.

In May 2022, Murugesan R. [7] proposed a novel and innovative method entitled 'E-SOFT' for determining the optimal assignment plans to the assignment problems based on zeros assignment approach.

In June 2022, Muugesan R. [8] introduced a new method named 'TWINS' which generates the OTP and also an alternative OTP to the given TSP, if it exists.

In this article, we have identified and established a set of TSPs which are having additional OTPs. The generation of the OTP and the additional OTP is done by applying the proposed T-SOFT method.

This article is well-designed as follows: Section 1 presents the basic information about the current research in TSP. Section 2 presents the algorithm of the existing E-SOFT method. Section 3 presents the algorithm of the proposed T-SOFT method for generating the OTP (and also an additional OTP, if it exists) to TSPs. Section 4 illustrates two benchmark TSPs having additional OTPs, from the literature. Section 5 tabulates a set of 10 benchmark TSPs (having additional OTPs) identified from the literature. Section 6 tabulates the possible OTPs generated by the proposed T-SOFT method on the 10 benchmark TSPs. Section 7 draws the conclusion.

Length of a Tour For a complete round tour or simply a tour to an n city TSP, the salesperson travels exactly n arcs (or n edges). Addition of the values (distances or costs) in every arc of a tour yields the length of the tour. The length of a tour may be in terms of time or distance or currency units.

Optimal Tour A tour with the minimum length is called a least length tour or an optimal tour. The length of an optimal tour is denoted by the symbol Z. It is clear that, $Z \leq$ Length of a tour.

Lower Bound (LB) Addition of the minimum values in every row of a TSP provides a lower bound (LB) for Z.

Tighter Lower Bound (TLB) It is well-known that the TSP is a restricted version (having added constraints) of the AP. Therefore, we relax it temporarily by removing the added constraints and solve it as an AP. If the optimal assignment plan to the AP is feasible to the TSP, then it would have been the optimal tour plan to the TSP. If it is infeasible to the TSP, then the overall total distance (or cost) corresponding to the optimal assignment plan is a lower bound to the optimal value Z. This lower bound is, in general, tighter than the existing LB for Z and therefore, it is called as tighter lower bound (TLB). Note that, $LB \leq TLB \leq Z$.

Upper Bound (UB) Sum of the average values in every row of a TSP gives an upper bound (UB) for Z. Note down that, always $Z \leq$ Length of a tour $\leq UB$.

Relation between the Bounds and the Length of a Tour
The relation between the various bounds and the length of a tour is given by
$LB \leq TLB \leq Z \leq$ Length of a tour $\leq UB$.
However, the bounding condition $TLB \leq Z$ is not true for very few instances.

Algorithm for the Existing "E-SOFT FOR AP"

This algorithm is quite different from the existing Hungarian algorithm. It inherits certain salient features from the SOFTMIN method [7] developed for solving transportation problems and has been extended for solving APs. The "Extended SOFTMIN" method is simply named as "E-SOFT" method [7].

In Hungarian method, individual assignments are made at the appropriate 0-entry cells based on covering all the 0-entries in a reduced cost matrix (RCM) by drawing minimum numbers of horizontal and vertical lines. But, in the E-SOFT method every individual assignment is placed at a 0-entry cell on the basis of the **S**um **O**f **F**irst **T**hree **min**imum (SOFTMIN) elements computed for every row and every column of the RCM derived from the given AP. The following are the sequence of three steps involved in it:

As the focused method requires the operations of 'row minimum subtraction' and 'column minimum subtraction' on the cost matrix of the given AP to generate the 'reduced cost matrix', we explain them as follows.

Row Minimum Subtraction (RMS) operation

Select the minimum element from each row and subtract it from each element in the corresponding row so that each row will contain at least one 0-entry.

Column Minimum Subtraction (CMS) operation

Select the minimum element from each column and subtract it from each element in the corresponding column so that each column will contain at least one 0-entry.

Reduced Cost Matrix (RCM)

The matrix derived by applying the RMS and CMS operations on the cost matrix of the given AP is called the reduced cost matrix (RCM). It is obvious that there will be at least one 0-entry in each row and in each column of an RCM. In an RCM, the cells with only 0-entries are called '0-entry cells'.

Step 1: Conversion into Minimization AP.

If the given AP with size n is of maximization type, then convert it into a minimization one.

Step 2: Derivation of an RCM.

On the AP with size n, perform the RMS operation followed by the CMS operation to obtain an RCM.

Step 3: Build the assignments one by one in the RCM by computing 'Soft Min' elements

1. For each row, find the **s**um **o**f **f**irst **t**hree (soft) **min**imum (min) elements. Write the resulting sum under the 'Soft Min elements' column by enclosing it in parentheses against the respective row. Similarly, do the same computation for each column.
2. Mark by *, the maximum among the 'Soft Min elements' computed for rows and columns, along the corresponding row(s) and/ column(s).
3. Making the assignments

a. Select the row or column which is marked by * and assign at the cell having unique 0-entry in that row or column.

b. If tie occurs among certain 0-enty cells in that selected row or column, then select the 0-entry cell which has the least original assignment cost figure for assignment.

c. If tie occurs among the least original assignment cost figure, then consider each such 0-entry cell for assignment as a separate case and finally choose the best assignment plan among them. Such a situation may produce an alternative assignment plan to the given AP.

Specific Cases

When an RCM is of size 2 × 2, then there can be three possibilities (two 0-entries, three 0-entries, four 0-entries) of 0-entry cells in it.

1. If the RCM has only two 0-entry cells, which are at diagonally opposite positions, then select both the cells for two individual assignments.
2. If the RCM has three 0-entry cells, then select only the two cells which are at diagonally opposite positions for two individual assignments.

3. If the RCM has all four 0-entry cells, then select any two cells which are at diagonally opposite positions for two individual assignments. Such a situation creates alternative assignment plans.

3. Algorithm of the proposed T-SOFT method

The E-SOFT method developed for solving the assignment problems has been extended for solving the traveling salesman problems as well. The extended 'E-SOFT for TSP' is simply termed as 'T-SOFT'. The first letter T in the term 'T-SOFT' denotes Traveling salesman problem. The T-SOFT method identifies the possible optimal tour plans to a given TSP in two phases.

Phase-I (Generating subtours)

Step 1: Solve the given TSP as an AP

Consider the given TSP as an AP by relaxing the additional conditions imposed on it. Solve the AP by applying the 'E-SOFT for AP'.

Step 2: Write the assignment plan as subtours

Write the obtained optimal assignment plan as optimal subtours (cycles) one by one and compute the length of each subtour.

Phase-II

(Linking of all the subtours together suitably to form a complete round tour)

Now we will link all the subtours together rightly to form a complete round tour. The 'linking operation' or the 'action of linking' is carried out by following the below steps:

Step 1: Consider the 'subtour of shortest length'. If more than one subtour of shortest length occurs, then consider the subtour arbitrarily.

Step 2: In the shortest length subtour, start the tour plan from the first city and travel ahead and identify the finally visited city (not the start city) and let it be i. In the i^{th} city (i^{th} row) of the corresponding assignment table (RCM), look for the city (row), not visited already, having the next available 0-entry (or 1-entry or next higher entry) for the linkage. Let it be j.

Step 3: Identify the subtour in which the city j lies and continue the journey from city j in the sequence given in the subtour in which j lies.

Step 4: If all the cities are visited only once, then return back to the start city; otherwise, spot the finally visited city in the subtour in which j lies. Let it be city k. In the k^{th} city (k^{th} row) of the corresponding assignment table (RCM), look for the city (row), not visited already, having the next available 0-entry (or 1-entry or next higher entry) for the linkage. Let it be l.

Step 5: Repeat Steps (3) and (4) until a complete round tour with size n (a feasible solution) has been obtained to the given TSP. Compute the length of the tour.

Step 6: In the shortest length subtour, start the tour plan from the second city and travel ahead and identify the finally visited city (not the start city) and let it be p. In the p^{th} city (p^{th} row) of the corresponding assignment table (RCM), look for the city (row), not visited already, having the next available 0-entry (or 1-entry or next higher entry) for the linkage. Let it be q. Repeat Steps (4) and (5) until a complete round tour with size n (a feasible solution) has been obtained to the given TSP. Compute the length of the tour.

Step 7: Repeat Steps (3) to (6) until all possible complete round tours with size n (a feasible solution) have been generated to the given TSP starting from the 'first one' to 'last but one' city in the shortest length subtour. Among the generated complete round tours, identify the tour with the least length and is the optimal tour plan to the given TSP.

Important Note

In Step 4 and Step 6, if tie occurs among the next available 0-entries (or 1-entries or next higher entries), then consider each such 0-entries (or 1-entries or next higher entries) as a separate case for linkage while forming a complete round tour and finally choose the best tour plan among them.

4. Illustrative TSP

Exact illustrative explanation makes the readers to be familiar with the proposed T-SOFT method systematically. Thereby, the objective of this article – finding an additional optimal tour plan – is achieved. Keeping in mind, two TSPs from the literature has been illustrated.

Example1:

Consider the following 6-city symmetric TSP, whose distance (in miles) matrix is shown in Table 1.

Table 1:- The given 5-city TSP.

	City					
City	**1**	**2**	**3**	**4**	**5**	**6**
1	--	1	5	4	--	--
2	1	--	2	2	--	2
3	--	2	--	1	4	1
4	4	2	1	--	2	2
5	--	--	4	2	--	3
6	--	2	1	2	3	--

Solution by the T-SOFT Method

Phase-I (Generating subtours)

By considering the given TSP as an AP by relaxing the additional conditions imposed on it we solve the AP by applying the 'E-SOFT for AP'. The reduced cost matrix (RCM) along with the generated optimal assignment plan is shown in Table 2. In each row and column, the 0-entry with the * symbol represent the assignment. Thereby, the optimal subtours generated along with their lengths are shown Table 3.

Table 2:- The RCM with a complete assignment plan.

	City					
City	**1**	**2**	**3**	**4**	**5**	**6**
1	--	0*	4	3	--	--
2	0*	--	1	1	--	1
3	--	1	--	0	2	0*
4	3	1	0	--	0*	1
5	--	--	2	0*	--	1
6	--	1	0*	1	1	--

Table 3:- Optimal subtours and their lengths.

Subtour	Length of Subtour (in miles)
1 → 2 → 1	1 + 1 = **2**
3 → 6 → 3	1 + 1 = **2**
4 → 5 → 4	2 + 2 = 4
Tighter Lower Bound (TLB)	8

The least length subtours are 1 → 2 → 1 and 3 → 6 → 3 with length 2 miles. First we consider the subtour 1 → 2 → 1 and then 3 → 6 → 3 to form the possible numbers of complete round tours.

Case (1): Consider the subtour 1 → 2 → 1

By applying the steps of Phase-II, one can obtain the following possible tours along with their lengths, which are shown in Table 4:

Table 4:- Generated tours and their lengths using the subtour 1 → 2 → 1.

Subtour	**Start city**	**Generated Tour**	**Length of Tour** (in miles)
1 → 2 → 1	1	1 → 2 → 3 → 6 → 5 → 4 → 1	1 + 2 + 1 + 3 + 2 + 4 = **13**
	2	2 → 1 → 4 → 5 → 6 → 3 → 2	1 + 4 + 2 + 3 + 1 + 2 = **13**

Writing the OTP
As of Table 4, we see that, only two possible tours are formed and both of the tours are with the same least length of 13 miles. Consequently, the OTPs are 1 → 2 → 3 → 6 → 5 → 4 → 1 and 2 → 1 → 4 → 5 → 6 → 3 → 2 (equivalently, 1 → 4 → 5 → 6 → 3 → 2 → 1) with Z= 13 miles.

Case (2): Consider the subtour 3 → 6 → 3
By applying the steps of Phase-II, we can get the following possible tours along with their length, which are shown in Table 5:

Table 5:- Generated tours and their lengths using the subtour 3 → 6 → 3.

Subtour	**Start city**	**Generated Tour**	**Length of Tour** (in miles)
3 → 6 → 3	3	3 → 6 → 5 → 4 → 1 → 2 → 3	1 + 3 + 2 + 4 + 1 + 2 = **13**
	6	6 → 3 → 2 → 1 → 4 → 5 → 6	1 + 2 + 1 + 4 + 2 + 3 = **13**

Writing the OTP
As of Table 5, we see that, only two tours are possibly formed and both of the tours are with the same least length of 13 miles. Consequently, the OTPs are 3 → 6 → 5 → 4 → 1 → 2 → 3 (equivalently, 1 → 2 → 3 → 6 → 5 → 4 → 1) and 6 → 3 → 2 → 1 → 4 → 5 → 6 (equivalently, 1 → 4 → 5 → 6 → 3 → 2 → 1) with Z= 13 miles.

Decision
Making use of both the subtours 1 → 2 → 1 and 3 → 6 → 3 with the same shortest length first in Phase-II have formed the same set of OTPs. Thereby, the OTPs generated on the whole by the proposed T-SOFT method are: 1 → 2 → 3 → 6 → 5 → 4 → 1; 1 → 4 → 5 → 6 → 3 → 2 → 1. Hence, the given TSP has two distinct OTPs. In other words, the given TSP has an additional OTP.

Example2:
Consider the following 6-city symmetric TSP, whose distance (in miles) matrix is shown in Table 6.
Table 6: The given 6-city TSP

	City					
City	**1**	**2**	**3**	**4**	**5**	**6**
1	--	12	29	22	13	24
2	12	--	19	3	25	6
3	29	19	--	21	23	28
4	22	3	21	--	4	5
5	13	25	23	4	--	16
6	24	6	28	5	16	--

Solution by the T-SOFT Method
Phase-I (Generating subtours)
By considering the given TSP as an AP by relaxing the additional conditions, we solve the AP by applying the 'E-SOFT for AP'. The RCM along with the generated optimal assignment plan is shown in Table 7. In each row and column the 0-entry with the * symbol represents the assignment. Thereby, the optimal subtours generated along with their lengths are shown Table 8.

Table 7:- The RCM with the optimal assignment plan.

	City					
City	**1**	**2**	**3**	**4**	**5**	**6**
1	--	0	1	10	0*	10
2	0	--	0*	0	21	1
3	1	0*	--	2	3	7
4	10	0	2	--	0	0*
5	0*	2	3	0	--	10
6	10	1	7	0*	10	--

Table 8:- Optimal subtours and their lengths.

Subtour	Length of Subtour (in miles)
1 → 5 → 1	13 + 13 = 26
2 → 3 → 2	19 + 19 = 38
4 → 6 → 4	05 + 05 = **10**
Tighter Lower Bound (TLB)	74

Among the generated three optimal subtours, the least length subtour is 4 → 6 → 4 with length 10 miles. Therefore, we consider this subtour first to form the complete round tour. By applying the steps of Phase-II, one can obtain the following possible tours along with their lengths, which are shown in Table 9:

Table 9:- Generated tours and their lengths using the subtour 4 → 6 → 4.

Subtour	**Start city**	**Generated Tour**	**Length of Tour** (in miles)
4 → 6 → 4	4	4 → 6 → 2 → 3 → 1 → 5 → 4	5+6 +19+29+13+4 = **76**
	6	6 → 4 → 2 → 3 → 1 → 5 → 6 6 → 4 → 5 → 1 → 2 → 3 → 6 6 → 4 → 5 → 1 → 3 → 2 → 6	5+3+19+29+13+16 = 85 5+4+13+12+19+28 = 83 5+4+13+29+19+6 = **76**

Writing the OTP

As of Table 9, we see that, there are two tours having the least length of 76 miles. Consequently, the OTPs are 4 → 6 → 2 → 3 → 1 → 5 → 4 (equivalently, 1 → 5 → 4 → 6 → 2 → 3 → 1) and 6 → 4 → 5 → 1 → 3 → 2 → 6 (equivalently, 1 → 3 → 2 → 6 → 4 → 5 → 1) with Z= 76 miles.

Decision

The OTPs generated on the whole by the proposed T-SOFT method are: 1 → 5 → 4 → 6 → 2 → 3 → 1; 1 → 3 → 2 → 6 → 4 → 5 → 1. Hence, the given TSP has two distinct OTPs. In other words, the given TSP has one more OTP.

Benchmark TSPs

In order to validate the objective of this article, we have identified and solved ten (eight classical and two non-classical) benchmark TSPs, in different relatively small sizes from a range of literature and textbooks, having additional OTPs, which are shown in Table 10. A TSP is said to be 'classical', if each pair of given cities is connected by a desired mode of travel.

Table 10:- List of benchmark TSPs having alternative optimal tour plans.

TSP	**TSP**
Problem 1 $[C_{ij}]$5×5= [-- 16 4 12 --; 16 -- 6 -- 8; 4 6 – 5 6; 12 -- 5 – 20; -- 8 6 20 --]	**Problem 6** $[C_{ij}]$5×5= [-- 17 16 18 14; 17 – 18 15 16; 16 18 – 19 17; 18 15 19 – 18; 14 16 17 18 --]
Problem 2 $[C_{ij}]$5×5= [-- 3 6 2 3; 3 -- 5 2 3; 6 5 – 6 4; 2 2 6 – 6; 3 3 4 6 --]	**Problem 7** $[C_{ij}]$6×6= [-- 13 2 15 15 15; 13 -- 14 1 12 12; 2 14 – 16 14 14; 15 1 16 – 10 10; 15 12 14 10 – 4; 15 12 14 10 4 --]
Problem 3 $[C_{ij}]$5×5= [-- 10 8 9 7; 10 -- 10 5 6; 8 10 – 8 9; 9 5 8 – 6; 7 6 9 6 --]	**Problem 8** $[C_{ij}]$6×6= [-- 1 5 4 -- --; 1 -- 2 2 -- 2; -- 2 – 1 4 1; 4 2 1 – 2 2; -- -- 4 2 – 3; -- 2 1 2 3 --]
Problem 4 $[C_{ij}]$5×5= [-- 10 12 14 8; 10 -- 13 8 9; 12 13 – 12 8; 14 8 12 – 11; 8 9 8 11 --]	**Problem 9** $[C_{ij}]$6×6= [-- 12 29 22 13 24; 12 -- 19 3 25 6; 29 19 – 21 23 28; 22 3 21 – 4 5; 13 25 23 4 – 16; 24 6 28 5 16 --]
Problem 5 $[C_{ij}]$5×5= [-- 8 4 9 9; 8 -- 6 7 10; 4 6 – 5 6; 9 7 5 – 4; 9 10 6 4 --]	**Problem10** $[C_{ij}]$7×7= [-- 86 49 57 31 69 50; 86 -- 68 79 93 24 5; 49 68 – 16 7 72 67; 57 79 16 – 90 69 1; 31 93 7 90 – 86 59; 69 24 72 69 86 – 81; 50 5 67 1 59 81 --]

Outcomes Analysis

On behalf of every one of the ten numbers of TSPs, listed in Table 10, the optimal subtours generated by Phase-I along with their lengths and the possible OTPs generated by Phase-II along with their least lengths as a result of the proposed T-SOFT method are shown in Table 11.

Table 11:- Optimal subtours and the resultant OTPs generated by the T-SOFT method.

Prob. No. #	Subtours generated by Phase-I	Length of Subtours	OTPs generated by Phase-II	Length of Tour (Z)
1.	1 → 3 → 4 → 1 2 → 5 → 2	21 **16**	 2 → 5 → 3 → 4 → 1 → 2 5 → 2 → 1 → 4 → 3 → 5	 47 47
2.	1 → 4 → 1 2 → 5 → 3 → 2 (Alternative subtours) 1 → 4 → 2 → 1 3 → 5 → 3	**04** 12 **07** 08	4 → 1 → 5 → 3 → 2 → 4 1 → 4 → 2 → 3 → 5 → 1	16 16
3.	1 → 3 → 1 2 → 4 → 5 → 2	**16** 17	1 → 3 → 4 → 2 → 5 → 1 3 → 1 → 5 → 2 → 4 → 3	34 34
4.	1 → 3 → 5 → 1 2 → 4 → 2 (Alternative subtours) 1 → 5 → 3 → 1 2 → 4 → 2	28 **16** 28 **16**	 2 → 4 → 3 → 5 → 1 → 2 4 → 2 → 1 → 5 → 3 → 4	 46 46
5.	1 → 3 → 2 → 1 4 → 5 → 4 (Alternative subtours) 1 → 2 → 3 → 1 4 → 5 → 4	17 **08** 18 **08**	 5 → 4 → 2 → 1 → 3 → 5 4 → 5 → 3 → 1 → 2 → 4	 29 29
6.	1 → 3 → 5 → 1 2 → 4 → 2	47 **30**	 2 → 4 → 3 → 1 → 5 → 2 4 → 2 → 5 → 1 → 3 → 4	 80 80
7.	1 → 3 → 1 2 → 4 → 2 5 → 6 → 5	04 **02** 08	 2 → 4 → 5 → 6 → 3 → 1 → 2 2 → 4 → 6 → 5 → 3 → 1 → 2	 44 44
8.	1 → 2 → 1 3 → 6 → 3 4 → 5 → 4	**02** **02** 04	1 → 2 → 3 → 6 → 5 → 4 → 1 2 → 1 → 4 → 5 → 6 → 3 → 2	13 13
9.	1 → 5 → 1 2 → 3 → 2 4 → 6 → 4	26 38 **10**	 4 → 6 → 2 → 3 → 1 → 5 → 4 6 → 4 → 5 → 1 → 3 → 2 → 6	 76 76
10.	1 → 3 → 5 → 1 2 → 6 → 2 4 → 7 → 4	87 48 **02**	 4 → 7 → 2 → 6 → 1 → 5 → 3 → 4 7 → 4 → 3 → 5 → 1 → 6 → 2 → 7	 153 153

Reasons for choosing the E-SOFT method for generating subtours of TSPs

By means of our research articles in AP, we have proved that E-SOFT method [7] is the most efficient one to solve APs. Therefore, in the proposed T-SOFT method, the E-SOFT method is chosen for generating the optimal subtours for a TSP.

Novelty in terms of methodology in the proposed T-SOFT method
The generated set of optimal subtours by Phase-I of the proposed T-SOFT method is connected together exactly in Phase-II to form OTPs by considering the 'subtour of shortest length' first. By starting the tour plan from the first (or next) city of this subtour and travel ahead along the sequence given in this subtour and connecting of all the remaining subtours together to form a tour plan is carried out based on the available 0-entry or 1-entry or the next available immediate higher entry among the upcoming unassigned cells in the associated reduced cost matrix. In this way, the optimal subtours are connected perfectly to form all possible tour plans by removing the cycles (subtours) easily.

How is the proposed T-SOFT method different from the existing methods in TSP?
In the proposed T-SOFT method, we have seen that how the cycles (subtours) are removed to form a single cycle (or a tour plan) rightly and easily. Thereby, the proposed T-SOFT method is entirely different from the existing methods and the difference found in each is given below:

1. In Hungarian Method: The given TSP considered as an AP is solved by the Hungarian method of assignment [3], thereby an optimal solution is obtained. If this optimal solution violates the condition that salesman can visit each city only once, then one looks for the 'next best' solution by bringing the next (non-zero) cost element 1 along with the zero elements into the solution. If more than one cost element 1 occurs, then each such 1 is considered separately until a feasible solution to the TSP is obtained.
2. In Ones Assignment Method: In Ones Assignment Method [3], priority rule plays an important role to make a tour.
3. In the New Revised Ones Assignment Method: In the New Revised Ones Assignment Method [5], no clear cut rule is given to make a tour from the solution consisting cycles.
4. In the Direct Approach: In the Direct Approach [7], assignments are made based on choosing a suitable assignment preference table consisting of absolute favorable costs of from cities or to cities and their frequencies.
5. In the Application of Hungarian Algorithm to solve TSP [4]: The given TSP considered as an AP is solved by the Hungarian method of assignment, by this means an optimal solution is obtained. If this optimal solution contains more cycles, then it is removed by constructing a Modifying Distance (Cost or Time) Matrix and then solving the same by Hungarian Algorithm in order to obtain an optimal solution with only one cycle. This algorithm removes the cycles generated by the Hungarian algorithm, but does not guaranteed to yield an optimal solution to a TSP.
6. In the TWINS method of generating alternative optimal tour plans to TSP [8]: An optimal tour plan is generated by the ESAN-ESAN method based on 'zeros assignment approach' and an alternative optimal tour plan is generated by the ESA-ESAN method based on 'ones assignment approach'. For generating the optimal subtours and thereby generating alternative optimal tour plans for a given TSP, the TWINS method requires zeros assignment approach and ones assignment approach whereas, the proposed T-SOFT method requires only the E-SOFT method based on zeros assignment approach.

Conclusion:-
In this research article, we have acknowledged a set of ten benchmark TSPs and established that each of the TSPs is having additional optimal tour plan. The optimal tour plan and the additional optimal tour plan have been generated through the proposed method named T-SOFT. Hence, it is explicit that by applying the T-SOFT method one can generate the possible optimal tour plans to a given TSP, provided it exists. For the concerned administration, the identified additional available optimal tour plans become the base for subsequent analysis and finally the decision itself.

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