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RESEARCH ARTICLE

REFLECTIVE INSIGHTS ON TEACHING TRIGONOMETRIC RATIOS OF COMPLEMENTARY ANGLESUSING THE FLIPPED CLASSROOM APPROACH

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Manuscript Info ***Abstract***

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Introduction:-

Based on the definition proposed by the Flipped Learning Network (2014), differing from the traditional teaching approach where teachers predominantly impart mathematical knowledge and end with a few questions as summative assessment in the brick-and-mortar classroom (i.e., group learning space), asking students to apply the knowledge in the post-class homework (i.e., individual learning space), flipped learning approach switches the order: students acquire basic knowledge individually through the pre-class instructional video along with online follow-up exercises, then learning more complex content and engaging in teacher-student and student-student problem-solving activities in the classroom(Lo et al., 2018).

Considering the progressively widespread implementation of the flipped learning approach during the post-epidemic era,which has shown evident effectiveness in improving students' learning (Yang & Chen, 2020), I have practiced this trial teaching by flipping the classroom. Within this context, Merrill's (2002) First Principle of Instruction has provided me with valuable inspiration in structuring my teaching boards, during which I have endeavored to apply Variation Theory (Kullberg et al., 2017) and scaffolding strategy in designing my teaching materials. In the following sections, I will explicit the design of my trial teaching grounded on these influential theories, and showcase the enhancements to be made based on the feedback of Dr. Lo and other peer prospective teachers, as well as my reflections after multiple reviews.

Background information

My chosen topic is Trigonometric Ratios of Complementary Angles, which serves nearly the finale of the trigonometry learned in junior secondary school. Targeting Form 3 students with average academic performance, to ensure most of them can comprehend the instructional videos, I have differentiated the learning objectives of out-of-class learning and in-class learning aligned with the Hong Kong's curriculum document (Curriculum Development Council, 2017) and the question types shown in the textbook.

Learning objectives of out-of-class dimension

1. Understand the relations between trigonometric ratios of complementary angles (CDC, 2017):
- $\sin\theta = \cos(90° - \theta)$
- $\cos\theta = \sin(90° - \theta)$
- $\tan\theta = \frac{1}{\tan(90° - \theta)}$

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2. Convert the trigonometric ratio of a given angle (e.g., sin 20°= cos ______).

3. Solve simple trigonometric equations (e.g., sin θ= cos a (a is a known angle); $\frac{1}{\tan(90°-\theta)}$ = tan a (a is a known angle)).

Learning objectives of in-class dimension

1. Solve more-advanced trigonometric equations (e.g., cos(aθ+ b)= sin (-mθ+n))

2. Evaluate the values of trigonometric expressions without using calculators.

3. Simplify trigonometric expressions.

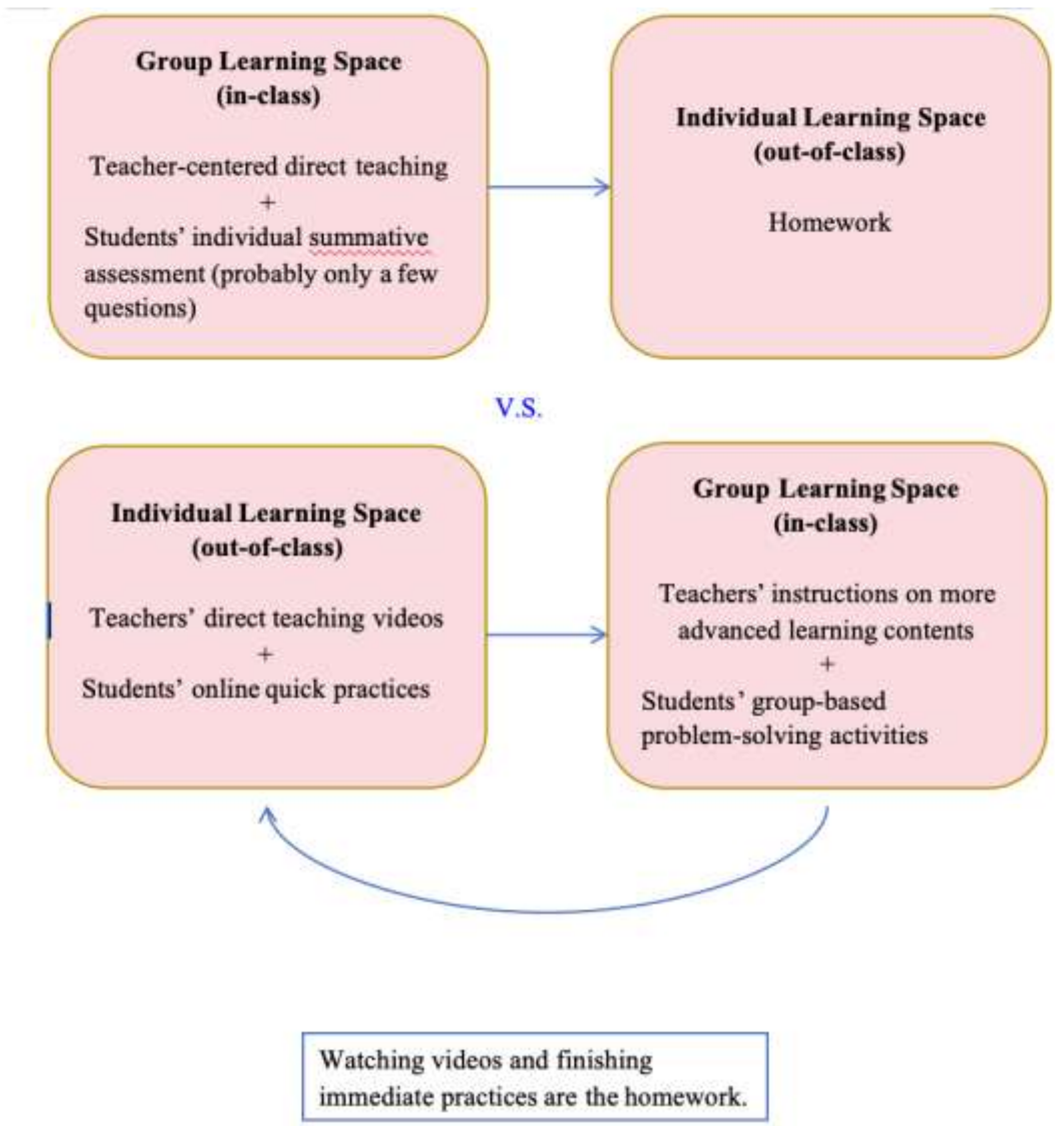


Figure 1:- Traditional classroom and flipped classroom in my understanding.

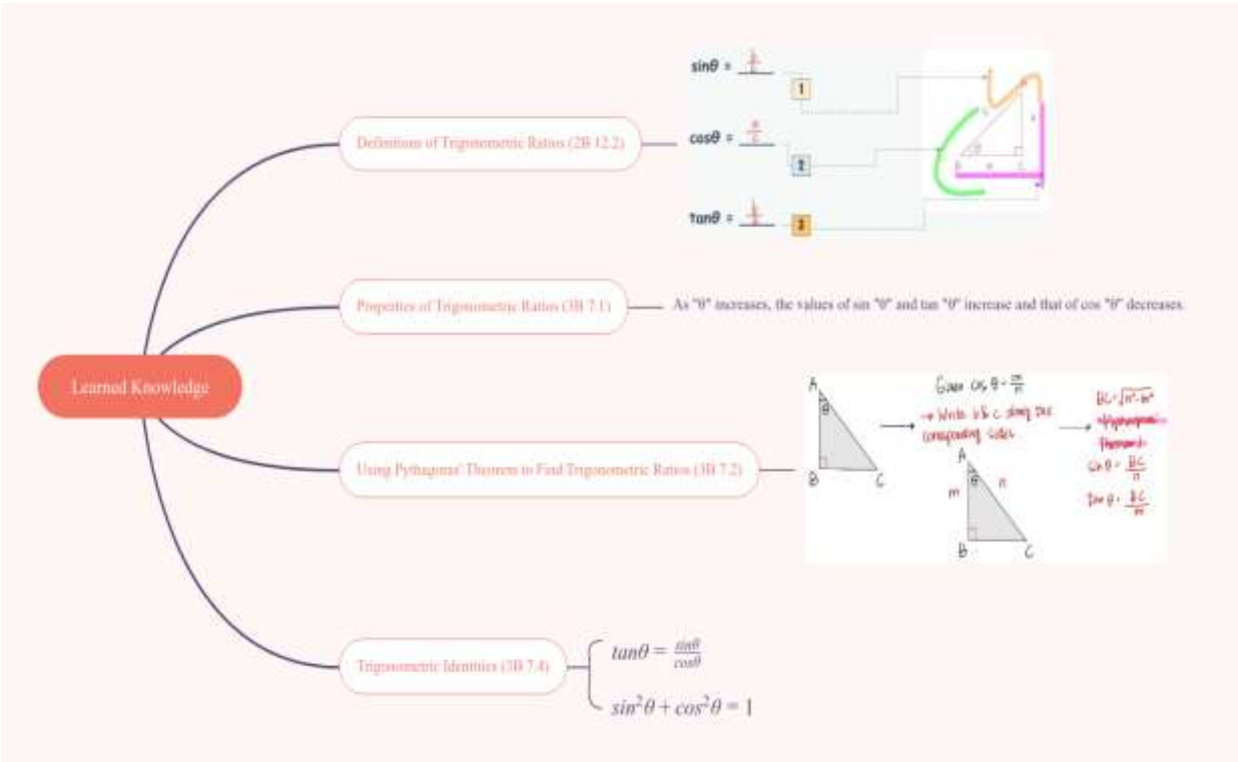


Figure 2:- Stockpile of students' knowledge prior to this topic.

Learning Unit	Learning Objective	Time	Remarks
27. Trigonometry	27.1 understand sine, cosine and tangent of angles between 0° and 90°	18	The trigonometric ratios of 0° and 90° are **not** required.
	27.2 understand the properties of trigonometric ratios		The properties include: For $0° < \theta < 90°$, • as θ increases, the values of $\sin\theta$ and $\tan\theta$ increase and that of $\cos\theta$ decreases • $0 < \sin\theta < 1$ • $0 < \cos\theta < 1$ • $\tan\theta > 0$ • $\frac{\sin\theta}{\cos\theta} = \tan\theta$ • $\sin^2\theta + \cos^2\theta = 1$ • $\sin(90° - \theta) = \cos\theta$ • $\cos(90° - \theta) = \sin\theta$ • $\tan(90° - \theta) = \frac{1}{...}$

36

Figure 3:- Curriculum requirements for this topic (CDC, 2017).

Instructional Video with rationales and improvements

The First Principle of Instruction, as outlined by Merrill (2002), encapsulates five principles of effective teaching and learning, namely revolving around real-world problems (i.e., problem-centered principle), the teaching process commence with activating students' existing knowledge (i.e., activation principle), followed by demonstration, application and integration.

In the instructional video, I primarily utilized the principles of activation, demonstration and application. It is worth mentioning that I adopted the video style of Khan-style with teacher's talking head in the realm of videotaping, which is recommended in Hew & Lo's (2020) study. The former's write-while-speaking function has an advantage over the static PowerPoint slides on unveiling the progressive development of mathematical knowledge (Greiffenhagen, 2014), as well as seamless integration with Mayer's (2014) signaling principle of multimedia instruction that Lo (2018) underscores in flipped learning scenarios, namely to highlight important mathematical concepts and instruct students to take notes. The latter can reduce the monotony of the video (Kizilcec et al., 2014) by augmenting the presence of the teacher, thereby improving students' attentiveness (McLaren et al, 2011).

Figure 4:- Khan-Style + my talking head.

Activation

Complementary angles is introduced to studentsin secondary 1, which is a seemingly distant memory for them. Thus, I first reviewed its definition with students (i.e., activation principle), which pave the way to prove the relations between the trigonometric ratios of complementary angles in a right-angled triangle. Accordingly, I strategically provided signal in the bottom-right corner of the PowerPoint, so as to facilitate students' note-taking.

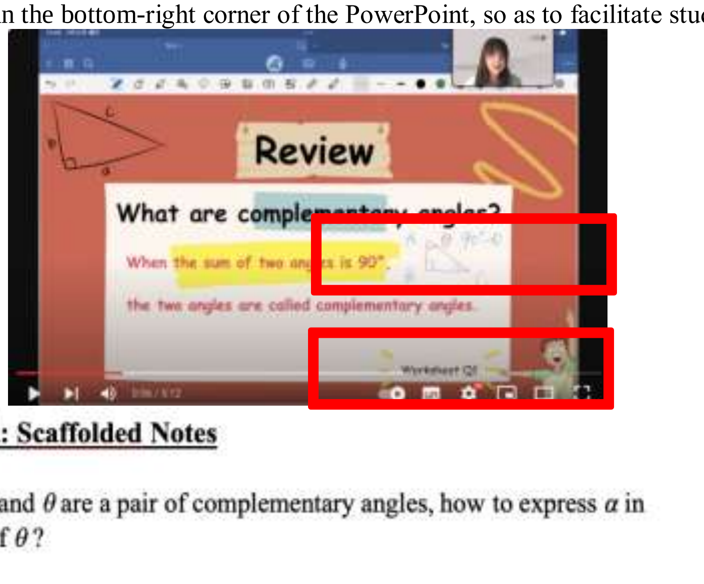


Figure 5:- Review of the definition of complementary angles.

Another crucial prerequisite is the definition of trigonometric ratios, as it is the stepping stone to proving the the identities. Considering that it is also compulsory in the curriculum (CDC, 2017), guided by Kolb's (1984) experiential learning theory which encompasses concrete experience, reflective observation, abstract conceptualization, and active experimentation, I decided to use a novel way to evoke it. Responding to the former two, I used the dynamic courseware GeoGebra to create an experimental environment for students to concretely experience the exploration of mathematical concepts (Leung, 2011); then detailed instructions were provided in the worksheet to facilitate students' reflective observation of mathematical relationships (Weinhandl et al., 2020) between trigonometric ratios of complementary angles. If students can successfully finish the observation tasks, they have in fact gained the essential mathematical concept of this lesson (i.e., abstract conceptualization).

2. Follow the instructions and finish this task step by step.
(a) Open the GeoGebra file:
https://www.geogebra.org/calculator/eaqabypm

(*You may either click the link or scan the QR code.)

(b) Choose the correct answer. Then press the button of "Check".
(c) Try until you get the full mark of six.
(d) Screenshot your finished task and submit it later.
(e) Complete the table of identities of trigonometric ratios of complementary angles below (θ is an acute angle).

$\sin\theta =$ ______ $(90° - \theta)$	$\sin(90° - \theta) =$ ______ θ
$\cos\theta =$ ______ $(90° - \theta)$	$\cos(90° - \theta) =$ ______ θ
$\tan\theta =$ ______	$\tan(90° - \theta) =$ ______

Figure 6:- Detailed descriptions for concrete experience and reflective observation on the worksheet.

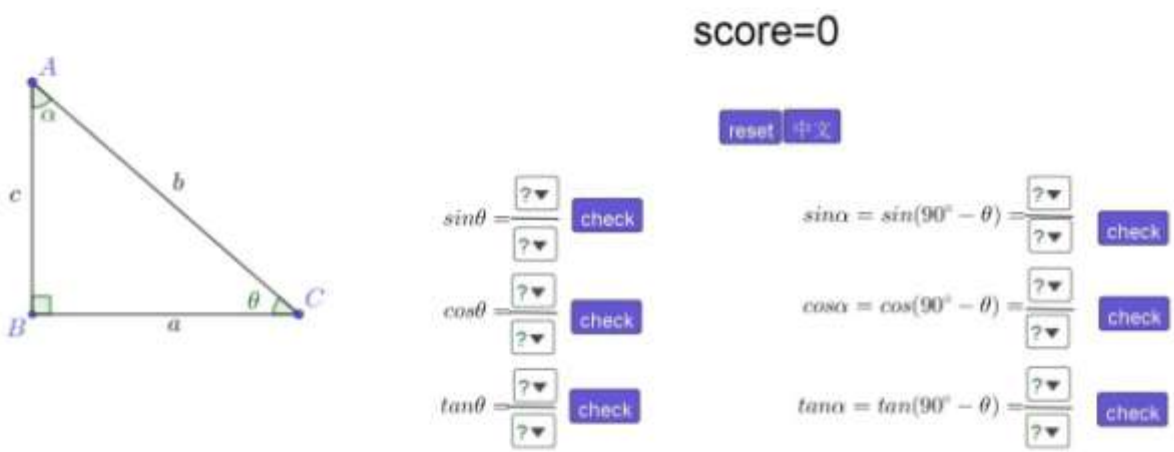


Figure 7:- Students use GeoGebra toreview old knowledge andexplore new knowledge.

Nevertheless, modifications were implemented in this particular section, according to Dr. Lo's comment. The conventional arrangement of "sine, cosine, tangent," would cause a "crossing" phenomenon. Given the average abilities of the targeting students, there may be a high possibility that students view it merely as an assessment of the definition of trigonometric ratio rather than an opportunity to discover trigonometric identities. Hence, the places of sin (90° - θ) and cos (90° - θ) were switched. The reorganization allowed for horizontal alignment, helping students easily find the connection between trigonometric ratios of complementary angles. Additionally, as the task fundamentally works for assessing whether students could grasp the definition of trigonometric ratios, I used the Variation Theory to vary the placement of the right triangle.

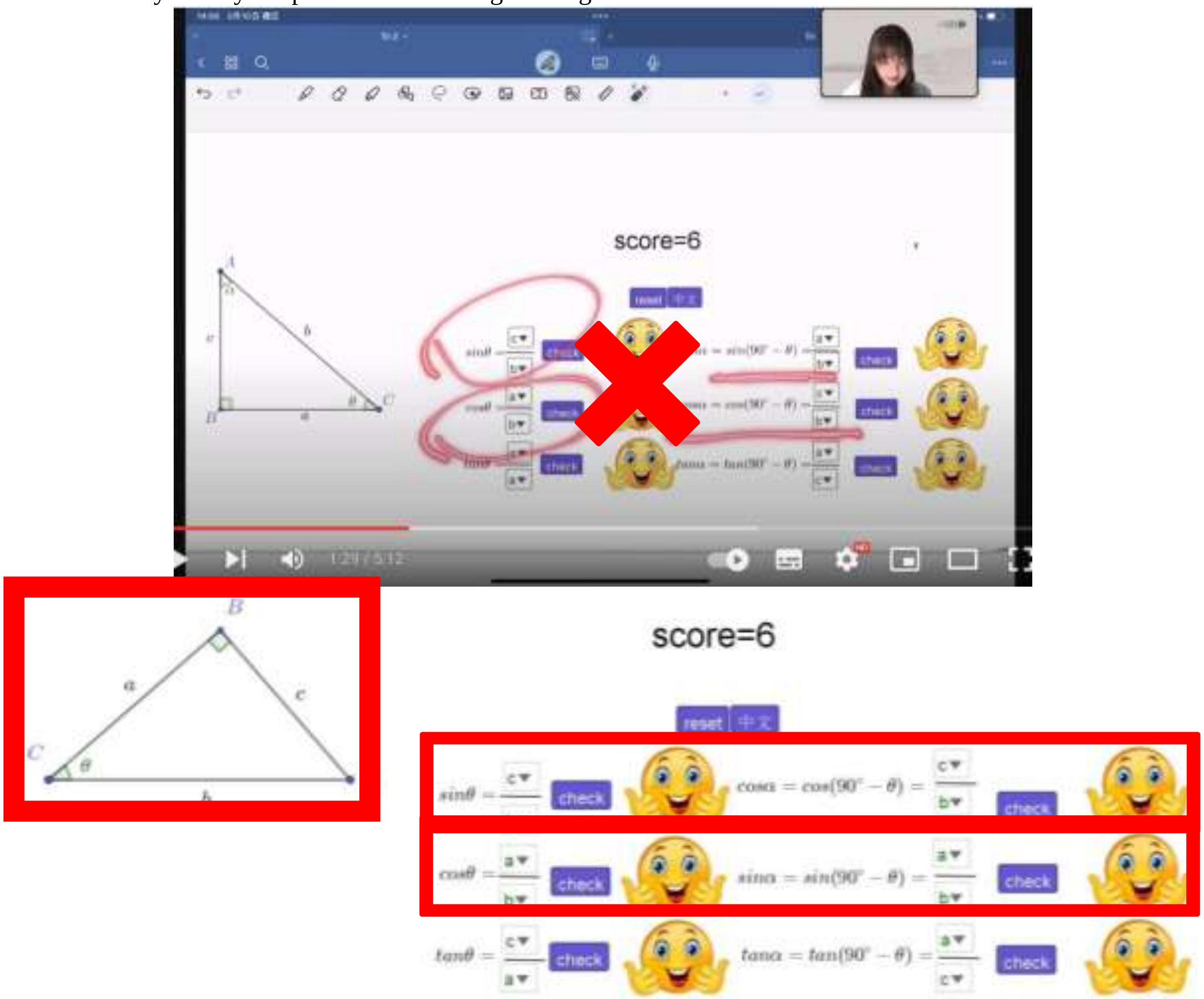
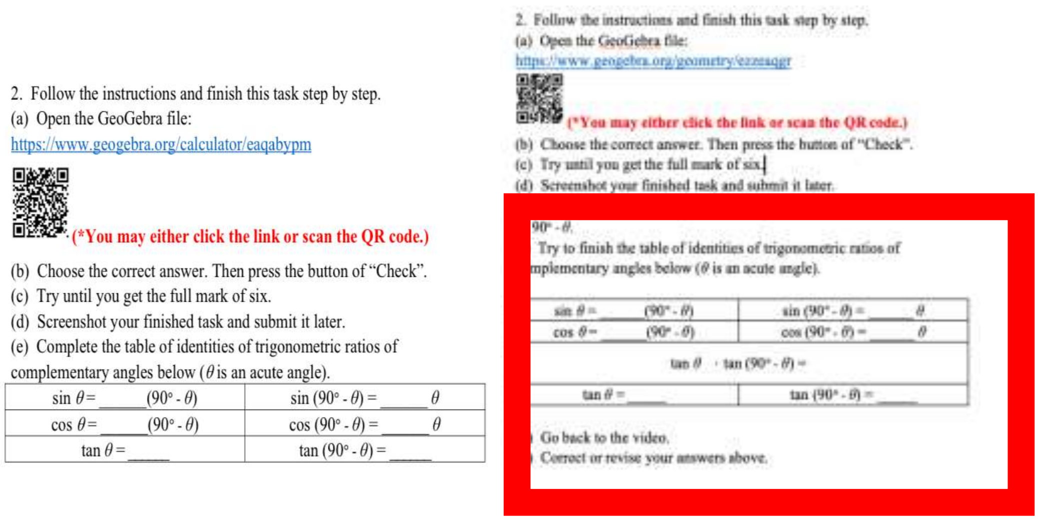


Figure 8:- The screenshot of the modified courseware.

Furthermore, modifications were implemented to the order of instructions on the worksheet. Originally, students would go back to the instructional video before being asked to fill out the table. Nevertheless, this method may not adequately stimulate reflective observations among students. Consequently, the worksheet was modified to ask students to initially fill out the table using their observations, meaning that my video has to be adjusted to include the entire presentation of the seven trigonometric identities, so as to allow students' self-correction and serve as an introduction to the new subject (i.e., demonstration principle).


2. Follow the instructions and finish this task step by step.
(a) Open the GeoGebra file:
https://www.geogebra.org/calculator/eaqabypm
(*You may either click the link or scan the QR code.)
(b) Choose the correct answer. Then press the button of "Check".
(c) Try until you get the full mark of six.
(d) Screenshot your finished task and submit it later.
(e) Complete the table of identities of trigonometric ratios of complementary angles below (θ is an acute angle).

$\sin\theta$ = ____ $(90° - \theta)$	$\sin(90° - \theta)$ = ____ θ
$\cos\theta$ = ____ $(90° - \theta)$	$\cos(90° - \theta)$ = ____ θ
$\tan\theta$ = ____	$\tan(90° - \theta)$ = ____

2. Follow the instructions and finish this task step by step.
(a) Open the GeoGebra file:
https://www.geogebra.org/geometry/ezzeaqgr
(*You may either click the link or scan the QR code.)
(b) Choose the correct answer. Then press the button of "Check".
(c) Try until you get the full mark of six.
(d) Screenshot your finished task and submit it later.

$90° - \theta$.
Try to finish the table of identities of trigonometric ratios of complementary angles below (θ is an acute angle).

$\sin\theta$ = ____ $(90° - \theta)$	$\sin(90° - \theta)$ = ____ θ
$\cos\theta$ = ____ $(90° - \theta)$	$\cos(90° - \theta)$ = ____ θ
$\tan\theta \cdot \tan(90° - \theta)$ =	
$\tan\theta$ = ____	$\tan(90° - \theta)$ = ____

Go back to the video.
Correct or revise your answers above.

Figure 9:- Modifications of worksheets to facilitate students' reflective observation.

Demonstration

The teaching of mathematics involves introducing both factual information and problem-solving skills (Hew & Lo, 2020). Thus, after students have learned the trigonometric identities, the teacher should demonstrate itsapplication with practical examples. The crux of the matter in the realm of this topic is conversion, where students frequently make mistakes based on my teaching experience. Common errors include the incorrect beliefs that "sin 2θ = 2 sin θ" and "sin (θ + 30°) = cos (90° - θ) - cos (90° - 30°)." Therefore, in reference to the approach of utilising incorrect responses to form a contrast with the correct ones (Ekdahl & Runesson, 2015), without changing the core idea of viewing the content in the blanket after the trigonometric ratios as a whole, I combined the aforementioned occurrences into sin (2θ + 30°) as a manifest of the generalization in Variation Theory. Afterwards, I presented the incorrect methods intentionally, clarified the errors, and explained the correct process, to avoid students making similar mistakes when facing such problems in the future.

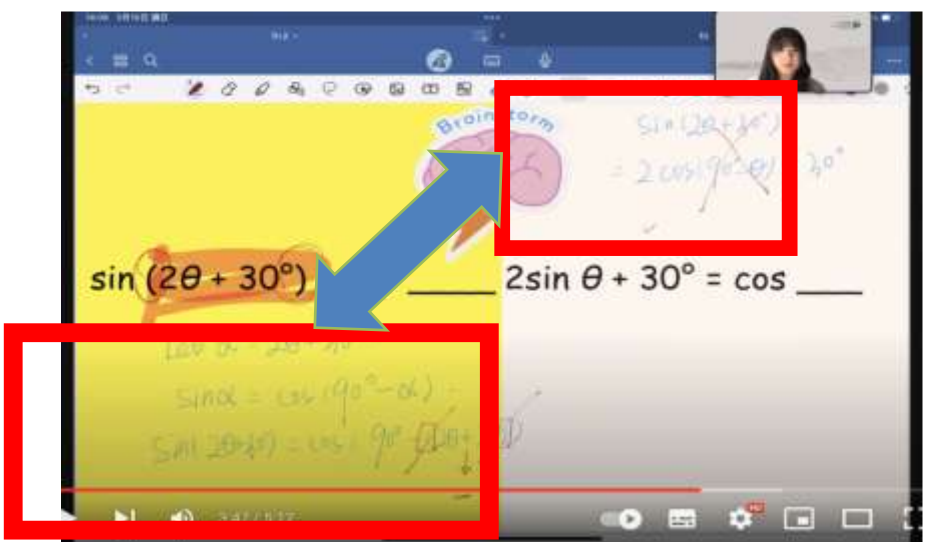


Figure 10:- Making contrast between wrong answers and the correct ones is an effective way to improve mathematical abilities (Ingram et al., 2013).

In the subsequent application of transforming trigonometric ratios, in order to reduce students' cognition load, I chose the question type of solving equations, which can avoid the usage of trigonometric identities(sin^2 θ + cos^2θ = 1, tan θ = (sin θ)/(cos θ)). In the exposition on the process of solving trigonometric equations, I comprehensively used students' existing knowledge of the properties of trigonometric ratios (i.e., sin x increases asx increases), to show students that givensin a = sin b, we can conclude that a = b. The reason for evoking the activation principle in the demonstration phase is that I hope students see the connection among different pieces of mathematical knowledge.Finally, using the concrete example cos θ = sin 35°, I demonstrated the process of solving trigonometric ratios: first converting sin 35° into cos(90°-35°) and then equalizing θ with(90°-35°).

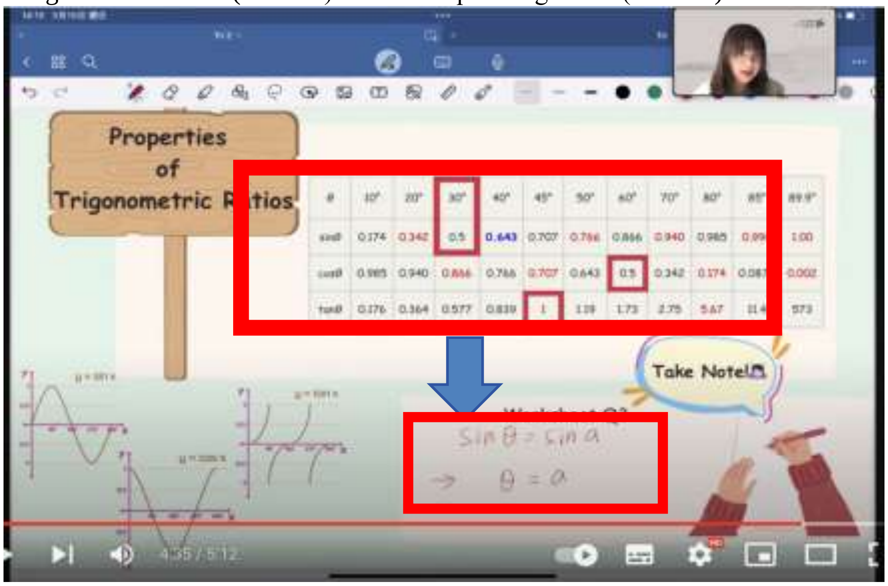
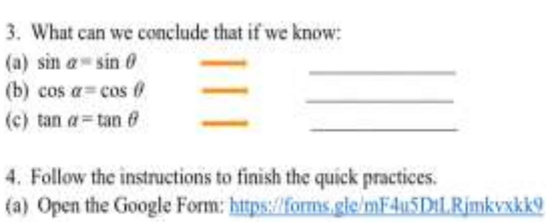


Figure 11:- The new knowledge (i.e., way to solve trigonometric equation) is built upon the old knowledge (i.e., properties of trigonometric ratios).

Nevertheless, after receiving feedback from Dr. Lo, I found the need to make changes of this teaching section. First, it was recognized that students might not make such kinds of mistakes during the transition process. Additionally, I realized that there had been a gap between the abstract identities and their practical use in solving equations, which may result in the situation that even though students know the key point is to convert both sides of equations into expressions with the same trigonometric ratios,they do not know how to do it! Third, according to the Variation Theory, my given counter-example has skipped numerous design steps and may be too complicated for beginners. Therefore, I will substitutethe part containingsin(2θ +30°) with two illustrative exercises: cos 40° = sin_______, tan 30° =$\frac{1}{\tan\ \_\_\_\_}$, aiding in familiarizing students with conversion. In addition, following the idea of separation in Variation Theory, another example of solving tan (90° - α) =$\frac{1}{\tan\ 65°}$ is added after cos θ = sin 35° to introduce the solving technique: transforming the trigonometric ratios of 90° - αback to those of α. Spaces for these four exampleswill be included in the teaching PowerPoint and worksheet, where students can follow the teacher's presentation and write down the completed solving procedures.



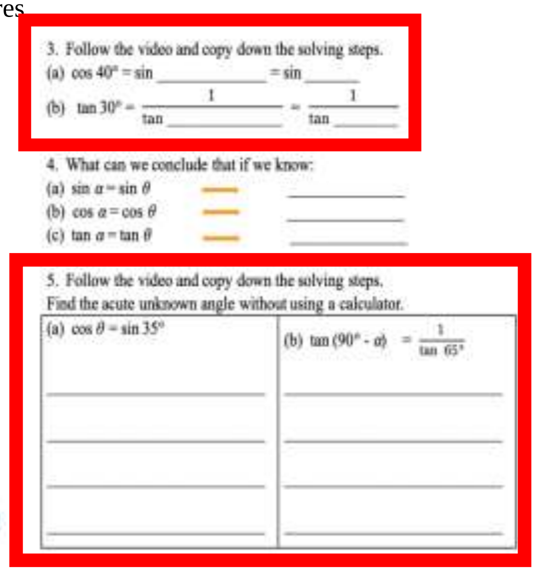


Figure 12:- Revised worksheet with spaces for the four examples.

Furthermore, removal of the graphs of trigonometric functions has been takento avoid students' cognitive pressure.

9. More about graphs of functions	9.1 sketch and compare graphs of various types of functions including constant, linear, quadratic, trigonometric, exponential and logarithmic functions	11	Comparison includes domains, existence of maximum or minimum values, symmetry and periodicity
	9.2 solve the equation $f(x) = k$ using the graph of $y = f(x)$		
	9.3 solve the inequalities $f(x) > k$, $f(x) < k$, $f(x) \geq k$ and $f(x) \leq k$ using the graph of $y = f(x)$		

Figure 13:- Graphs of trigonometric functions will be taught in secondary 4 (CDC, 2017).

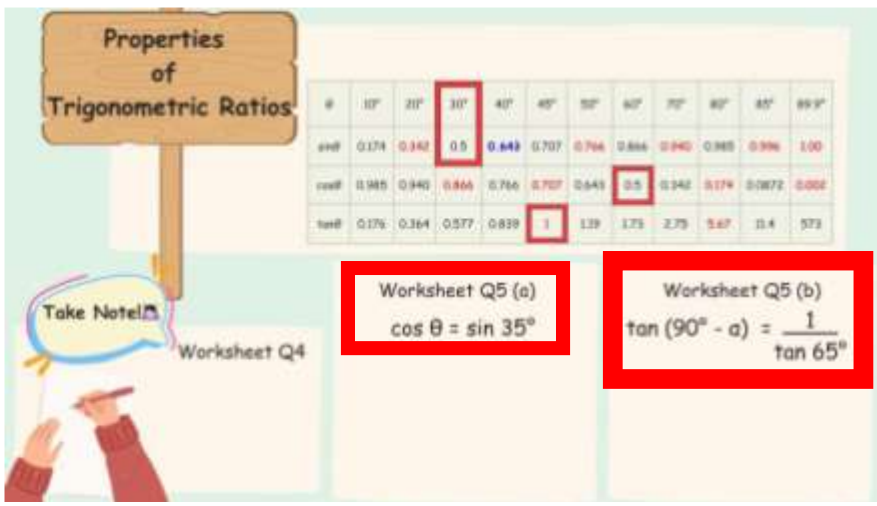


Figure 14:- The teaching PowerPoint after deleting graphs and adding examples.

Application

After going through the instructional video, it is crucial for teachers to provide students with relevant questions to apply the knowledge they have just gained (Lo et al., 2018) (i.e., Merrill's application principle and Kolb's active experimentation phase). Based on the advantage of online quizzes in providing immediate corrective feedback to improve students' metacognitive awareness (Lo et al., 2023), I created three questions using Google Forms, where the first two evaluated students' concept knowledge of trigonometric identities, and the third one resembled the example. Then I 'force' students to retake the test until they attain full marks. Integrating such a self-correction element, this assessment can support students' self-regulated learning in finding solutions and the ideas involved (Anghileri, 2006).Additionally, students are expected to show their problem-solving process on a worksheet, as multiple-choice questions may not fully capture students' mathematical thinking. In this case, students' process of conversion can more significantly imply whether students master the pre-class materials than the final answer. Hence, this method, along with the "words to teacher", which provides students with a platform to express their uncertainties, allows the teacher to identify any conceptual misunderstandings before class and make decisions about addressing these issues or introducing more complex concepts (Lo et al., 2018).

Figure 15:- Students need to i) complete the Google Form, ii) self-correct to obtain full marks, iii) write down the solution stepson the worksheet.

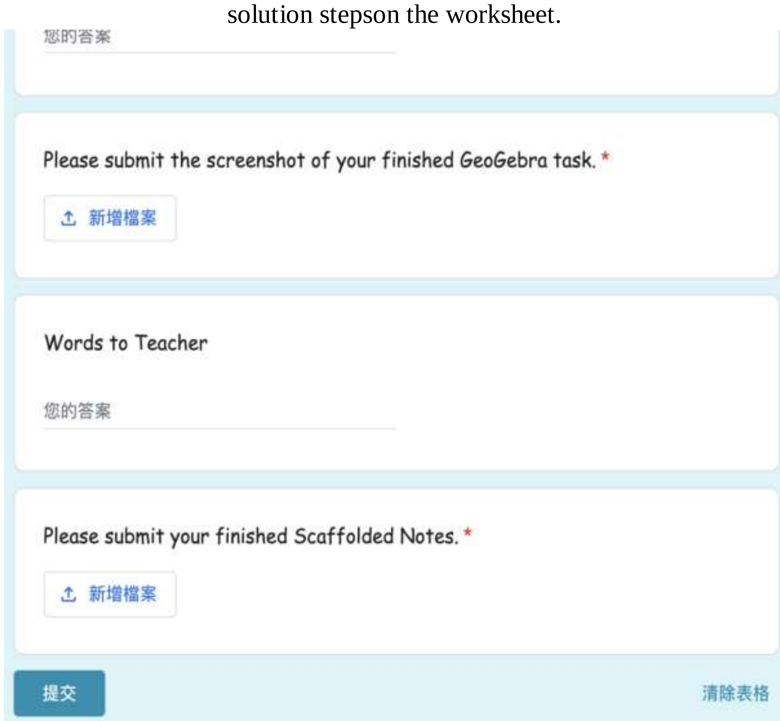


Figure 16:- Words to Teacher.

However, the adjustments to the examples demonstrated in the video will result in modifications here. The original first question actually examined students' concept knowledge of sin (90° - θ) = cos θ ,where option A of the original secondquestion served the same function, so the former will be excluded. Then, due to the resemblance between cos θ = sin 53° (Q3) and cos θ = sin 35° (Example 5a), I will utilize the idea of separation to alter the question to sin α = cos 53° to evaluate students' ability to convert cosine to sine. Lastly, expanding on the previous example of tan (90°

- α) $= \frac{1}{\tan\ 65°}$, I will modify the right-hand side to $\frac{1}{\tan\ (-\alpha+65°)}$. Although the fundamental principle is still converting the trigonometric ratio of (90° - α) back to the one of α, I begin to direct students to the dimension of involving unknowns on both sides of the equation.

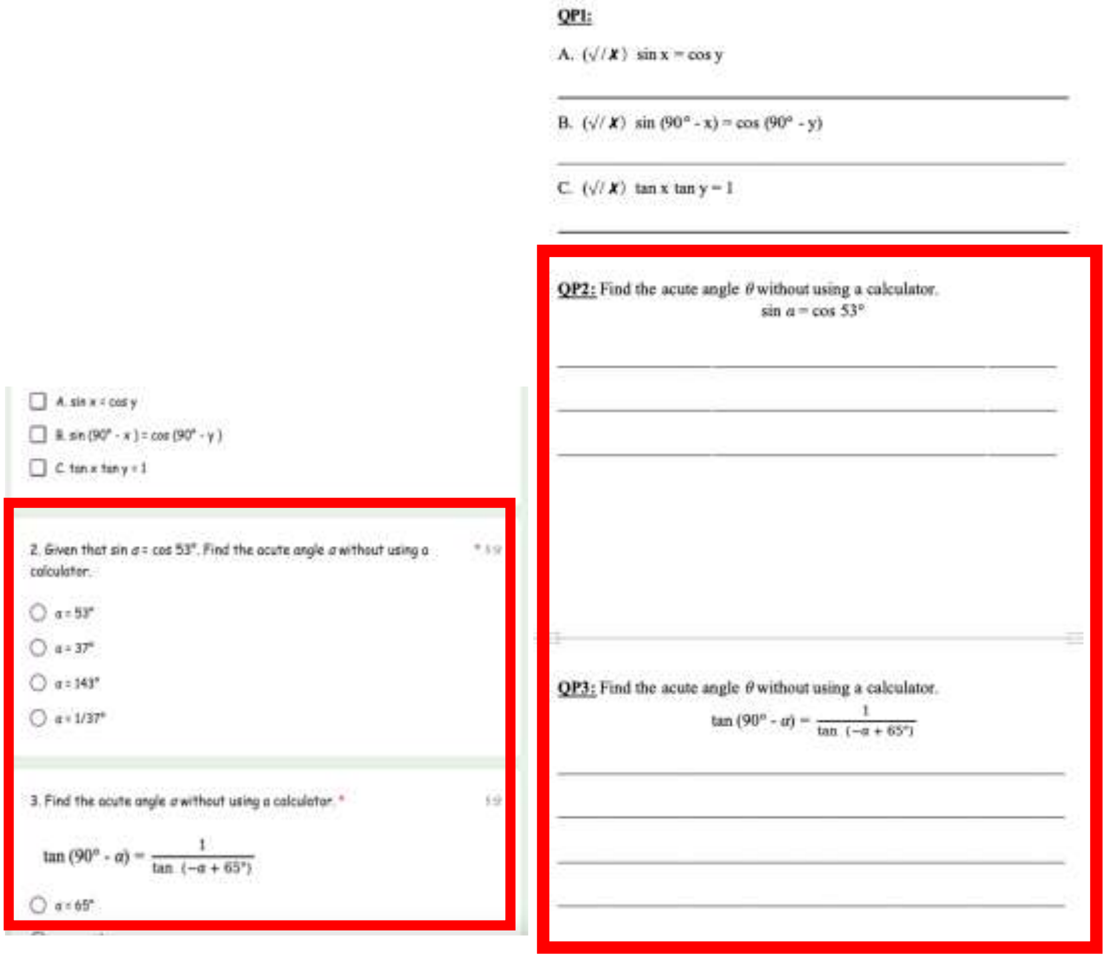


Figure 17:- Revised Google Form and worksheet.

In-class worksheet with rationales and improvements

My initial design mainly focused on engaging students in problem solving. Yet, according to Lo and Hew (2017), in-class learning in the flipped classroom also encompasses concise retrospective learning and mini-didactic lecture. Honor to the word limit, I here briefly them as reviewing the seven trigonometric identities of complementary angles and the two trigonometric identities that are necessary for the exercises (i.e., activation principle), as well as resolving students' misunderstandings arising from the online practices (i.e., demonstration principle), if any.

Part B: In-class Exercise

Review:

Identities of Trigonometric Ratios of Complementary Angles	
sin θ = ______(90° - θ)	sin (90° - θ) = ______ θ
cos θ = ______(90° - θ)	cos (90° - θ) = ______ θ
tan θ · tan (90° - θ) =	
tan θ = ______	tan (90° - θ) = ______
Identities of Trigonometric Ratios	
$\sin^2\theta + \cos^2\theta$ = ______	tan θ = ________

Important Marks:

____

____

____

____

Figure 18:- Revised in-class worksheet with review and problem resolution.

Application & Integration

Unfortunately, I am unable to create real-world problems related to the trigonometric ratios of complementary angles, which restricts the implementation of the problem-centered principle, while the principles of application and integration can be used through students' completing in-class exercises individually, group discussions and presentations. In this student-centered setting, the teacher plays an important role as a facilitator:offering individualized scaffolding-based support (Lo et al., 2018); participating in group discussions to assess understanding and clarify concepts (Kostaris et al., 2017) (i.e., demonstration principle); conducting productive classroom talk through probing for reasoning and encouraging students to restate presenters' ideas, so as to promote mathematical understanding (Resnick et al., 2018). This teaching strategy ensures that the objectives of problem-solving are students rather than the teacher (Mason et al., 2013). And the problems to be solved are designed based on the Variation Theory.

Solving trigonometric equation

Since extremely similar problems have been presented in the pre-class phase, the teacher can evaluate the need for the first question based on students' performance.

The second question starts varying from sin α = cos 53°. By using separation, I add an extra angle component to the left side. Though the underlying skill is the same, students start to realize that there is no longer a mere unknown angle after the trigonometric ratio. The following is tanθ = $\frac{1}{\tan\ (\theta - 40°)}$. In my very first thought, further to the last question, I wanted to put the unknown angles on both sides. However, after several revisits, I found that students might begin from the left by using tanθ = $\frac{1}{\tan\ (90° - \theta)}$, which could not satisfy my design purpose to foster students' senses of integrality. Therefore, I will modify the question into tan2θ = $\frac{1}{\tan\ (\theta - 40°)}$. Finally, I generate the equation as sin (θ + 30°) = cos (-2θ + 75°). No matter how many changes I have made, the critical solving skill is still converting trigonometric ratios to the same one. As this question might be difficult for the targeting students, a hint of setting $\alpha = -2\theta + 75°$ will be displayed on the worksheet as a scaffold for students' understanding of the entirety.

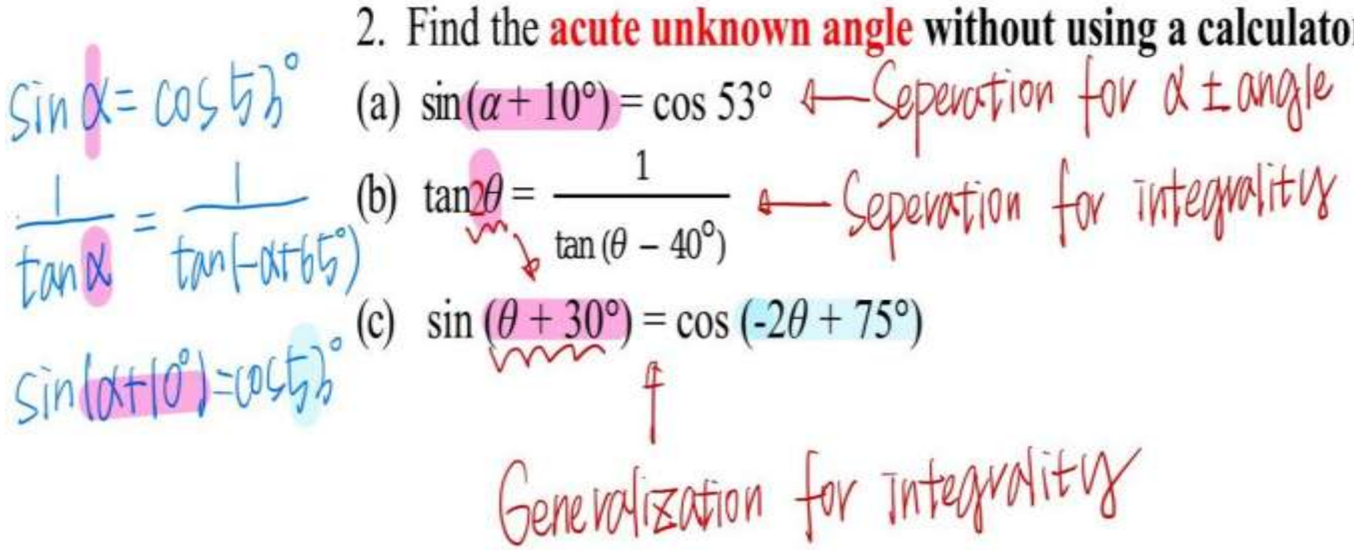


Figure 19:- The modified worksheet with a brief explanation on the utilization of Variation Theory.

Notably, after group presentations, the teacher may ask the presenters to synthesize the solving procedure for the whole class.

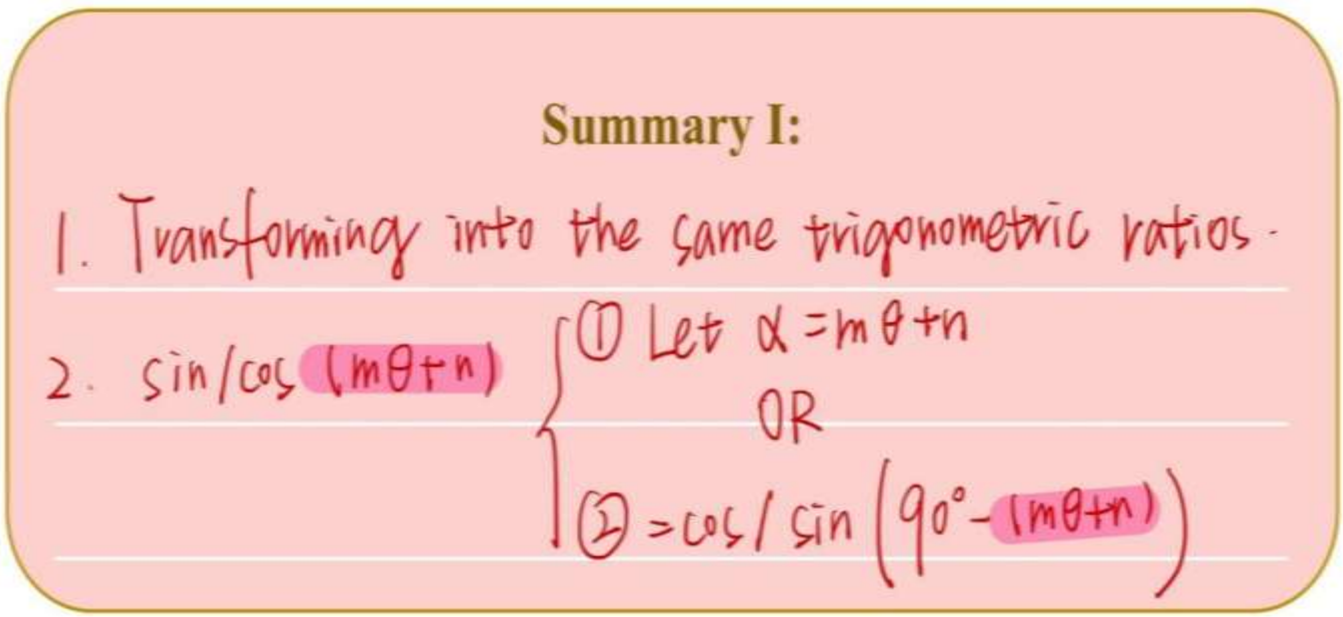


Figure 20:- A summary of the key steps in solving trigonometric equations.

Calculation

The key strategy of calculation is the conversion of different angles to the same one. The expression sin 73° + cos 17° is initially given, and the separation method is used to vary the components' index from 1 to 2, allowing students to apply the learned trigonometric identities to this topic. Later, the question is broadened to evaluate thorough expressions of sine, cosine, and tangent with the help of trigonometric identities. Ultimately, a more intricate situation, including the transformation of two pairs of complementary angles, is presented.

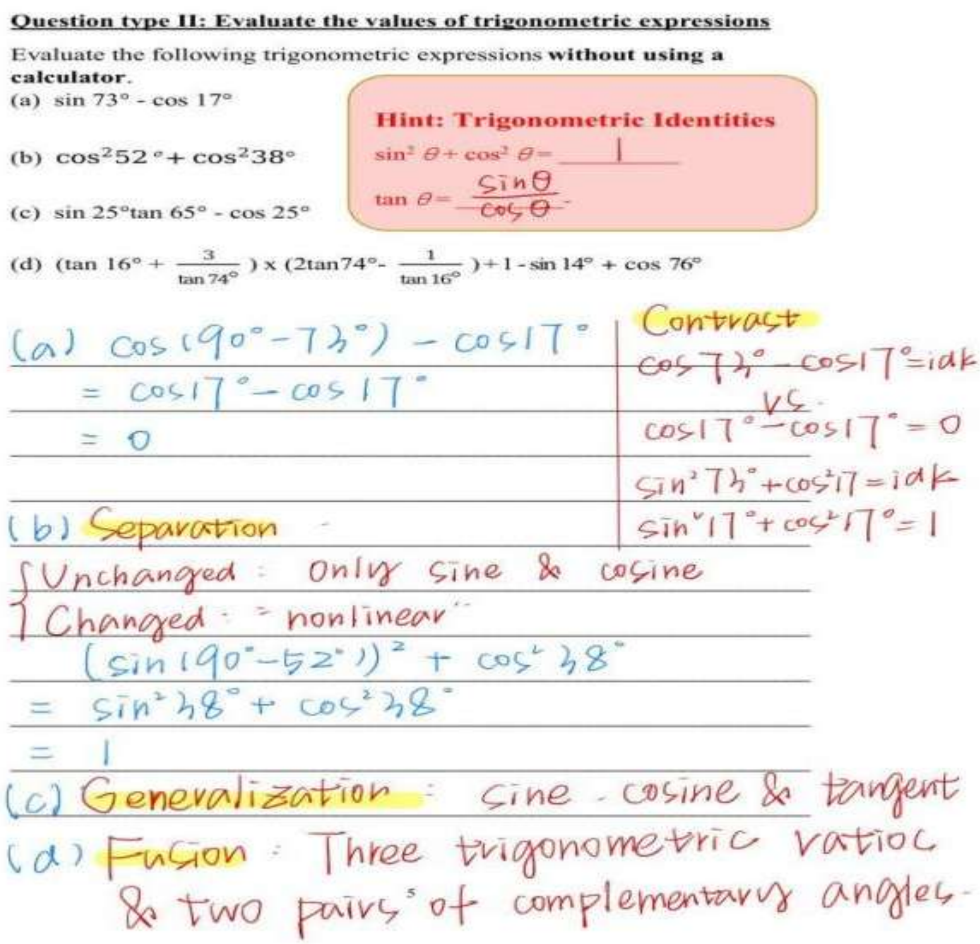


Figure 21:- Detailed exposition of my design of questions.

Simplifying

Inspired by the fusion phase of Variation Theory, the challenging question of simplifying is a mixture of the knowledge covered above. Considering the pressure of lengthy questions on students, I would boldly try gamifying this question on Quizizz by specifying the questions into smaller pieces.

Step 1: Students need to identify which trigonometric expressions should be transformed and what they will be converted into.

Figure 22:- The included knowledge has been demonstrated in in-class exercise *II(c)* and *Example 5b*.

Step 2: The teacher can use the perfect square formula $(a-b)^2 = a^2 - 2ab + b^2$ as parallel modelling to give students hints (Anghileri, 2006) on expanding $(\cos\theta - \sin\theta)^2$.

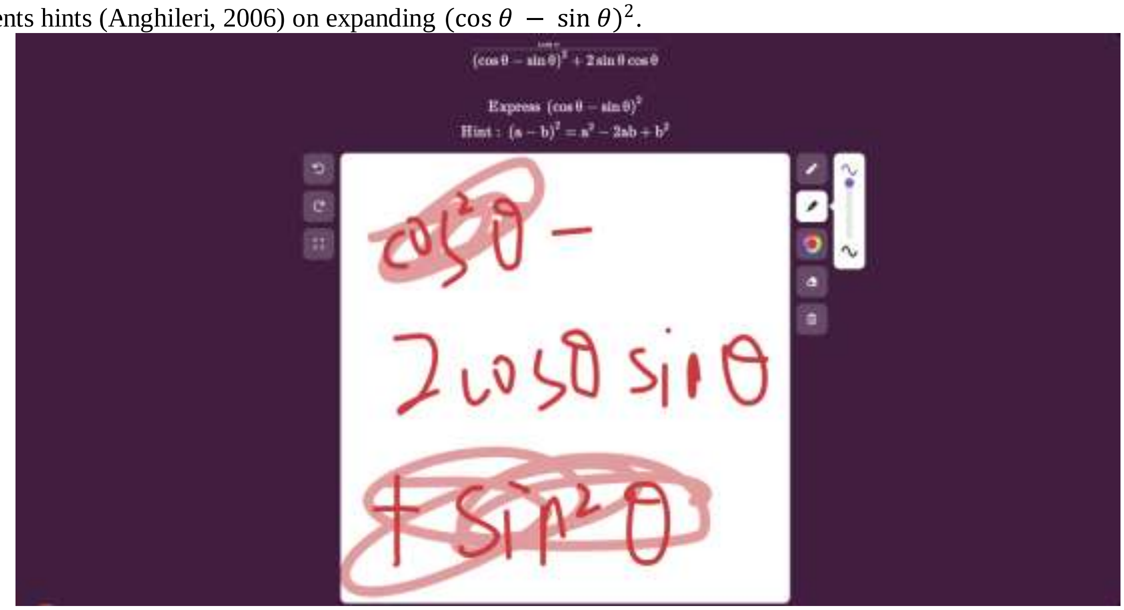


Figure 23:- The corresponding question on Quizizz.

Step 3: The teacher can switch back to PowerPoint and let students write down the completed solving process.

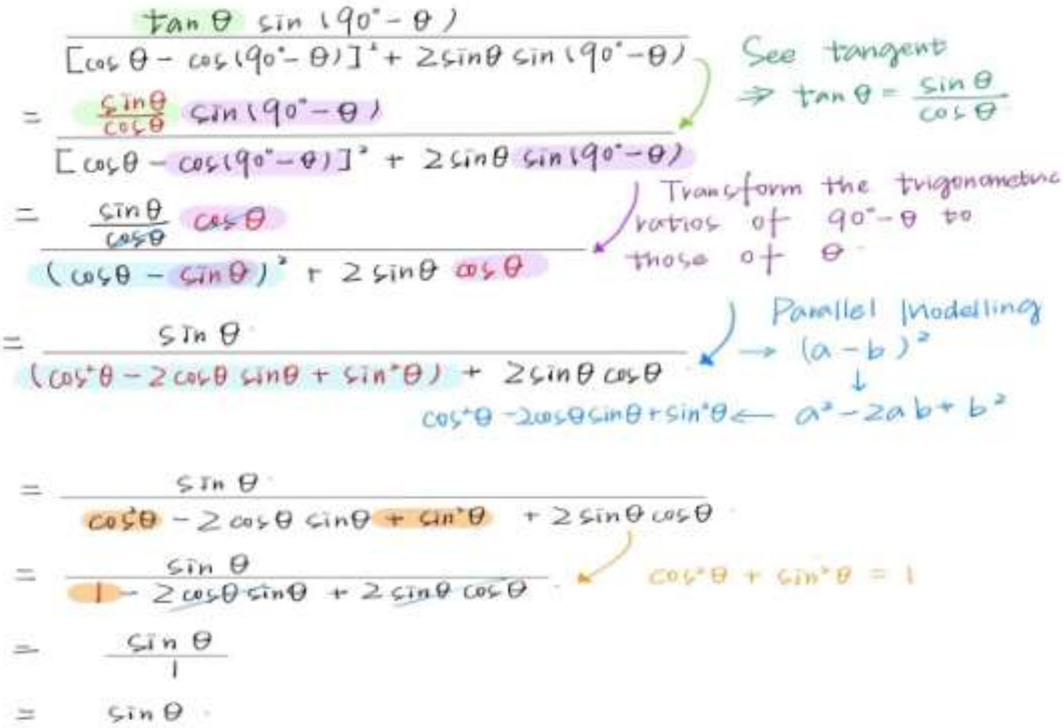


Figure 24:- Scaffolding the solving process with different highlighters.

At the end of this lesson, the teacher can direct students to take notes about the summary of this lesson.

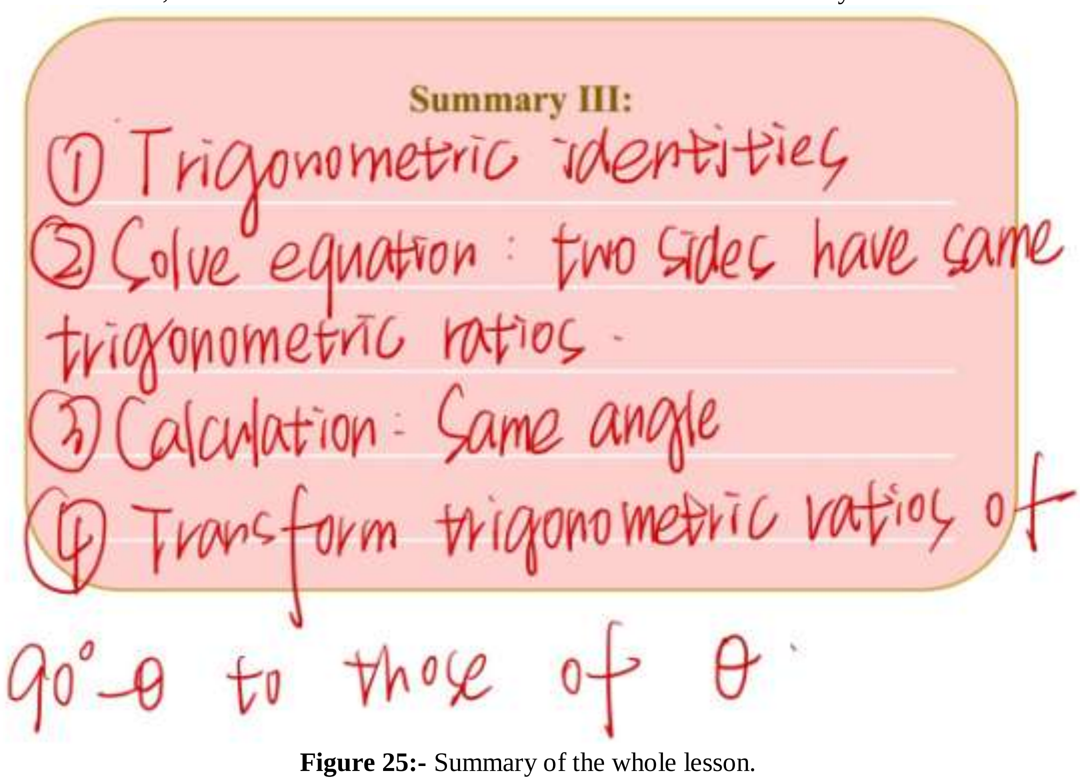


Figure 25:- Summary of the whole lesson.

Conclusion:-

To summarise, following Merrill's First Principle of Instructions, this lesson on trigonometric ratios of complementary angles divides the teaching and learning into the out-of-class dimension and the in-class dimension. Meanwhile, the structure of teaching materials are guided by Variation Theory. Although not without its flaws, I am sorry that it is the "local optimal" teaching plan I can conceive as far.

Appendix 1:- The mind map of my designed flipped classroom on the Trigonometric Ratios of Complementary Angles.

Appendix 2:- My revised worksheet with answers.

Trigonometric Ratios of Complementary Angles

Worksheet

Name: ___________ Class: ___________ SID: ___________

Part A: Scaffolded Notes

1. If α and θ are a pair of complementary angles, how to express α in terms of θ?

____

2. Follow the instructions and finish this task step by step.

(a) Open the GeoGebra file:
https://www.geogebra.org/geometry/ezzeaqgr

(*You may either click the link or scan the QR code.)

(b) Choose the correct answer. Then press the button of "Check".

(c) Try until you get the full mark of six.

(d) Screenshot your finished task and submit it later.

(e) Observe the relationships between trigonometric ratios of θ and those of $90° - \theta$.

(f) Try to finish the table of identities of trigonometric ratios of complementary angles below (θ is an acute angle).

$\sin\theta =$ ______ $(90° - \theta)$	$\sin(90° - \theta) =$ ______ θ
$\cos\theta =$ ______ $(90° - \theta)$	$\cos(90° - \theta) =$ ______ θ
$\tan\theta \cdot \tan(90° - \theta) =$	
$\tan\theta =$ ______	$\tan(90° - \theta) =$ ______

(g) Go back to the video.

(h) Correct or revise your answers above.

3. Follow the video and copy down the solving steps.

(a) $\cos 40° = \sin$ ______________ $= \sin$ _______

(b) $\tan 30° = \dfrac{1}{\tan \_\_\_\_\_\_\_\_\_\_\_\_\_\_} = \dfrac{1}{\tan \_\_\_\_\_\_\_\_}$

4. What can we conclude that if we know:

(a) $\sin \alpha = \sin \theta$ ⟶ _______________

(b) $\cos \alpha = \cos \theta$ ⟶ _______________

(c) $\tan \alpha = \tan \theta$ ⟶ _______________

5. Follow the video and copy down the solving steps.
Find the acute unknown angle without using a calculator.

(a) $\cos \theta = \sin 35°$	(b) $\tan (90° - \alpha) = \dfrac{1}{\tan 65°}$
____________________________	____________________________
____________________________	____________________________
____________________________	____________________________
____________________________	____________________________

6. Follow the instructions to finish the quick practices.

(a) Open the Google Form: https://forms.gle/mF4u5DtLRjmkvxkk9

(b) Try until you get the full mark of 15.

(c) Come back to the worksheet and show your steps below.

(d) If you have any confusion about the instructional video or the quick practice, write down your questions in the "Words to Teacher" in: https://forms.gle/4mbtP7yFJtcXEcCV6

(*This code is different from the above one. Make sure you have assessed to two different Google Forms.)

(e) Scan/ Take photos of your **Scaffolded Notes**.

(f) Submit the files: (i) GeoGebra screenshot; (ii) Scaffolded Notes to the above Google Form.

QP1:

A. (√/ ✗) sin x = cos y

B. (√/ ✗) sin (90° - x) = cos (90° - y)

C. (√/ ✗) tan x tan y = 1

QP2: Find the acute angle θ without using a calculator.

$$\sin \alpha = \cos 53°$$

QP3: Find the acute angle θ without using a calculator.

$$\tan (90° - \alpha) = \frac{1}{\tan \ (-\alpha + 65°)}$$

Part B: In-class Exercise

Review:

Identities of Trigonometric Ratios of Complementary Angles	
$\sin\theta =$ cos $(90° - \theta)$	$\sin(90° - \theta) =$ cos θ
$\cos\theta =$ sin $(90° - \theta)$	$\cos(90° - \theta) =$ sin θ
$\tan\theta \cdot \tan(90° - \theta) =$ 1	
$\tan\theta = \frac{1}{\tan(90° - \theta)}$	$\tan(90° - \theta) = \frac{1}{\tan\theta}$
Identities of Trigonometric Ratios	
$\sin^2\theta + \cos^2\theta =$ 1	$\tan\theta = \frac{\sin\theta}{\cos\theta}$

Important Marks:

eg. $\tan(90° - \alpha) = \frac{1}{\tan(-\alpha + 65°)}$

$\frac{1}{\tan\alpha} = \frac{1}{\tan(-\alpha + 65°)}$

$\alpha = -\alpha + 65°$

$\alpha = 32.5°$

Question type I: Solving trigonometric equations

1. Find the **acute angle** θ **without using a calculator**.
   (a) $\sin\theta = \cos 13°$
   (b) $\sin(90° - \theta) = \cos 13°$
   (c) $\tan\theta = \frac{1}{\tan 65°}$
   (d) $\tan(90° - \theta) = \frac{1}{\tan 65°}$

Hint:

You may write down the identities of trigonometric ratios of complementary angles along the answer area first.

2. Find the **acute unknown angle** without using a calculator.

(a) $\sin(\alpha + 10°) = \cos 53°$

(b) $\tan 2\theta = \frac{1}{\tan(\theta - 40°)}$

(c) $\sin(\theta + 30°) = \cos(-2\theta + 75°)$

Hint:

You may set $\alpha = -2\theta + 75°$

(a) $\sin(\alpha + 10°) = \cos 53°$

$\sin(\alpha + 10°) = \sin(90° - 53°)$

$\alpha = 27°$

2

(b) $\tan 2\theta = \frac{1}{\tan(\theta - 40^\circ)}$

$\frac{1}{\tan(90^\circ - 2\theta)} = \frac{1}{\tan(\theta - 40^\circ)}$

...

(c) $\sin(\theta + 30^\circ) = \cos(-2\theta + 75^\circ)$

$\sin(\theta + 30^\circ) = \sin(90^\circ - (-2\theta + 75^\circ))$

$\theta + 30^\circ = 90^\circ - (-2\theta + 75^\circ)$

...

Summary I:

① Transfore both sidec into the came trigonometric ratioc

② $\sin/\cos(m\theta + n) = \cos/\sin(90^\circ - (m\theta + n))$

Question type II: Evaluate the values of trigonometric expressions

Evaluate the following trigonometric expressions **without using a calculator**.

(a) $\sin 73^\circ - \cos 17^\circ$

(b) $\cos^2 52^\circ + \cos^2 38^\circ$

(c) $\sin 25^\circ \tan 65^\circ - \cos 25^\circ$

(d) $(\tan 16^\circ + \frac{3}{\tan 74^\circ}) \times (2\tan 74^\circ - \frac{1}{\tan 16^\circ}) + 1 - \sin 14^\circ + \cos 76^\circ$

Hint: Trigonometric Identities

$\sin^2\theta + \cos^2\theta = 1$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

(a) $\sin 73^\circ - \cos 17^\circ = \cos(90^\circ - 73^\circ) - \cos 17^\circ$

$= 0$

(b) $\cos^2 52^\circ + \cos^2 38^\circ = \sin^2(90^\circ - 52^\circ) + \cos^2 38^\circ$

$= 1$

(c) $\sin 25^\circ \tan 65^\circ - \cos 25^\circ$

$= \sin 25^\circ \dfrac{\sin 65^\circ}{\cos 65^\circ} - \cos 25^\circ$

$= \sin 25^\circ \dfrac{\cos 25^\circ}{\sin 25^\circ} - \cos 25^\circ$

$= 0$

(d) ...

$= (\tan 16^\circ + 3\tan 16^\circ) \times (2\tan 74^\circ - \tan 74^\circ) + 1$

$= 5$

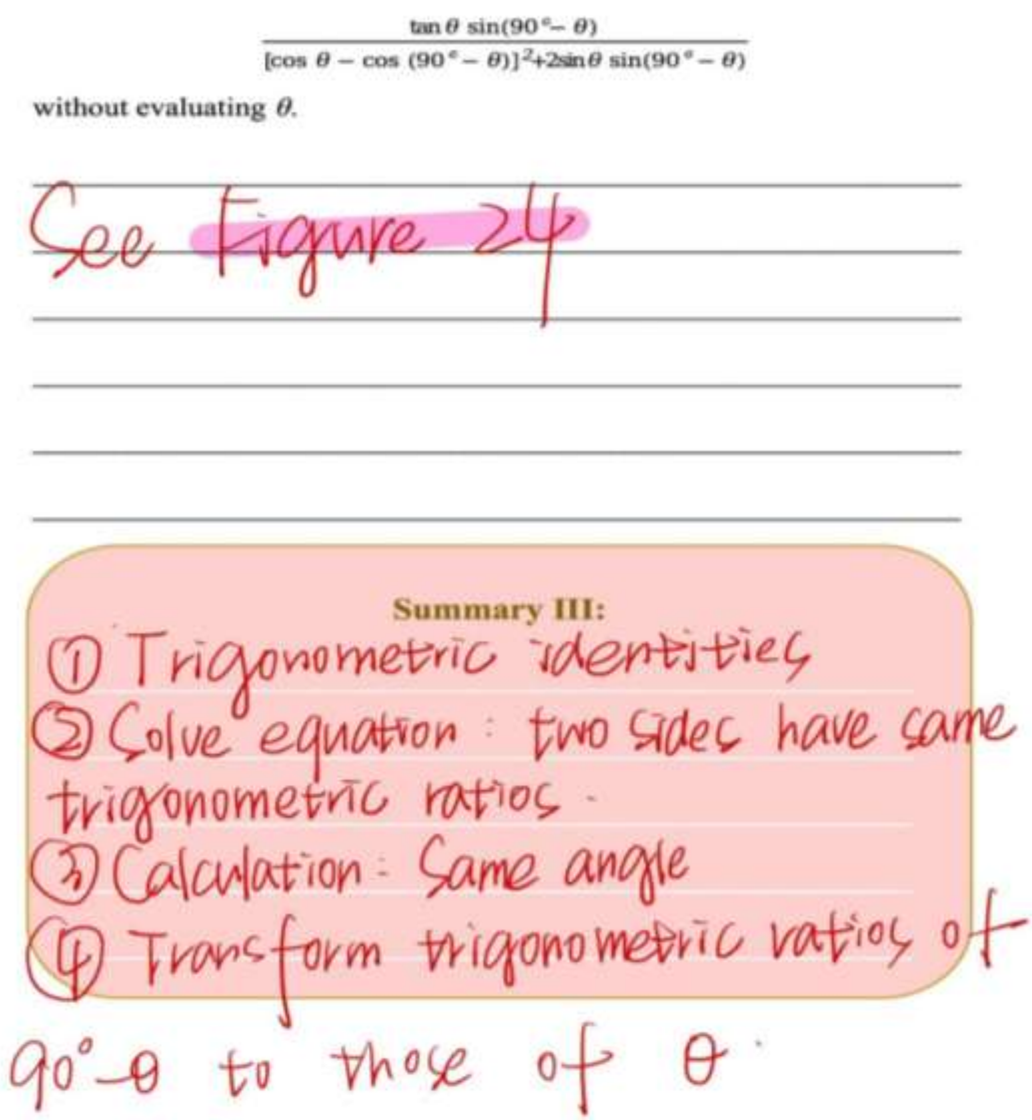
Summary II:

① Transform into the trigonometric ratios of the same angle.

② See $\tan\theta$, transform to $\dfrac{\sin\theta}{\cos\theta}$.

Question type III: Simplify the values of trigonometric expressions

Given that $\cos\theta = \frac{1}{8}$, find the value of



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