Journal Homepage: -www.journalijar.com

INTERNATIONAL JOURNAL OF ADVANCED RESEARCH (IJAR)

Article DOI:10.21474/IJAR01/20523
DOI URL: http://dx.doi.org/10.21474/IJAR01/20523

ISSN NO. 2320-5407

RESEARCH ARTICLE

COMPUTATION OF PREMIUM FOR CLOUD INSURANCE UNDER THE POISSON MODEL

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Manuscript Info

Manuscript History
Received: 28 December 2024
Final Accepted: 31 January 2025
Published: February 2025

Key words:-
Cloud, Cloud Insurance, Service Level Agreement, Premium Principles, Poisson model for outages

Abstract

Cloud services are now indispensable to operate businesses; run educational institutions manage finances and even for governance of nations. Although cloud has simplified extremely complex operations, with additional comfort arises the need of extra caution. Both the consumer of the cloud and the Cloud Service Provider (CSP) are vulnerable to threats such as cloud outages, hacking, cyber thefts and disruptions in internet access. As a result, resorting to insurance to protect their financial status is a natural solution. The insurer on the other hand has to cleverly decide the premium to be charged particularly when he sells a policy to the CSP in order to prevent any capital erosion as claims can be massive in magnitude. In this paper, we consider various premium principles that can be adopted by the cloud insurer to decide an accurate premium depending on the situation at hand. We then utilize theseprinciplesto determine the premium for an insurer who assumes the celebrated Poisson model for outages.

Introduction:-

The modern day world boasts of a digital revolution so much so that every piece of information that we come across today can be backed up or secured using what is known as the cloud. Cloud has indeed become sacrosanct for all enterprises, governments, banks, educational institutions and even a common man who keeps a record of his personal information like photographs, receipts, travel history and even maintains a diary using the cloud.The 'Remote Working' regime post COVID-19 has magnified the dependence on IT infrastructure, services and the cloud (c.f. Tsohou et al. (2023)). However extreme convenience and ease are generally accompanied by threats so much so that even a small hiccup can jeopardize the entire set up. The same is true in respect of the cloud as the speed with which the IT industry and cloud services are evolving is mammoth but cybercrimes have exacerbatedwith an even greater rate. Whether they are ransomware attacks, espionage attempts, data breaches or mega outages, all have the potential to cause turmoil in the cloud services. As a result to safeguard the interests of the buyers of the cloud services, a well laid out contract called a 'Service Level Agreement' (SLA) is chalked out between the CSP and the client. This contract offers guarantee compensation to the users in the event of failure of cloud services. The compensation offered in the case of long outages may be large enough to turn the balance sheet of the CSP upside down. He therefore resorts to insurance. Insurers on the other side have to be alert enough to be able to survey the cyber landscape in order to charge the correct amount of premium from the CSP so as to be able to protect their financial integrity. Catastrophic cyber events can actually wipe out an insurer from the market as the

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cost of a massive outage to a hyperscale cloud provider could possibly inflate to trillions and the insurance industry would not be able to bear the economic costs of such an outage (c.f. Carnegie Endowment (2021)).

Cloud insurance or cyber insurance is still in its embryonic stages and the insurance industry is still figuring out the methodology of setting the premiums that they need to charge for the cyber class of risks. A 2020 report by Allianz Global Corporate & Specialty (c.f. Allianz (2020)) highlights that cyber claims have been bulging steadily both in terms of their number and complexity. Further GAO (2021) emphasizes that there is an upward trend in the premiums charged by insurance companies for offering cyber insurance to its clients as shown in Figure 1 below. The report also notes that the demand for cyber insurance has inflated as the involved players are now in a better position to understand and respond to the rising cyber risks. In fact as noted in Aon (2022), the US domestic market recorded a huge direct written cyber insurance premium of $7.2 billion in 2022.

In spite of playing such a significant role in the cyber security practices, the literature on determination of premium for the cloud insurer is negligible. To the best of our knowledge, this paper is indeed the first attempt to utilize well known premium principles available in actuarial literature to set up the premium for the cloud insurer.

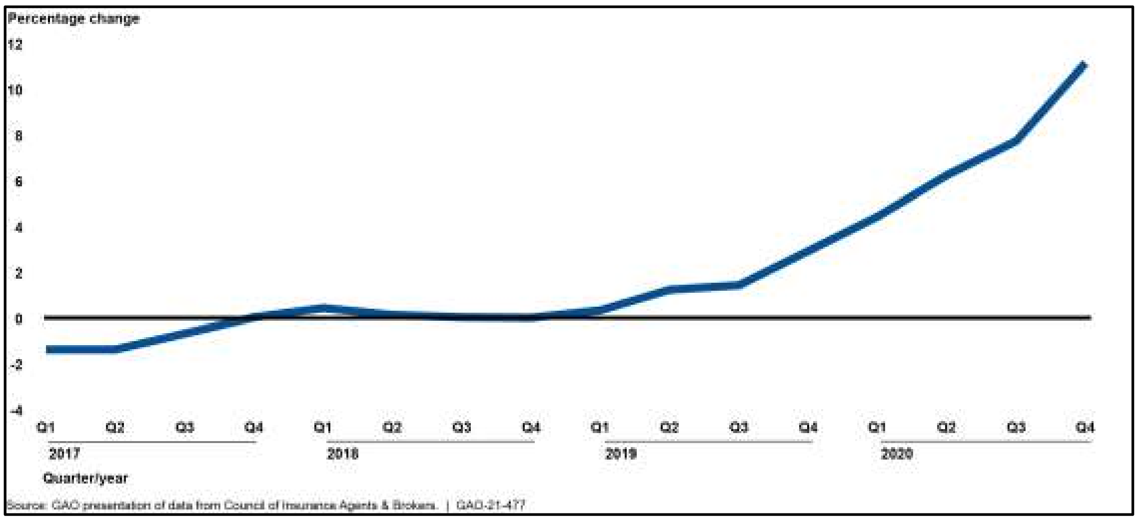

Figure 1:- Change in Cyber Insurance Premiums, 2017–2020.

In the passing it is very important to decipher what exactly the word premium means for an insurance company. A premium is the payment that an insurance seeker; in our case the CSP makes for complete or partial; in our case complete protection or insurance cover against a risk. We generally denote a risk by X and when we consider the risk X, it actually means the non-negative random variable of all possible claims that arise from this risk and are presented to the insurance company. In the present context, we are dealing with the risk of cloud outages. Also when a cloud outage occurs, it results in a loss for the CSP in terms of the compensation amount that it would need to pay to its customers according to the clauses of the SLA. The CSP then considers the cumulative losses over a period and reports them as claims to the insurance company to which it paid a premium for indemnifying itself against outages. As a result the premium charged by the insurance company must be some function of X (c.f. Dickson (2005)).In fact, we denote the premium as π_X and a rule that designates a numeric value to π_X is referred to as a premium calculation principle. A premium principle is thus presented as $\pi_X = f(x)$; where $f(x)$ denotes some function of x. In this paper we undertake the task of determining the correct premium π_X for the insurer providing coverage to the CSP.

The paper is organized as follows: Section 2 presents the most well-known Poisson model that describes the functionality of the cloud. Section 3 glances on six well known premium principles and utilizes them to determine the premium formulae for the insurer who assumes a Poisson model for the outages. Section 4 then showcases the outlined theory for the Poisson model for the number of outages by presenting powerful numerical results. Section 5 concludes the paper and highlights directions for future work.

The Poisson Model for Cloud Outages:-

Any organization or infrastructure works smoothly when the cloud is 'ON' and everything comes to a halt when the cloud is gone or 'OFF'. This principle is the key to devising models for cloud outages. As noted in the previous statement, we consider only two states for the working of the cloud, the functional or the 'ON' state nomenclated as 'S' and the non-functional or the 'OFF' state denoted by 'D'. These are also referred to as 'Uptime' and 'Downtime' for the cloud working as shown in Figure 2. We ignore here the intermediate state set-up which is referred to as 'Graceful Degradation', where some features of the cloud are operational while others fail to function. So in our case, the cloud keeps oscillating between the ON and OFF states. In order to present fruitful results on the working of the cloud, we have to consider appropriate distributions for the working and the non-working states.

One can lay hand on a very scanty availability of statistical models in the literature that describe the Uptime or ON period and the downtime or OFF period. These are summarized below (c.f. Naldi and Mazzoccoli (2018) and Mastroeni et al. (2019)):
(i) The Poisson Model (the Markov Model or Exponential-Exponential Model)
(ii) The Exponential-Pareto Model
(iii) The Pareto-lognormal Model

These three models vary in terms of the probability distributions that describe the duration of the ON and OFF states. Out of these models, the Poisson model is the most popular and is described below.

The Poisson model is the simplest out of the three where the duration of both the ON and OFF states of the cloud are described by an exponential distribution (c.f.Mastroeni and Naldi (2011), Naldi (2018) andNaldi and Mazzoccoli(2018)).We denote the parameters of the exponential distribution in the ON and OFF state asλ_S and λ_D respectively. Clearly then the cumulative distribution functions (c.d.f.s) are given by:

$$F_S(x) = 1 - e^{-x\lambda_S} \; ; \; x \geq 0 \tag{1}$$

and

$$F_D(x) = 1 - e^{-x\lambda_D} \; ; \; x \geq 0 \tag{2}$$

In this case, the number of outages say N over a time interval say T follow Poisson distribution with parameter $\lambda_S T$. The use of Poisson distribution for number of outages has also been outlined in service level agreement decision making (c.f. Franke et al. (2013)).This implies

$$P[N = n] = e^{-\lambda_S T} \frac{(\lambda_S T)^n}{n!} \; ; \; n = 0,1,2, \ldots, \tag{3}$$

which is essentially the probability law for the Poisson Distribution.

The covenants of the 'Service Level Agreement' dictate that the remedial damage amount or compensation to be paid to the consumer of the cloud by the CSP should be proportional to the number of outages in a specified interval say T, not taking into account their duration. As a result

$$X = cN \tag{4}$$

where X is defined in the previous section,c is the reparationto bepaid for each outage and N is the number of outages. As seen above, Nadheres to a Poisson law with parameter $\lambda_S T$. For brevity, we denote $\lambda_S$by λ. We further note that for a Poisson distribution with parameter λT, the mean and variance coincide with the parameter. This implies:

$$\mathrm{E}[X] = c\lambda T \tag{5}$$

and

$$\mathrm{V}[X] = c^2 \lambda T. \tag{6}$$

where E[X] and V[X]denote respectively the expectation and variance of the random variable X. We now turn our attention to the main focus of this paper viz. the premium principles and utilize them to compute the premium for the Poisson model of outages.

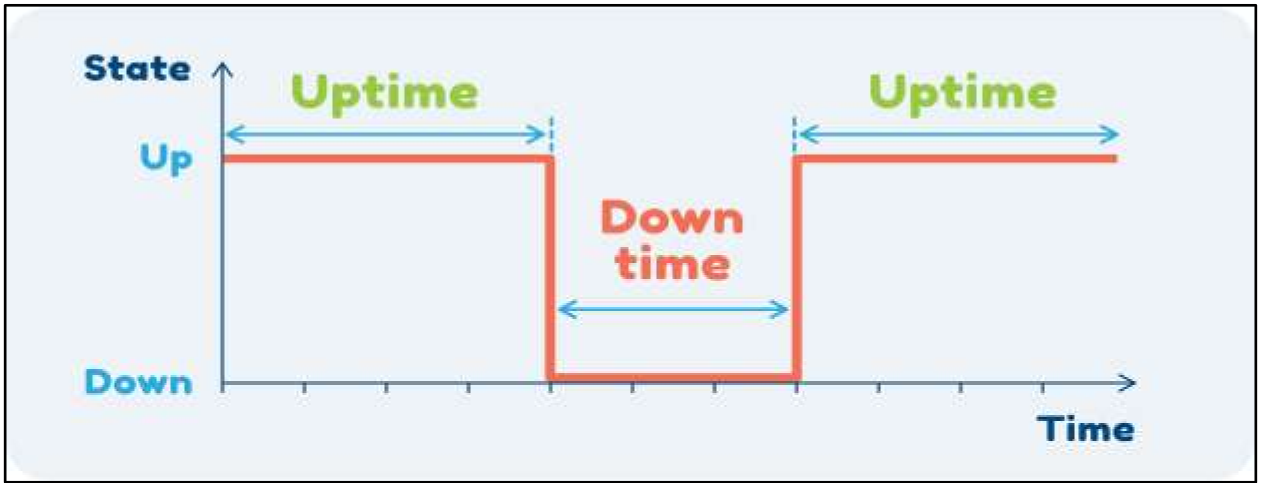

Figure 2:- Cloud State Sequence.

The Premium Principles and their Application to the Poisson Model for Cloud Outages:-
The actuarial literature offers a plethora of premium principles for computation of the premium for the insurer. We stick to six popularly used premium principles stated below one by one. Good references for premium principles are Dickson (2005) and Lima Ramos (2017).

1. The Pure Premium Principle
Definition: The pure premium principle defines

$$\pi_X = \mathrm{E}[X] \tag{7}$$

It is the simplest premium principle but unattractive for the insurer due to absence of any loading for profit or for unfavourable claim experience which is quite common for a cyber-insurer.

*Pure Premium for the Poisson Model:*Using equations (5) and (7), the pure premium in the case of Poisson model of outages is

$$\pi_X = c\lambda T \tag{8}$$

As a result the pure premium is directly proportional to c, i.e., the compensation offered by the CSP to the customer for each outage as well as the average number of outages during time T viz. λT. This seems obvious because as the compensation offered by the CSP for each outage increases or the average number of outages explode, the claim amount would bulge and so the insurer is liable to charge a higher premium. We explore the dependence of the pure premium on c and λTnumerically in the next section.

2. The Expected Value Principle
Definition: The expected value principle declares

$$\pi_X = (1+\alpha)\mathrm{E}[X] \tag{9}$$

where $\alpha > 0$ is termed 'the Premium Loading Factor' with $\alpha E[X]$ representing the loading.

Again, like the pure premium principle, it is a very simple principle. However it suffers from the drawback that it ignores the variability of the risk distribution apparently assigning the same premium to all risks that have equal means. Accounting for the dispersion of the risk distribution is crucial when dealing with highly uncertain events such as cloud outages.

Premium for the Poisson Model: Using equations (5) and (9), the expected value premium principle yields the following premium in the case of Poisson model of outages:

$$\pi_X = (1+\alpha)c\lambda T \quad (10)$$

Thus, the premium is directly proportional to the loading factor α, the compensation offered by the CSP to the customer for each outage c, as well as the average number of outages during time T viz. λT, all connected by a linear relationship. We undertake the dependence of the premium on α, c and λTnumerically in the next section.

3. The Variance Principle
Definition: The variance principle sets

$$\pi_X = \mathrm{E}[X] + \alpha \mathrm{V}[X] \quad (11)$$

where $\alpha > 0$ is again 'the Premium Loading Factor' as in the Expected Value Principle with $\alpha V[X]$ representing the loading which is proportional to $V[X]$.

The variance principle eliminates the drawback of the expected value principle and incorporates the variability of the risk distribution leading to a more accurate pricing. However, even risks having identical first two moments may be worlds apart in the insurer's perspective especially when losses follow a highly skewed distribution. This advocates considering premium computation principles that incorporate the entire probability distribution of the risk X rather than encompassing just a few statistical properties (c.f. Denuit (1999)). This is particularly true in the context of unpredictable and emerging risks such as cyber risks. Many such principles exist in the actuarial literature such as the Orlicz principle (c.f. Haezendonck, J., and Goovaerts, M. (1982)), the Esscher Principle (c.f. Esscher (1932)) and the risk adjusted premium principles (c.f. Wang (1996)). Out of these, we undertake the Esscherpremium principle as the last premium calculation principle in our discussion here.

Premium for the Poisson Model: Using (5), (6) and (11), the variance principle yields the following premium in the case of Poisson model of outages:

$$\pi_X = (1+\alpha c)c\lambda T \quad (12)$$

Equation (12) displays a quadratic relationship between the premium π_X and the compensation offered for each outage *c*. This means that the insurer is very sensitive to increases in the remedial amount offered by the CSP to the user for each outage. Weinvestigate the dependence of the premium on α, c and $\lambda T$with numbers in the next section.

4. The Standard Deviation Principle
Definition: The standard deviation principle defines

$$\pi_X = \mathrm{E}[X] + \alpha\sqrt{\mathrm{V}[X]} \quad (13)$$

where $\alpha > 0$ is again 'the Premium Loading Factor' with$\alpha\sqrt{V[X]}$ representing the loading which is proportional to the standard deviation of the loss distribution.

The standard deviation principle echoes the variance principle and incorporates the variability of the risk distribution leading to a more reliable premium value.

Premium for the Poisson Model: Using (5), (6) and (13), the standard deviation premium principle leads to the following premium in the case of Poisson model of outages:

$$\pi_X = (\sqrt{\lambda T} + \alpha)c\sqrt{\lambda T} \quad (14)$$

We consider the behaviour of the premium with changing values ofα, c and λT in the next section.

The first four principles are based on the moments of the loss distribution. The next principle delves on 'Expected Utility Theory': an extremely important concept in actuarial science and insurance.

5. The Principle of Zero Utility
The utility concept was a brainchild of Daniel Bernoulli (1738) (c.f. Bernoulli (1953)) which led to the germination of 'Expected Utility Theory'. Bernoulli had the belief that the value that an individual attaches to a given monetary amount is not the monetary value itself but indeed its 'moral' value, importance or utility. Quoting him

"(…) an item's value must not be idealized upon its price but the utility it provides to his owner. An item's price depends on the item alone, it is the same to everyone; its utility though depends on the individual circumstances of the subject who assesses it." (Bernoulli, 1738).

This led to the formulation of the utility function (u.f.) $u(x)$, which can be portrayed as a function that numeralizes the value or utility that an individual or an organization assigns to the monetary amount x(c.f.Preet Singh Kapoor and Jain (2011)). More precisely, utility is a function $u: \mathbb{R}_0^+ \to \mathbb{R}_0^+$ that declares a utility to each monetary amount of the domain.The utility function complies with the principle of non-satiation, i.e.$u'(x) > 0$. This means that $u(x)$is an increasing function of wealth x such that individuals have preference for more wealth to less. In insurance and financial sector, the investor preferences are influenced by their attitude towards risk, which can be realized in terms of properties of utility functions. Three possibilities arise: risk-averse, risk-neutral or risk-loving investors (c.f. Dickson (2005)). For a risk-averse (risk-loving) investor an incremental increase (decrease) in wealth is of less interest than an incremental decrease (increase). Accordingly, the utility function $u(x)$ is strictly concave (convex), that is, $u''(x) < (>)0$ for a risk averse (risk loving) investor. A risk- neutral investor is unaffected by risk and for him, $u'(x) > 0$and$u''(x) = 0$. The form of the utility function can be chosen to model an individual's preferences according to whether or not, he likes, dislikes or is indifferent to risk. The greater the curvature of $u(x)$, the more marked will be the risk aversion calculated by the coefficient of risk aversion defined as:

$$r(x) = -\frac{u''(x)}{u'(x)} \tag{15}$$

$r(x)$is also known as the Arrow-Pratt measure of absolute risk-aversion (ARA) (c.f. Arrow (1971) and Pratt (1964)).The absolute risk aversion can be increasing or decreasing abbreviated as IARA and DARA respectively based upon on the nature of$r(x)$. In case if $r(x)$is a constant and does not vary with the monetary amount x, then the utility function is said to be adhering to 'Constant Absolute Risk Aversion (CARA)'.

Further, thereare several utility functions that are used in actuarial, financial and economic literature.The most popular among them are (c.f. Dickson (2005) and Lima Ramos (2017)):

1. Exponential with parameter β: $u(x) = -exp(-\beta x)$; $\beta > 0$.

2.Quadratic with parameterβ: $u(x) = x - \beta x^2$; $x < \frac{1}{2\beta}, \beta > 0$.

3. Logarithmic with parameterβ: $u(x) = \beta \log x$; $x > 0, \beta > 0$.

4.Fractional with parameter β: $u(x) = x^\beta$; $x > 0, 0 < \beta < 1$.

5.Linear: $u(x) = x$.

The CARA property defined above is indeed characteristic of the exponential utility function and due to this reason exponential utility function has been extensively harnessed in the cyber insurance literature (c.f. Mazzoccoli and Naldi (2020), Naldi and Mazzoccoli(2018), Martinelli et al. (2017), Marotta et al. (2017) and Böhme and Schwartz (2010)).

We now define the principle of zero utility.
Definition: The principle of zero utility stipulates that the minimum premium π_X that an insurer with initial wealth, *w,* should allocate to provide full insurance coverage against a risk *X* should satisfy the following equation.

$$u(w) = \mathrm{E}[u(w + \pi_X - X)] \tag{16}$$

As a result the premium will in general depend on the insurer's surplus or wealth *w*. However an exception occurs when we employ the exponential utility function given in (1) in the list of u.f.s above. Then

$$\pi_X = \frac{1}{\beta} \ln M_X(\beta) \tag{17}$$

where $M_X(\beta)$ denotes the moment generating function (m.g.f.) of the loss random variable X with parameter β such that:

$$M_X(\beta) = \mathrm{E}\left[e^{\beta X}\right] \tag{18}$$

provided the expectation exists.It is important to note that β also happens to be the coefficient of risk aversion defined above. In this case the principle of zero utility is called the exponential principle. As this principle is based on the m.g.f. of the loss distribution, it embodies more information about *X* than any principle considered till now.

To compute the premium for the Poisson model of cloud outages we will consider the exponential u.f. and therefore stick to the exponential principle.

Premium for the Poisson Model: Using equations (17) and (4), the exponential premium principle yields the following premium in the case of Poisson model of outages:

$$\pi_X = \frac{\lambda T(e^{c\beta} - 1)}{\beta} \tag{19}$$

where equation (19) is obtained by utilizing the basic properties of m.g.f. and the fact that the m.g.f. of a Poisson random variable say Y having a Poisson distribution with parameter λ is given by:

$$M_Y(t) = e^{\lambda(e^t - 1)} \tag{20}$$

where *t* is parameter of the m.g.f. In our case $t = c\beta$.

Equation (19) offers an interesting relationship between the premium $\pi_X$and the average number of outages during time *T* viz. λT, c, i.e., the compensation offered by the CSP to the customer for each outage and the parameter β of the exponential u.f. It is clear that as c and β alleviate, an exponential magnification is recorded in the premium.We undertake the dependence of the premium on these quantities numerically and graphically in the next section.

We now move to the Esscher premium principle which embodies the m.g.f. of the loss distribution in determination of the premium for the insurer.

5. The EsscherPrinciple

The Esscher premium principle proposed by Bühlmann, H. (1980) based on the Esscher transform developed by the famous Swedish actuary F. Esscher in 1932 (c.f. Esscher (1932)) involves a transform that modifies the probability law of the risk distribution and then sets the premium to be the pure premium for the transformed distribution (c.f. Dickson (2005)).

Suppose that the risk random variable *X* is a continuous random variable on$(0, \infty)$ with probability density function (p.d.f.) *f*. We then define the p.d.f. for a new transformed random variable $\tilde{X}$ as g by applying the following modification:

$$g(x) = \frac{e^{hx} f(x)}{\int_0^\infty e^{hy} f(y) dy}, \qquad h > 0 \tag{21}$$

where the denominator is essentially the m.g.f. of the original random variable *X*, with parameter h, i.e.,$M_X(h)$, with m.g.f defined in equation (18). The density g is indeed a weighted version of density f. This can be seen by rewriting equation (21) as:

$$g(x) = w(x)f(x) \tag{22}$$

where

$$w(x) = \frac{e^{hx}}{M_X(h)}. \tag{23}$$

Further, one can see that as $h > 0$, therefore $w'(x) > 0$ and hence increasing weights are allocated with alleviation in *x*. Next from equation (21), the distribution function (c.d.f.) of the transformed random variable $\tilde{X}$ is given by:

$$G(x) = \frac{\int_0^x e^{hy} f(y)dy}{M_X(h)}, \qquad h > 0. \tag{24}$$

In fact the distribution function *G* is known as the Esscher transform of the original distribution function *F* with parameter *h* and it is from here that this premium principle derives its name. Next the m.g.f. of the transformed random variable$\tilde{X}$ takes the following form:

$$M_{\tilde{X}}(t) = \frac{M_X(t+h)}{M_X(h)}. \tag{25}$$

Finally the expectation of the transformed random variable $\tilde{X}$ is given by:

$$\mathrm{E}[\tilde{X}] = \frac{\int_0^\infty xe^{hx} f(x)dx}{\int_0^\infty e^{hy} f(y)dy} = \frac{E[Xe^{hx}]}{E[e^{hx}]} = \frac{E[Xe^{hx}]}{M_X(h)}. \tag{26}$$

This means that if one can recognize the distribution of the transformed random variable $\tilde{X}$, by using the uniqueness theorem of m.g.f., one can easily lay hand on $\mathrm{E}[\tilde{X}]$.

We now define the Esscher premium principle.

Definition: The Esscher premium principle prescribes

$$\pi_X = \frac{E[Xe^{hx}]}{E[e^{hx}]} = \frac{E[Xe^{hx}]}{M_X(h)}, \tag{27}$$

which is essentially$\mathrm{E}[\tilde{X}]$, as can be seen from equation (26). It can be easily seen that the Esscher principle augments the probabilities of large values and reduces the probabilities of small values.

Premium for the Poisson Model: We first obtain the m.g.f. of the transformed random variable $\tilde{X}$ in the Poisson case. Utilizing equations (25) and (4), and the basic properties of m.g.f., we see that:

$$M_{\tilde{X}}(t) = \frac{M_{cN}(t+h)}{M_{cN}(h)} = \frac{M_N(c(t+h))}{M_N(ch)} = e^{\lambda T e^{ch}(e^{ct}-1)} = M_{N'}(ct) = M_{cN'}(t) \tag{28}$$

where $M_{N'}(ct)$ denotes the m.g.f. with parameter *ct* of a Poisson random variable *N'*with parameter $\lambda T e^{ch}$. Further it is straightforward to see that

$$\tilde{X} = cN' \tag{29}$$

and so

$$\mathrm{E}[\tilde{X}] = c\lambda T e^{ch} \tag{30}$$

Using equations (26), (27) and (30), the Esscher premium principle yields the following premium in the case of Poisson model of outages:

$$\pi_X = c\lambda T e^{ch}, h > 0. \tag{31}$$

Equation (31) offers a fascinating relationship between the premium $\pi_X$and the average number of outages during time T viz. λT, c, i.e., the refund offered by the CSP to the customer for each outage and the parameter h of the Esscher transform. It is clear that as c and $h$bulge, an exponential amplificationwould be seen in the premium. We explore the dependence of the premium on these quantities numerically and graphically in the next section.

Premium Calculation Examples for the Poisson Model for Cloud Outages:-

Having presented six most popularly used premium calculation principles, we now turn our attention to determination of premium in each case.

For the premium loading factor we use: $\alpha = 0.1$. We present the variation in premium with the change in the values of the quantities involved. We undertake the behavior of the premium in response to λT and $c$forthe pure premium and expected value principles together.

1. The Pure Premium and the Premium based on the Expected Value Principle

We first portray the pure premium and the premium based on the expected value principle by fixing the compensation per outage, i.e., we set$c = 100$ and vary the average number of outages during the time T, i.e. $\lambda T$from 0 to 50 in Figure 3 below. It is evident from this figure that for a fixed $\alpha$and c, as the value of λT increases, the difference between pure premium and the premium calculated by the expected value principle starts inflating.

Next in Figure 4 we depict the aforesaid premiums by fixing $\lambda T = 10$ and varying c from 100 to 1000 and a similar trend as in Figure 3 starts emerging.

2. The Premium based on the Variance Principle

We follow the same pattern for the premium based on the variance principle. We first illustrate the behaviour of this premium for a constant compensation of 100 for an outage, i.e. $c = 100$ by increasingλT from 0 to 50 and present the results in Figure 5. A clear increasing linear trend is visible. Next in Figure 6 we depict the variance premiums by fixing $\lambda T = 10$ and varying c from 100 to 1000 and obtain a parabolic curve as from equation (12) it can be seen that the variance premium is essentially a second degree polynomial in c.

3. The Premium based on the Standard Deviation Principle

We follow essentially a similar approach for the premium based on the standard deviation principle. We first consider how the premium varies with altering $\lambda T$for a constant remedial amount of 100 for an outage, i.e. $c = 100$ and the findings are displayed in Figure 7. A clear increasing trend is visible. Next in Figure 8 we demonstrate the premiums based on standard deviation principle for varying c and a constant $\lambda T = 10$ and observe an increasing linear trend which is obvious from equation (14).

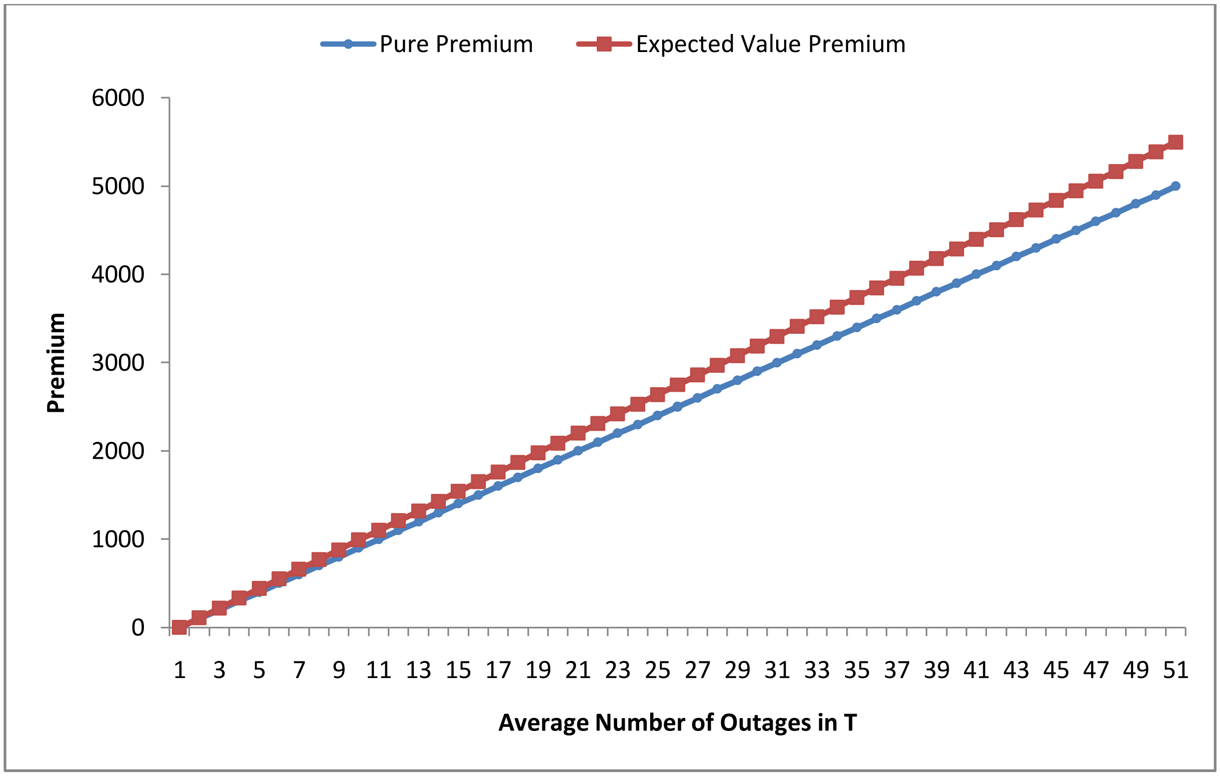

Figure 3:- Pure Premium and Expected Value Premium for $c = 100$ and $\alpha = 0.1$.

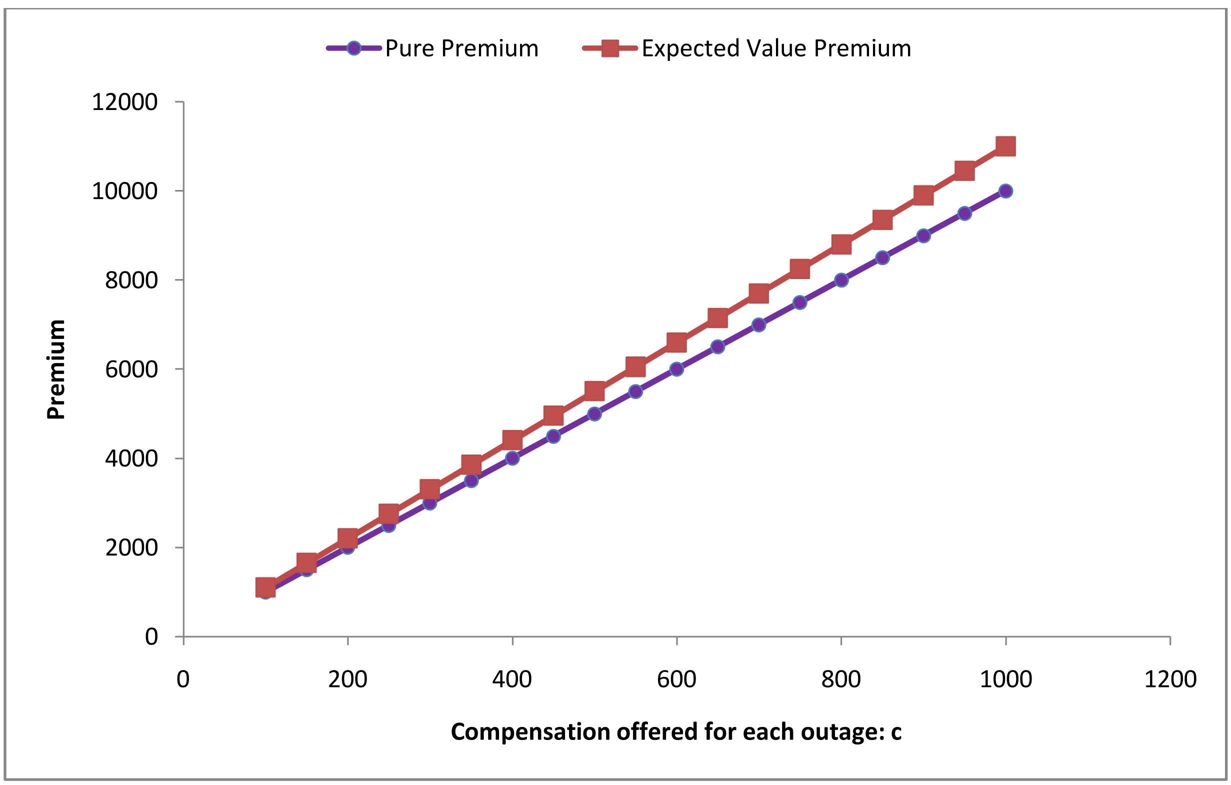

Figure 4:- Pure Premium and Expected Value Premium for $\lambda T = 10$and $\alpha = 0.1$.

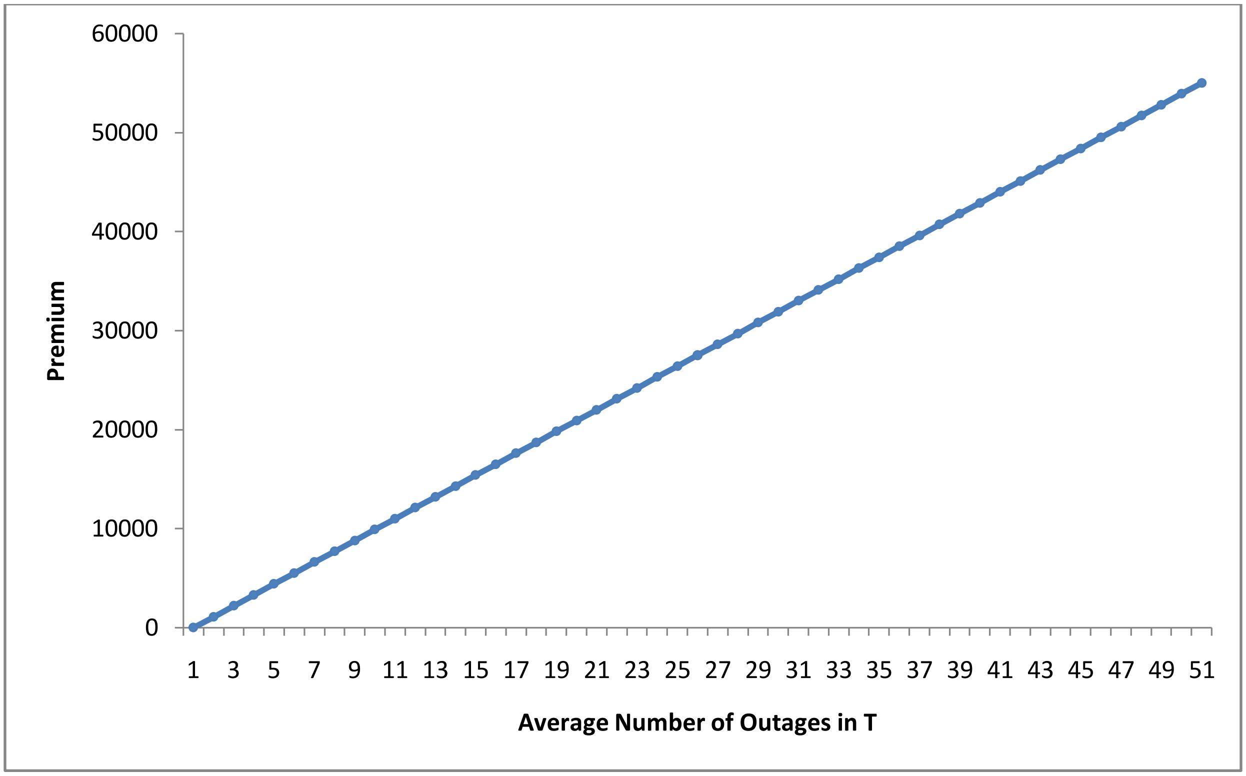

Figure 5:- Premium based on the Variance Principle for $c = 100$ and $\alpha = 0.1$.

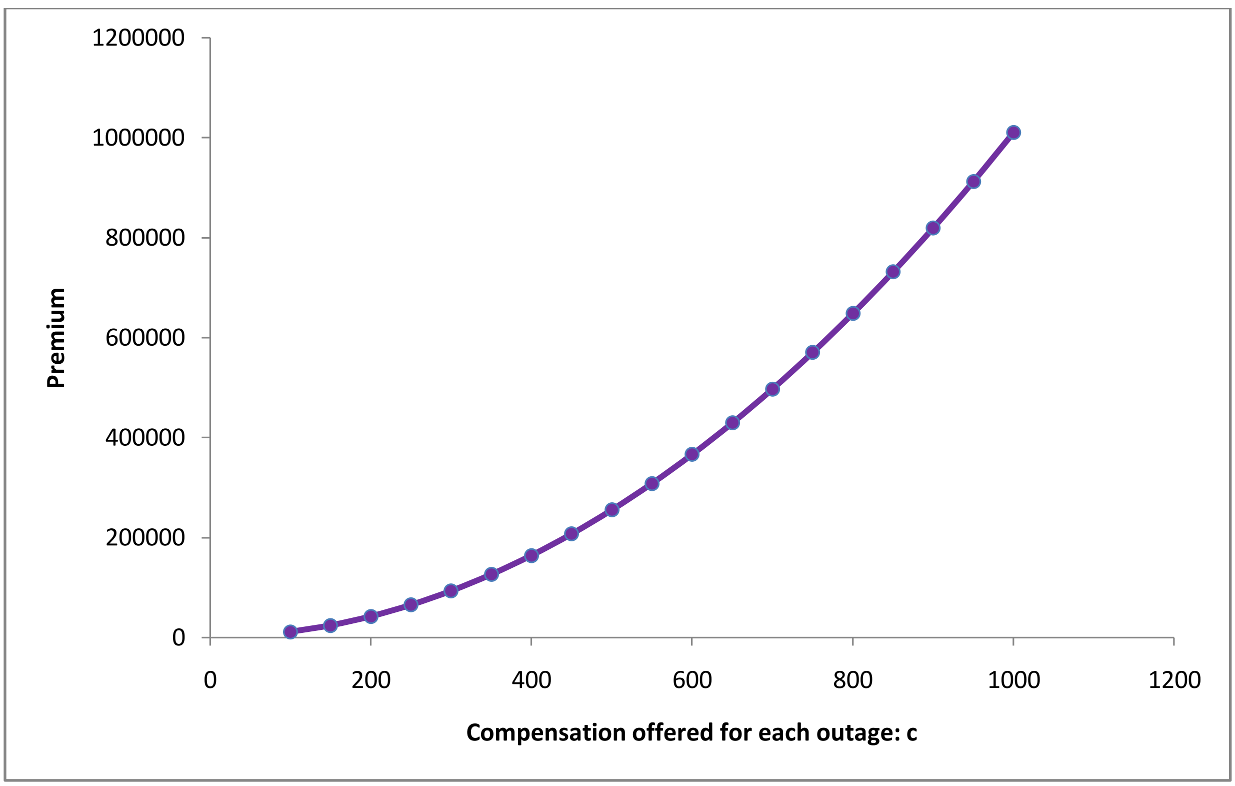

Figure 6:- Premium based on the Variance Principle for $\lambda T = 10$and $\alpha = 0.1$.

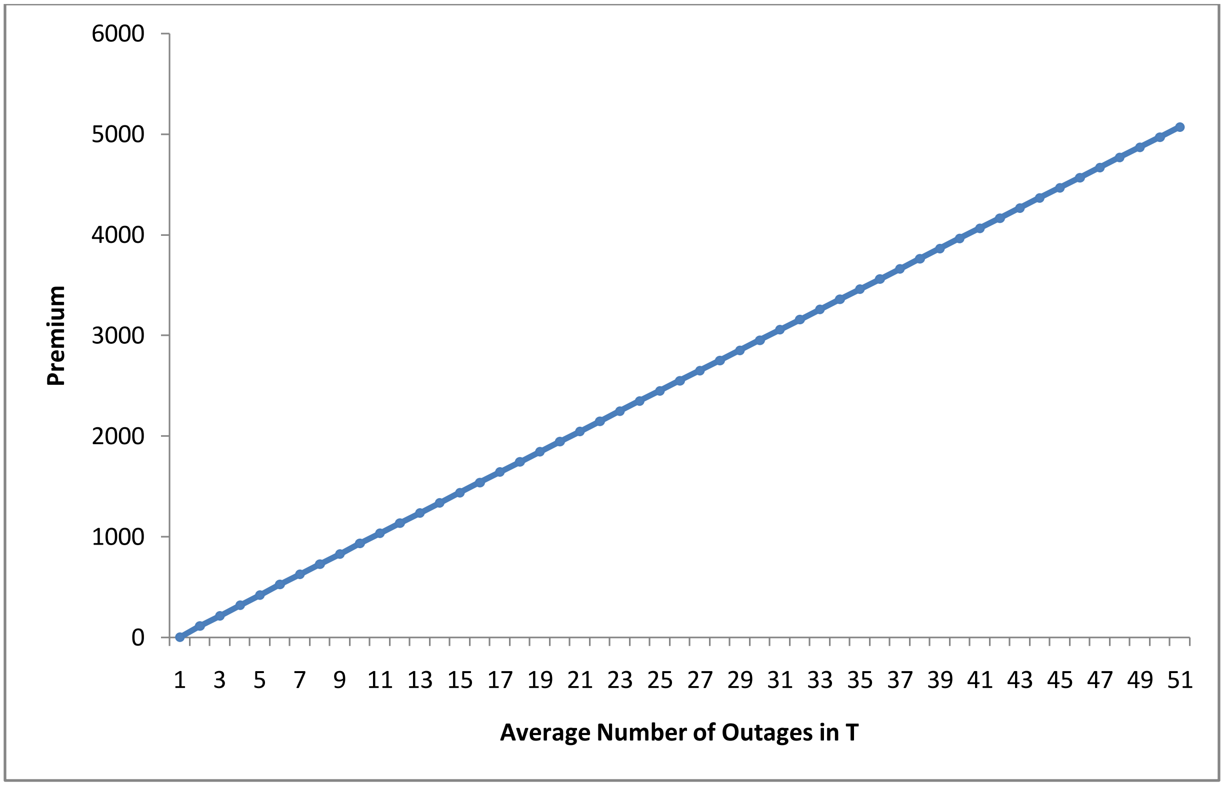

Figure 7:- Premium based on the Standard Deviation Principle for $c = 100$ and $\alpha = 0.1$.

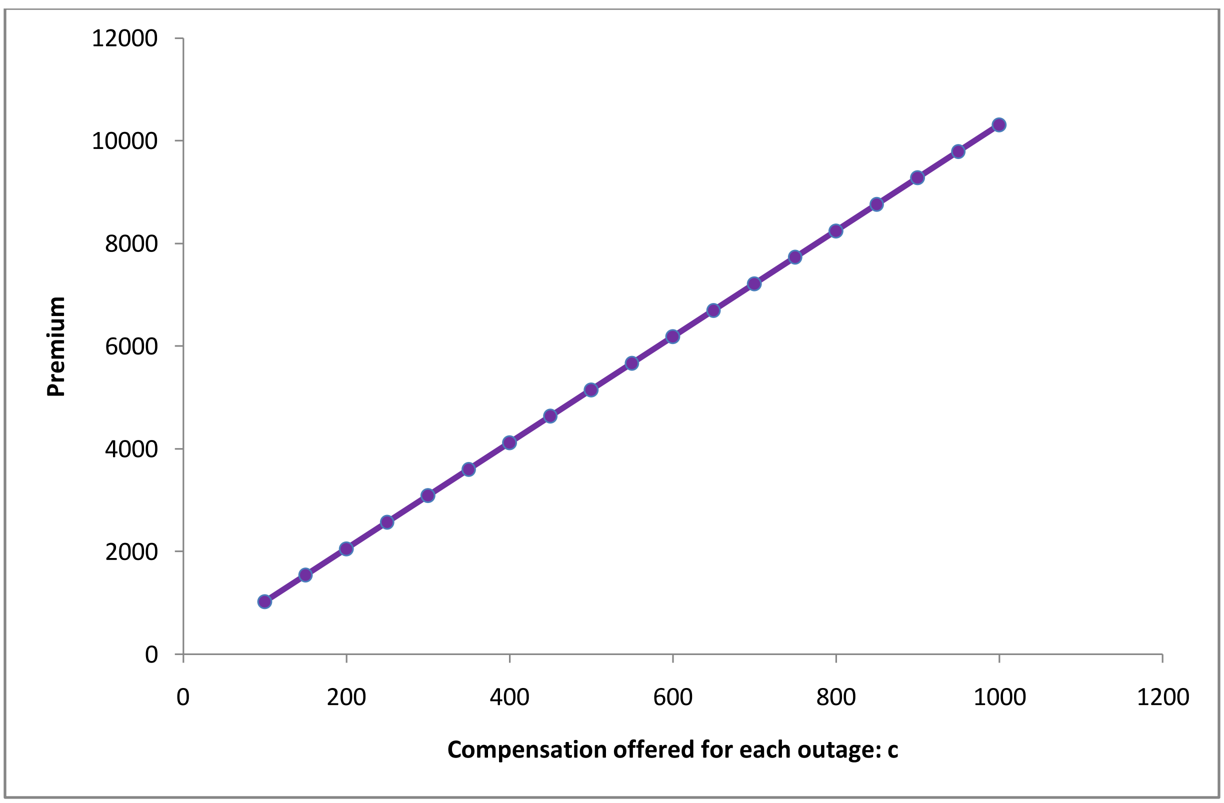

Figure 8:- Premium based on the Standard Deviation Principle for $\lambda T = 10$and $\alpha = 0.1$.

4. The Premium based on the Exponential Principle

Here we consider an insurer who resorts to the exponential utility function with parameterβ defined above. We invoke equation (19) and at first examine the response of the premium to a change in the average number of outages during the time T, i.e. $\lambda T,$ keeping the other two quantities viz. the compensation per outage, c and the parameter of the exponential utility function viz. β fixed such that we set $c = 100$ and $\beta = 0.01$. As dictated by equation (19), a clear increasing linear trend is visible in Figure 9.

Next we display the behaviour of the premium to the altering values of the parameterβ of the exponential utility function keeping $\lambda T$and c fixed. As noted before, β also signifies the risk aversion coefficient of the insurer. As β appears in the exponent in equation (19), the premium starts to explode exponentially with an elevation in its values. This observation is apparent in Figure 10.

Finally in Figure 11,we portray the demeanour of the exponential premium to the variation in the compensation offered for each outage. Interestingly just like the previous case an exponential increase in premium is recorded as the values of c increase from 100 to 650 which is obvious as an insurer is likely to study the service level agreement minutely to estimate the magnitude of claims that can arise and only then will prescribe the premium. A higher compensation amount would mean higher claims and therefore higher premiums.

5. The Premium based on the Esscher Principle

We follow the same approachas above for the premium based on the Esscher principle by replacing the parameter$\beta$of the exponential utility function by the parameter h of the Esscher transform. Invoking equation (31), we decipher similar behavioural pattern as above as can be seen from Figures 12-14. This is indeed attributed to a striking similarity in equations (19) and (31).

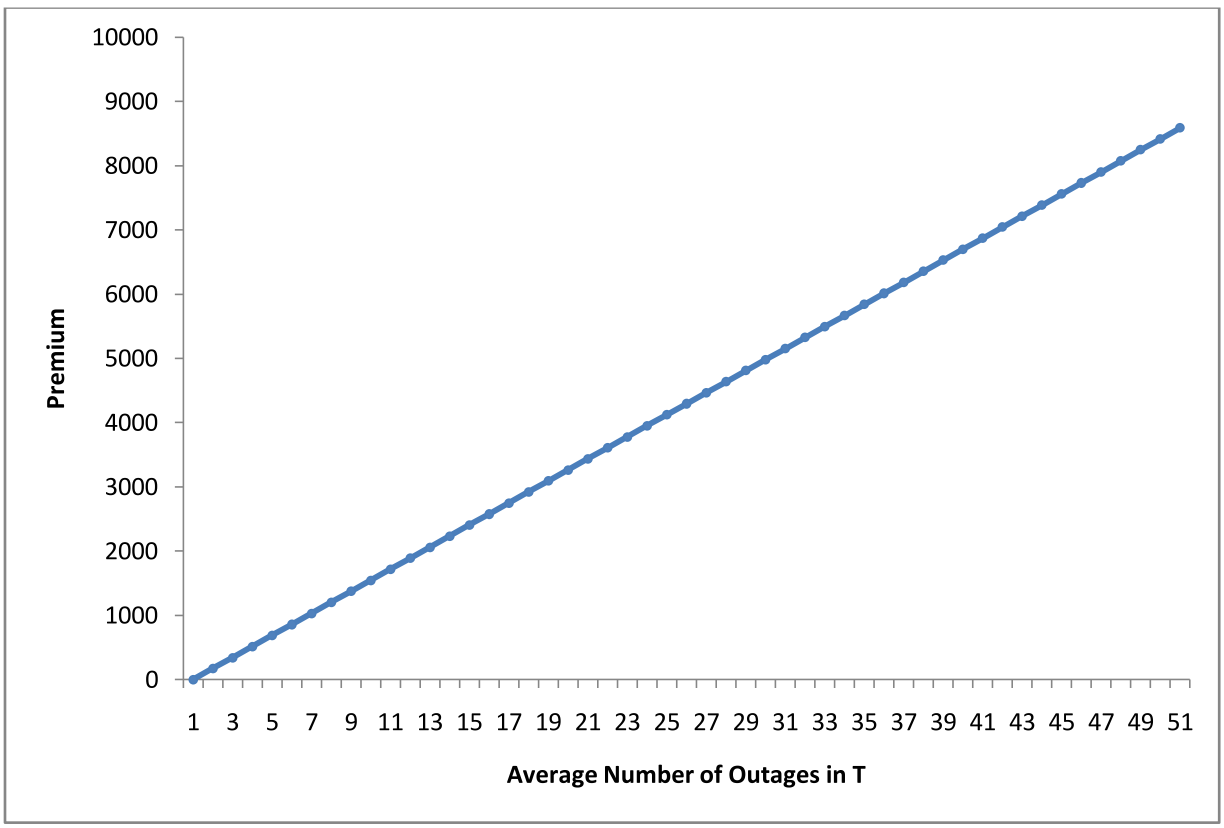

Figure 9:- Premium based on the Exponential Principle for $c = 100$ and$\beta = 0.01$.

Conclusion:-

In this paper we have presented a methodology for determination of premium for cloud insurance invoking six popular premium principles. Cloud insurance is a nascent area for insurers and this paper offers ground breaking research to help them devise competent policies with optimum premiums, safeguarding them against a possible financial avalanche in case of catastrophic claims arising due to multiple cloud outages. Depending on the information about past history of the cloud service provider (CSP), the insurer can cleverly compute the premium by choosing the most appropriate principle. In this research we have exploited the Poisson model of cloud outages. In our forthcoming research we are working on the exponential-Pareto and the Pareto-lognormal models of cloud outages.

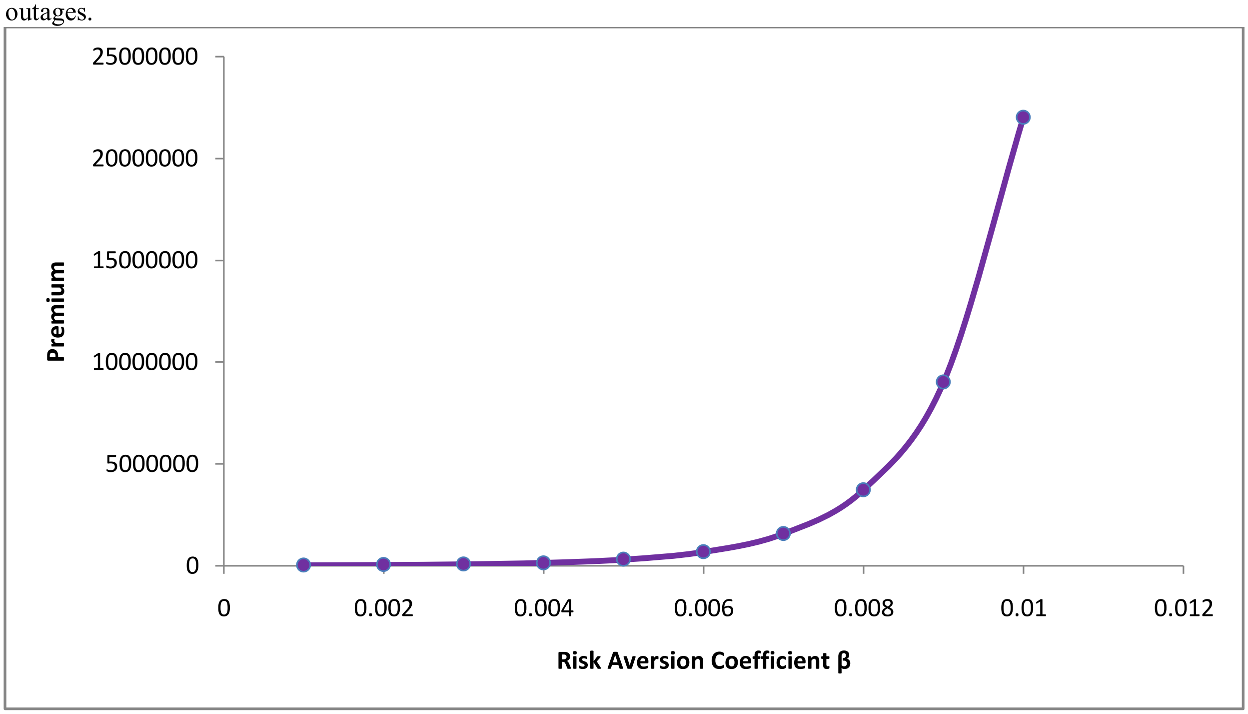

Figure 10:- Premium based on the Exponential Principle for $\lambda T = 10$and $c = 1000$.

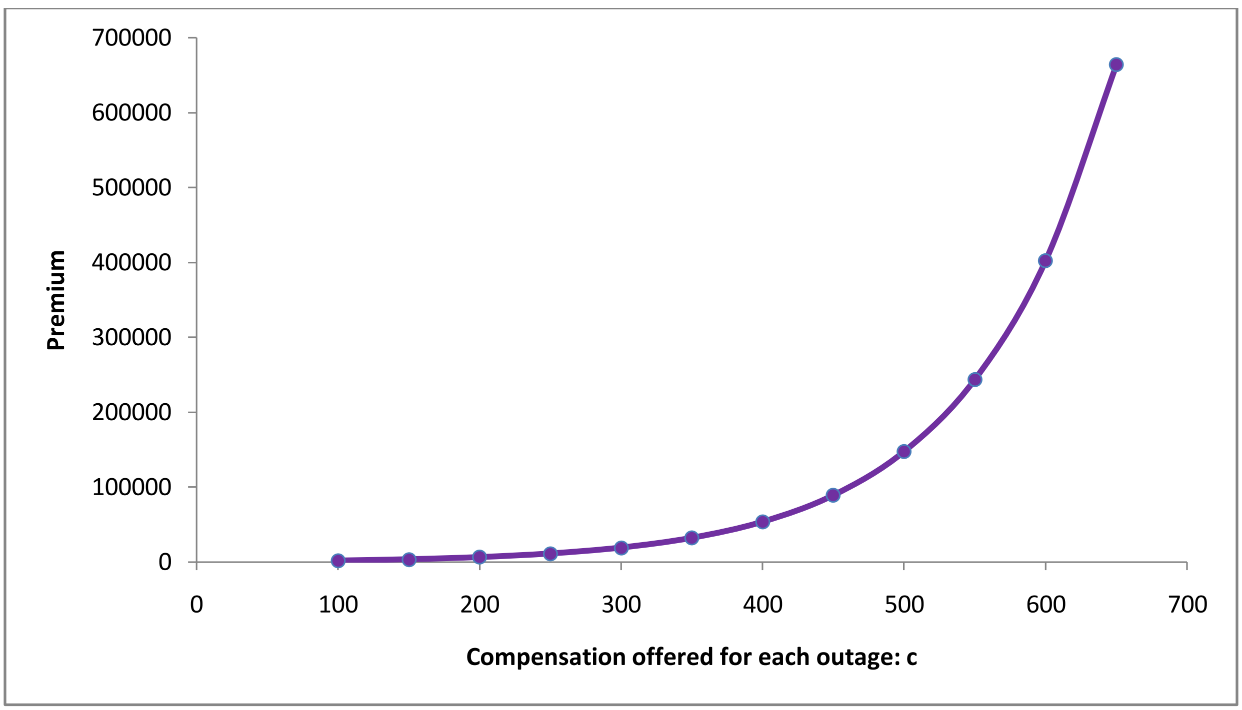

Figure 11:- Premium based on the Exponential Principle for $\lambda T = 10$and $\beta = 0.01$.

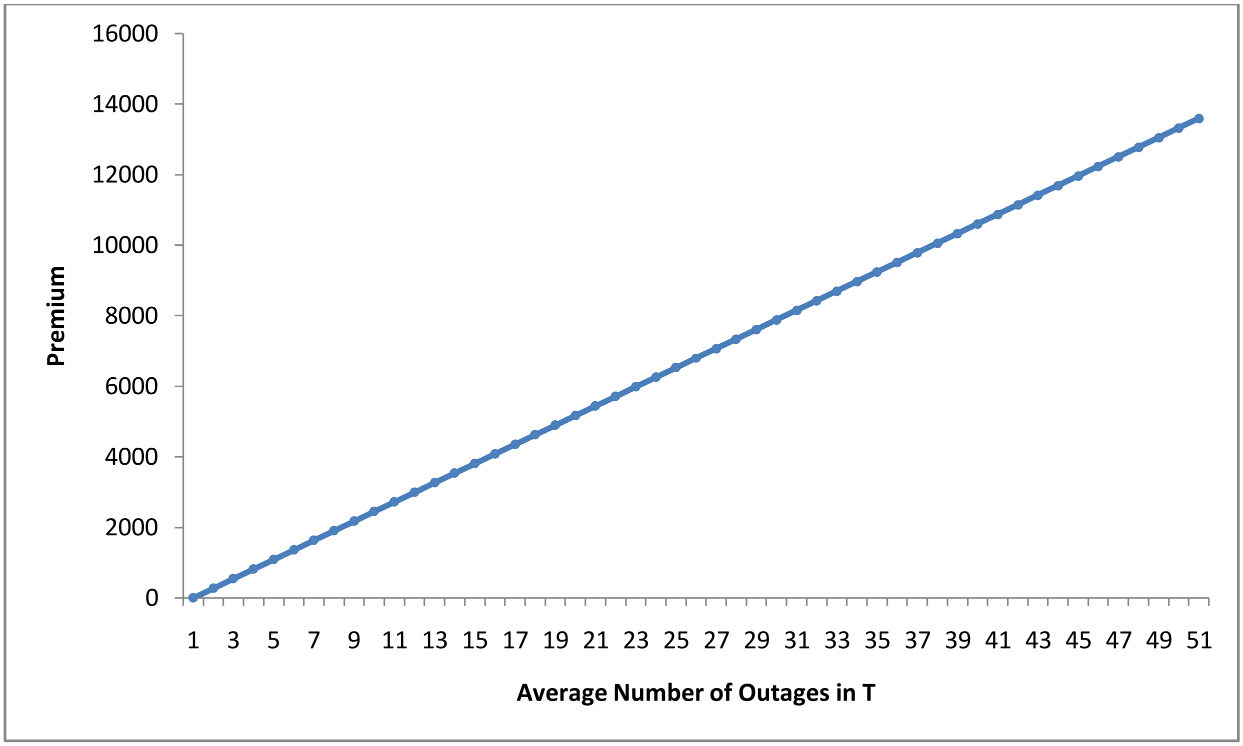

Figure 12:- Premium based on the Esscher Principle for $c = 100$ and $h = 0.01$.

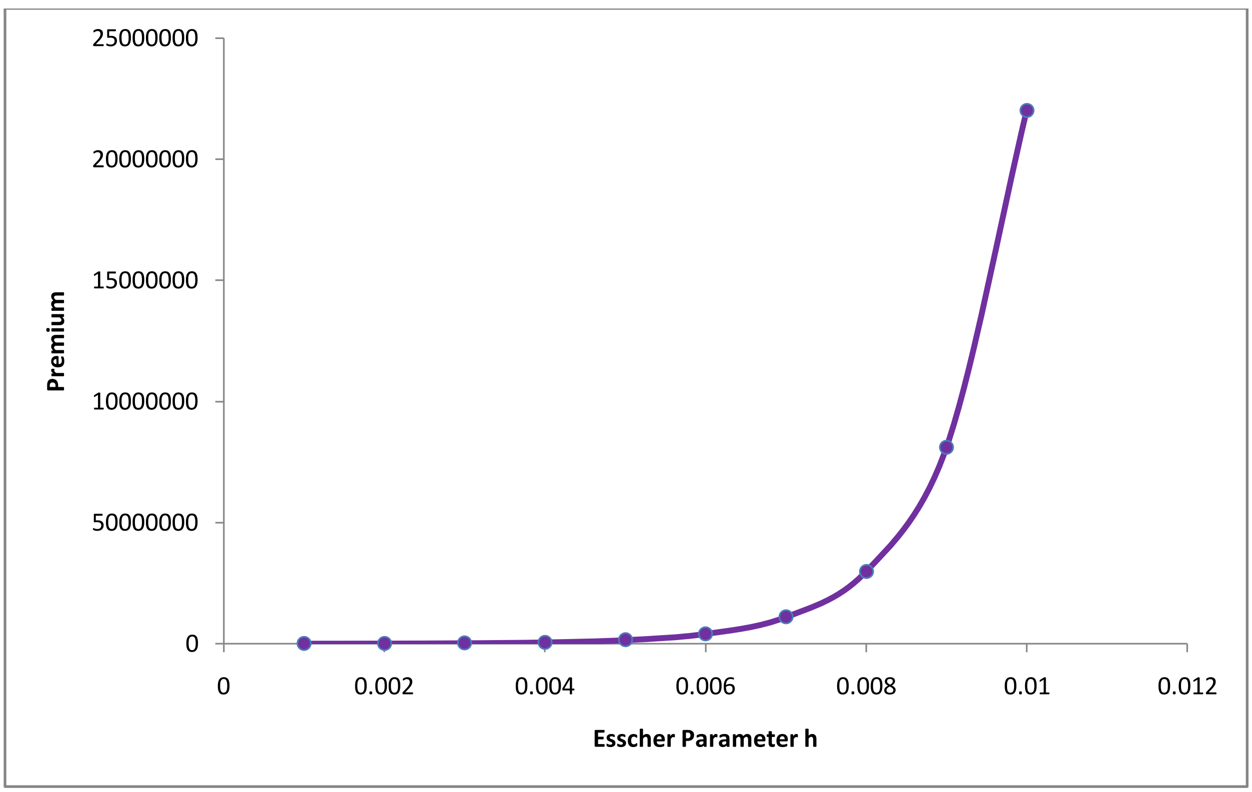

Figure 13:- Premium based on the Esscher Principle for $\lambda T = 10$and$c = 1000$.

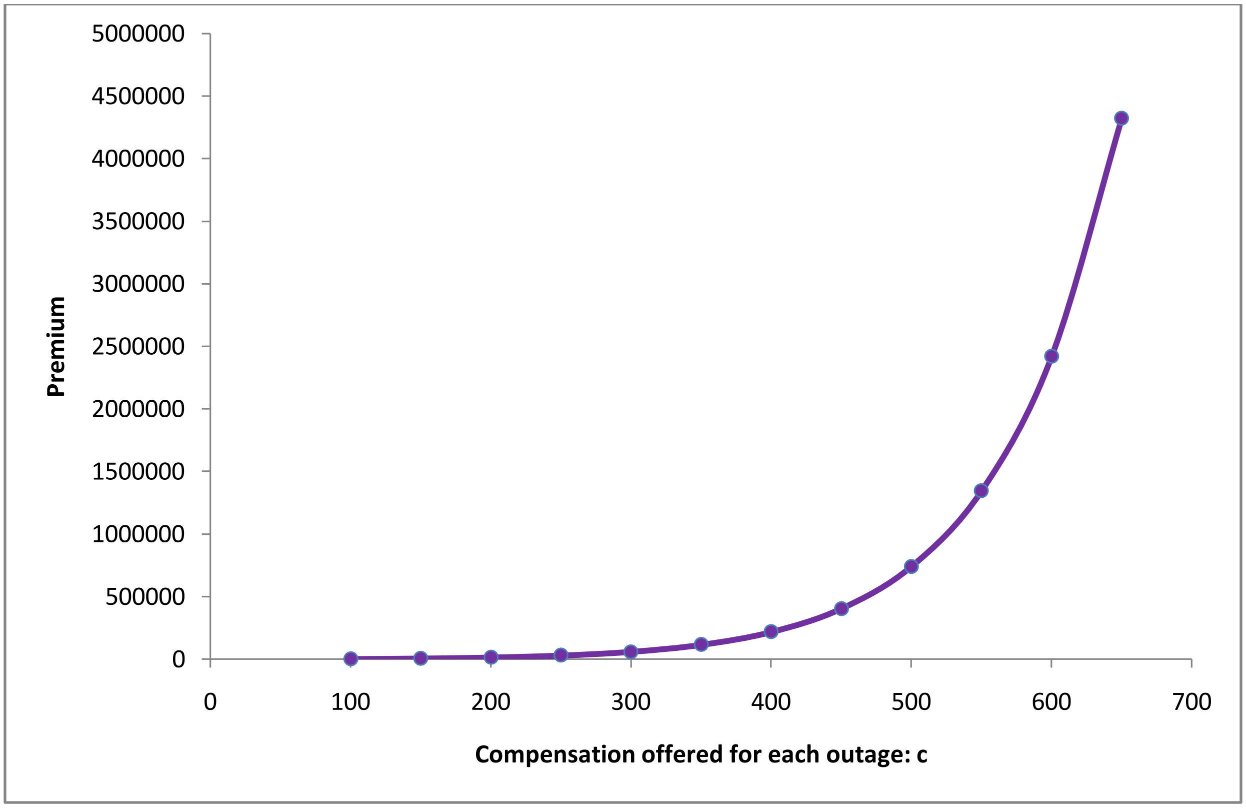

Figure 14:- Premium based on the Esscher Principle for $\lambda T = 10$and$h = 0.01$

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