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RESEARCH ARTICLE

MATHEMATICAL MODELLING IN WAITING QUEUE IN SHOPPING MALL

Tijeshwari Bisen and Arun Garg
Department of Mathematics, Madhyanchal Professional University, Bhopal.

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Abstract

This project is an attempt to analyze the use of queuing theory in a local shopping mall. Also considers the mall as a multi server queuing system following Poisson arrival and Exponential service based system. A comparison of the results using MATLAB pertaining to double server and multiple server queuing system is also provided.

Introduction:-

The queuing theory is one of the most celebrated problems of operation research which has attracted the attention of researchers, scientists, mathematics and social scientists. A lot of research work has been dedicated to the application of this theory in health care systems, construction industries, human resource management, transportation, traffic and many other such systems.

Queuing theory is a mathematical approach to study of the waiting lines. Long waiting time in any health care centre affects the improvement of the centre as well as the nation's economy. Therefore, to reduce the waiting time of arriving customers is a major challenge for services not only in India but all over the world especially in developing countries. While considering improvement in services, centre must measures the cost of providing a given level of service against the potential costs from having customer wait. Queuing theory has increasingly become a universal tool of management for decision making in a Local shopping mall.

Description of the Models

Double Channel Queuing System

Consider a double server queuing system, (M/M/2) in which arriving customers is following Poisson's process with the arrival rate λ and the service process is following the exponential distribution with the service rate μ. Here customers are identified as arriving customers. The services in all phases are independent and identical and only one customer at a time is in the service mechanism.

When a customer enters the system and at a time if the system is free, his/her service time starts at once and when the system is not free, the customer joins the queue and wait for their turn/number for service. After completion of services, the customer is free from queue if there is not any further extended service facility. If the server is busy then the arriving customers goes to orbit and becomes of repeated calls. This pool of source of repeated calls may be viewed as a sort of queue.

Corresponding Author:- Tijeshwari Bisen
Address:- Department of Mathematics, Madhyanchal Professional University, Bhopal.

The time it takes to service every customer is an exponential random variable with parameter μ. A pictorial representation of a double server queuing system, (M/M/2) is given below in which customers are standing in queue, waiting for the server to be free for providing service.

In the situation of congestion of customers in a health care system, there is a very less probability for the customers arriving in end to get treatment as there is a double server rendering services. In worst situation, customers may leave the system without being serving.

Multichannel Queuing System

The multichannel queuing model is known in the Kendall's notation as the M/M/m model, where M signifies a Poisson distribution and m is the number of parallel service channel in the system. This is commonly used to analyze the queuing problem. This model commutes the average wait time and queue lengths, given arrival rate, number of servers and service rates. This particular model applies, in which there is multiple channel served by a double queue at a bank teller or many airline ticket counters. The output of the model are as follows:

1. Expected waiting time per customer in the system.
2. Expected waiting time customers in the queue.
3. Expected number of customer in the system.
4. Expected number of customer in the queue.

The exact calculation of these measures requires knowledge of the probability distribution of the arrival rate and service times. Moreover, successive inter-arrival times and service times are assumed to be statistically independent of each other. In this system, there are multiple servers with all sharing common waiting line a waiting line is crated when all the servers are busy in rendering service. As soon as one server becomes free, a customer is dispatched from the waiting line using dispatching discipline in force for being served. There are obvious from the pictorial representation of the multichannel queuing system which is given below.

Model Assumptions

This research is based on the following assumptions.

1. The finding obtained after investigate from one unit of the Billing counter should be valid in the other units.
2. The customers are almost well familiar with the organization system of the shopping mall.
3. The arrival rate of the customers to queue and service rate are compatible to poison distribution or in other words the time interval between two consecutive arrivals and time services both follow exponential distribution.
4. The queuing theory discipline is such that the first customer goes to the server which is ready for service.
5. In case of multichannel queuing system, it is assumed that none of the servers are unattended.

Assumed Parameters in Queuing Model

- n = Number of customers (units) in the system.
- c = number of parallel servers.
- λ = It is the mean rate of arrivals per unit of time in the system.
- μ = It is the average number of customers served per unit time in the system.
- $c\mu$ = Serving rate when c>1 in a system.
- ρ = Utilization factor.
- P_0 =Steady state probability of all idle servers in the system.
- P_n = Steady satiate probability exactly n customers in the system.
- L_q = Average number of customers in the queue.
- L_s = Expected number of customers in the system.
- W_q =Service time.
- W_s = The expected time a customer spends in the system.

Data Analysis

The memory less property is utilized to define the state of the queuing system. To determine the performance measures, first find the probability of having n number of customers in the queuing system.

Probability of having 1 customer (i.e. $n = 1$) in the service system is:

$$P_1 = \rho P_0$$

Similarly,

$$P_2 = \rho P_1$$
$$P_2 = \rho^2 P_0 \dots\dots\dots$$
$$P_n = \rho^n P_0$$

The probability value is 1. i.e.

$$\Sigma_{n=0}^{\infty} P_n = 1$$
$$P_0 + P_1 + P_2 + \cdots = 1$$
$$P_0 + \rho P_0 + \rho^2 P_0 + \cdots = 1$$
$$(1 + \rho + \rho^2 + \cdots)P_0 = 1$$

Where $(1 + \rho + \rho^2 + \cdots)$ is an infinite series, Sum of infinite series can be written as,

$$(1|1 - \rho)$$

Hence,

$$P_0 = 1 - \rho$$

i.e., the probability of no customer in the system.

To determine performance measures L_s, L_q, W_s, W_q in the queuing system determine the average number of customers in the system.

The average number of customers L_s in the system can be written as,

$$L_s = \Sigma_{n=0}^{\infty} n P_n$$

Where $P_n = \rho^n P_0$ and $P_0 = 1 - \rho$

$$L_s = (1 - \rho)\Sigma_{n=0}^{\infty} n \rho^n = \rho(1 - \rho)\Sigma_{n=0}^{\infty} n \rho^{n-1}$$

Then,

$$\frac{\partial \rho^n}{\partial \rho} = n \rho^{n-1}$$
$$L_s = \rho(1 - \rho)\frac{\partial}{\partial \rho}\Sigma_{n=0}^{\infty} \rho^n = \rho(1 - \rho)\frac{\partial}{\partial \rho}(1|1 - \rho)$$
$$L_s = \frac{\lambda}{\mu - \lambda}$$

Using little's law according to which the average number of customers in the service system is the product of arrival rate and average time a customer's spends in the system Average time a customer spends in the system, W_s can be written using little's law as given below

$$W_s = \frac{L_s}{\lambda}$$

Determine L_s and know λ, hence

$$W_s = \frac{\lambda}{\mu - \lambda} \cdot \frac{1}{\lambda}$$
$$W_s = \frac{1}{\mu - \lambda}$$

Average time a customer spends in the queue W_q , can be determined by subtracting expected service time or average service time from average time a customer spends in the system W_s

$$W_q = W_s - \text{expected service time}$$
$$= W_s - \mu = \frac{1}{\mu - \lambda} - \mu$$
$$W_q = \frac{\lambda}{\mu(\mu - \lambda)}$$

Using little's law to determine the average number of customers in the queue

$$L_q = W_q \lambda$$
$$L_q = \frac{\lambda^2}{\mu(\mu - \lambda)}$$

Model Parameters

Traffic Intensity

It is obtained from dividing the average arrival rate λ (in time) to the average service rate μ.

i.e., $\rho = \lambda / \mu$

Whenever λ is larger, the arrival of customers will increase and the system will work harder and queue will be longer. On the contrary, whenever λ is smaller, the queue will be shorter but in this case the use of system will be low. If the arrival rate of customers in the system were more than service rate. i.e. $\lambda > \mu$ then $\rho > 1$, which means the system capacity is less than the arriving customers; therefore the queue length is increased. In this queuing system the average arrival rate is less than the average service rate i.e. $\lambda > \mu$.

Average Waiting Time in Queue
The average waiting time in queue (before service is rendered) is equal to the average time which a customer's waits in the queue for getting services. Its formula is,

$$\frac{\rho}{\mu(1-\rho)} = \frac{\lambda}{\mu(\mu-\lambda)}$$

Average Time Spend in the System
The average time spent in a system (on queue and receiving service) is equal to the total time that a customer's spends in a system which includes the waiting time and service time. Its formula is,

$$\frac{1}{\mu(1-\rho)} = \frac{1}{(\mu-\lambda)}$$

Average Number of Customers in the System
The average number of customers' in the system is equal to the average number of customers' who are in the line or server. It is defined as

$$\frac{\rho}{(1-\rho)} = \frac{\lambda}{(\mu-\lambda)}$$

Average Queue Length
The average queue length is composed of the average number of customers' who are waiting in the queue. It is defined as

$$\frac{\lambda^2}{\mu(\mu-\lambda)}$$

The Probability of not Queuing on the Arrival

$$= 1-\rho$$

Results and Discussions:-

Results of M/M/2
The arrival time as well as the time service began and ended for 125 customers' in the local shopping mall. There are two types of services; return and buying . A total of 22 days were used for the data collection. On the basis of actual observed collected data, find the
Total waiting time of 650 customers' for 22 days = 2000 minutes
Total service time of 650 customers' for 22 days = 1750 minutes
Using the model parameters for the double channel queuing model, arrive the following results:
The arrival rate,

$$\lambda = \frac{total\ number\ of\ patients}{Total\ waiting\ time}$$

$$\frac{650}{2000} = 0.33 \quad (1)$$

The service rate,

$$\mu = \frac{total\ number\ of\ patients}{Total\ service\ time} = \frac{650}{2000} = 0.371(2)$$

The average waiting time in a queue,

$$= \frac{\lambda}{\mu(\mu-\lambda)}$$

$$= 18.85 \equiv 19\text{Minutes} \quad (3)$$

The average time spent in a system,

$$\frac{1}{\mu-\lambda} = 21.54 \equiv 22\text{minutes} \quad (4)$$

Average number of customers in the system,

$$\frac{\lambda}{\mu-\lambda} = \frac{0.0789}{0.09469-0.0789} \equiv 7 \quad (5)$$

Average queue length (Average number of customers in the queue),

$$\frac{\lambda^2}{\mu(\mu-\lambda)} = \frac{\rho^2}{1-\rho}$$

$$= 6.13 \equiv 6 \quad (6)$$

The probability of queuing on arrival i.e. traffic,

$$= \lambda / \mu$$

$$= 0.88 \quad (7)$$

The probability of not queuing on the arrival,

$$= 1-\rho$$

$$= 1-0.88$$

$$= 0.12 \qquad (8)$$

Discussions:-

A double channel queuing system is used to represent the Local shopping mall where doctor is treated as a double server and the mode parameters are applied for calculations. The study for this case is on the basis of actual observed data collection in 22 days of service for 650 customers'. The traffic intensity, ρ = λ/μ = 0.12 obtained in (7) shows the probability of customers' queuing on arrival. This reveals the congestion of customers' waiting for billing as billing Clark is engaged.

This represents the inadequate service system of the shopping mall. Also from the results (3) and (4) it is obvious that the average time spent in the shopping mall is greater than the average time spent in the queue before providing service. Thus, there will always be a queue of customers' in the mall which is also very clear from the results (5) and (6). The result (8) shows that there is a very less possibility services to new arriving customers'.

Multi-Channel Queuing Theory Model (M/M/c: FCFS/∞/∞)

Multi- channel queuing theory treats the condition in which there are several service stations in parallel and each element in the waiting line can be served by more than one station.

Each service facility is prepared to deliver the same type of service. The new arrival selects one station without any external pressure. When a waiting line is formed, a double line usually breaks down into shorter lines in front of each service station. The arrival rate λ and service rate μ are mean values from poison distribution and exponential distribution respectively. Service discipline is first-come first served and customers are taken from a double queue i.e., any empty channel is filled by the next customer in line.

M/M/c Queuing Model

The first known values in a calculation of performance measure is,

(i) Traffic intensity(ρ)

(ii) Probability of the system should be idle P_0 The traffic intensity is,

$$\rho = \lambda / c\mu$$

Whenever λ is larger, the arrival of customers' will increase and the system will work harder and queue will be longer. On the contrary, whenever λ is smaller, the queue will be shorter but in this case the use of system will be low. If the arrival rate of customers' in the system were more than service rate. i.e. λ>μ then ρ > 1, which means the system capacity is less than the arriving customers'; therefore the queue length is increased. In this queuing system the average arrival rate is less than the average service rate i.e. λ >μ.

The probability that the system should be idle.

$$P_0 = \left[\Sigma_{n=0}^{\infty} \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n + \frac{1}{c!}\left(\frac{\lambda}{\mu}\right)^n \frac{c\lambda}{c\mu-\lambda}\right]^{-1}$$

The average number of customers' in the system,

$$L_s = \frac{(\lambda.\mu)\left(\frac{\lambda}{\mu}\right)^c}{(c-1)!(c\mu-\lambda)^2} \quad P_0 + \frac{\lambda}{\mu}$$

The average number of customers' waiting in the queue,

$$L_q = L_s - \text{Average number being served}$$

$$= L_s - c\left(\frac{\lambda}{c\mu}\right)$$

$$L_q = \frac{(\lambda.\mu)\left(\frac{\lambda}{\mu}\right)^c}{(c-1)!(c\mu-\lambda)^2} \quad P_0$$

Average waiting time a customer spends in the system,

$$W_s = \frac{L_s}{\lambda}$$

$$W_s = \frac{\mu\left(\frac{\lambda}{\mu}\right)^c}{(c-1)!(c\mu-\lambda)^2} \quad P_0 + \frac{1}{\mu}$$

Average waiting time of a customer in the queue,

$$W_q = \frac{L_q}{\lambda}$$

$$W_q = \frac{\mu\left(\frac{\lambda}{\mu}\right)^c}{(c-1)!(c\mu-\lambda)^2} \quad P_0$$

Utilization factor, $\rho = \frac{\lambda}{c\mu}$

Average number of idle servers, = c – (average number of customers served)

Probability that a customer has to wait,

$$p(n \geq c) = \frac{\mu\left(\frac{\lambda}{\mu}\right)^c}{(c-1)!(c\mu-\lambda)} \quad P_0$$

Probability that a customer enters the service without waiting,

$$1 - p(n \geq c) = 1 - \frac{\mu\left(\frac{\lambda}{\mu}\right)^c}{(c-1)!(c\mu-\lambda)} \quad P_0$$

Results of M/M/3:-

The arrival time as well as the time service began and ended for 650customers in the Local shopping mall "Venkatesh Nursing Home". There are two types of services; consultancy and surgery. A total of 22 days were used for the data collection. On the basis of actual observed collected data, find the

Total waiting time of 650 customers for 22 days = 2000 minutes (9)

Total service time of 650 customers for 22 days = 1750 minutes (10)

Consider 3 servers.

Using the model parameters for the double channel queuing model, the following results:

The arrival rate,

$$\lambda = \frac{total\ number\ \ of\ customer}{Total\ waiting\ \ time} = \frac{650}{2000} = 0.33 \quad (11)$$

The service rate,

$$\mu = \frac{total\ number\ \ of\ patients}{Total\ waiting\ \ time} = \frac{650}{1750} = 0.37 \quad (12)$$

The probability that the system should be idle,

$$P_0 = \left[\Sigma_{n=0}^{2} \frac{1}{n!}\left(\frac{0.325}{0.3714}\right)^n + \frac{1}{3!}\left(\frac{0.325}{0.3714}\right)^3 \frac{3(0.3714)}{3(0.3714)-0.325}\right]^{-1} = 0.42 \ (13)$$

The average number of customers' in the system,

$$L_s = \frac{(0.325).(0.3714)\left(\frac{0.325}{0.3714}\right)^3}{(3-1)!(3(0.3714)-0.325\ )^2}(0.42) + \frac{0.325}{0.3714}$$

$$= 0.90' = 1 \quad (14)$$

The average number of customers' waiting in the queue,

$$L_q = \frac{(0.325).(0.3714)\left(\frac{0.325}{0.3714}\right)^3}{(3-1)!(3(0.3714)-0.325\ )^2}(0.42) \equiv 0.03 \quad (15)$$

The average waiting time of customers' in the system,

$$W_s = \frac{(0.3714)\left(\frac{0.325}{0.3714}\right)^3}{(3-1)!(3(0.3714)-0.325\ )^2}(0.42) + \frac{1}{0.37}$$

$$\equiv 2.78 \text{ minute} \quad (16)$$

The average waiting time of customers' in the queue,

$$W_q = \frac{(0.3714)\left(\frac{0.325}{0.3714}\right)^3}{(3-1)!(3(0.3714)-0.325\ )^2}(0.42) \equiv 0.08 \text{ minute} \quad (17)$$

The traffic intensity is,

$$\rho = \frac{0.325}{3(0.3714)} = 0.29. \quad (18)$$

The probability of not queuing on the arrival,

$$= 1 - \rho$$
$$= 1 - 0.29$$
$$= 0.71. \quad (19)$$

Comparing the Results of M/M/1 and M/M/3

Comparison	**M/M/2**	**M/M/3**
Traffic intensity	0.44	0.29
average number of customers' in the system	1	1
average number of customers' in the queue	1	0
average waiting time of customers' in the system	4 minutes	3 minutes

average waiting time of customers' in the queue	2 minutes	1 minutes
The probability of new customers Arrival	0.91	0.71

The waiting time in M/M/3 is less than the M/M/2. The average number of customers in the queue is also less. The probability of new customer's arrival is large in the multi server.

Average Waiting Time in a System of Double Server and Multi Server by Using MATLAB

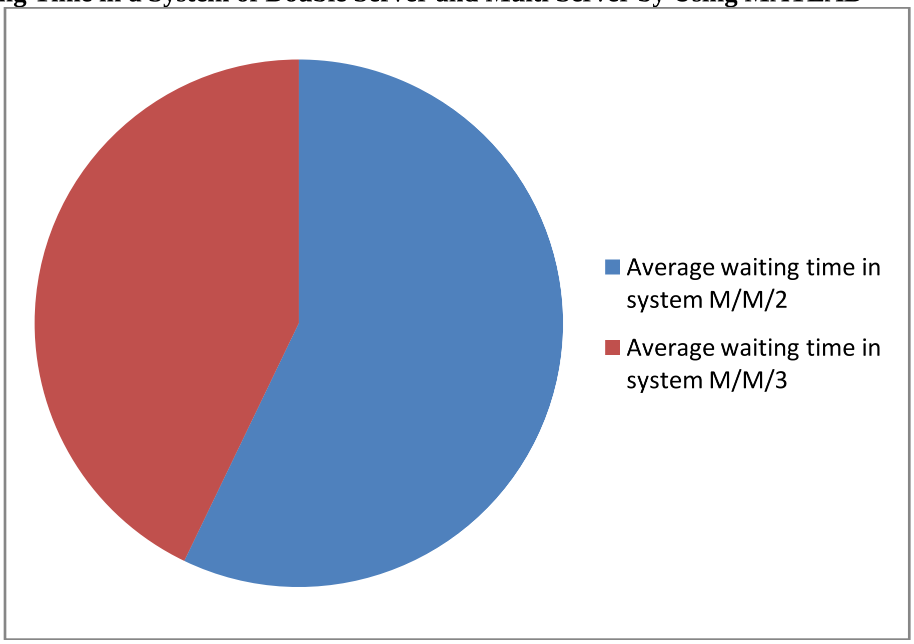

Fig 1:- Average Waiting Time in a Queue of Double Server and Multi Server by Using MATLAB.

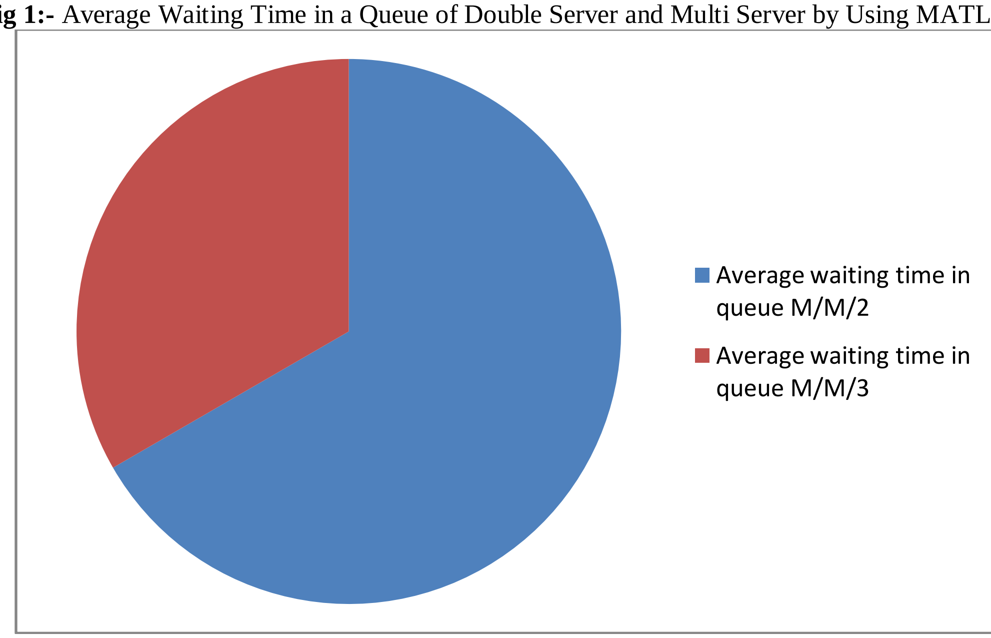

Fig 2:- New Customer's Arrivals Probability of Double Server and Multi Server by Using MATLAB.

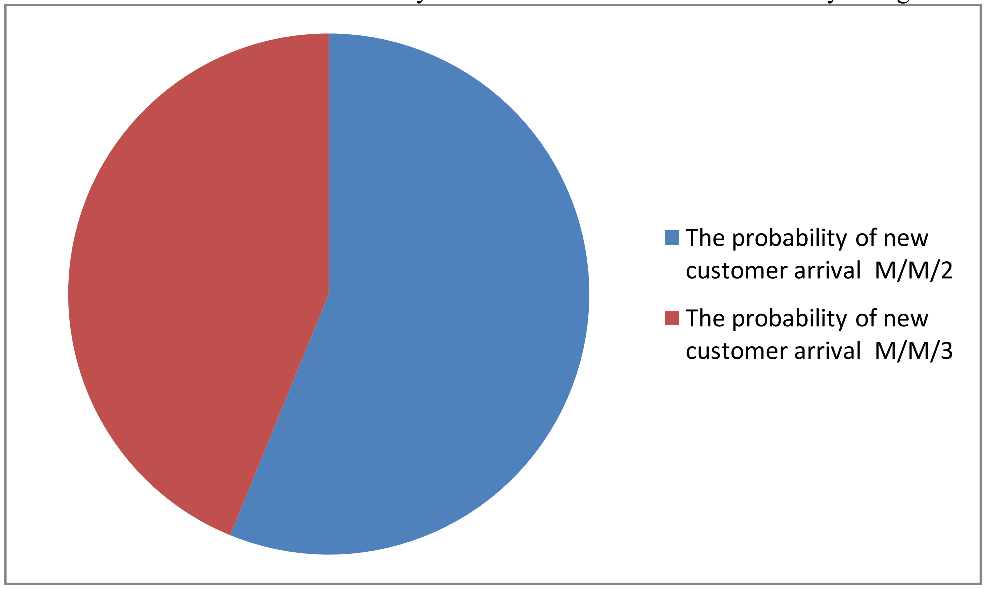

Fig 3

Conclusion:-

In this paper, multi-server queuing system was discussed. In this case the waiting time in a queuing model (M/M/2) was more than that of queuing model (M/M/3) and observed that new customer's arrival probability of double server queuing system was less than that of multi-server queuing system. The multi-server queuing system increases the efficiency of the hospital and reducing time compared to the double server queuing system.

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