ISSN (O): 2320-5407 Int. J. Adv. Res. 13(04), Apr-2025 1198-1204

Journal Homepage: -www.journalijar.com

INTERNATIONAL JOURNAL OF ADVANCED RESEARCH (IJAR)

ISSN NO. 2320-5407

Article DOI:10.21474/IJAR01/20831
DOI URL: http://dx.doi.org/10.21474/IJAR01/20831

## RESEARCH ARTICLE

# HOM-ANTICENTER SYMMETRIC ALGEBRAS AND HOM-MOCK LIE ALGEBRAS: MATCHED PAIRS AND MANIN TRIPLES

**Gbêvèwou Damien Houndedj[1,2] and Bakary Kourouma[1]**

1. Gamal Abdel Nasser University of Conakry, Republic of Guinea, Department of Mathematics, BP 1147 Conakry, Guinea.
2. University of Abomey-Calavi, International Chair in Mathematical Physics and Applications, ICMPA-UNESCO Chair, 072 BP 50, Cotonou, Rep. of Benin.

## Manuscript Info

*Manuscript History*
Received: 17 February 2025
Final Accepted: 20 March 2025
Published: April 2025

*Key words:-*
Hom-Anticenter-Symmetric Algebras, Hom-Mock Lie Algebras, Bimodules, Matched Pairs, Manin Triples

## Abstract

This work develops the hom-algebra theory for anticenter-symmetric algebras by introducing the concept of a hom-anticenter-symmetric algebra. We give their bimodules, dual bimodules and matched pairs. The relation of the bimodules and matched pairs of hom-anticenter-symmetric algebras between those of hom-Mock Lie algebras is discussed. Finally, we establish the Manin triple of hom-anticenter-symmetric algebras.



## Introduction:-

Mock-Lie algebras are commutative algebras satisfying the Jacobi identity. These algebras have been introduced into the literature under various names (see [9] and [3]). Moreover, recently, in [6], anticenter-symmetric Jacobi-Jordan algebras, or simply anticenter-symmetric algebras, have been defined while verifying that bimodules, matched pairs, and Manin triples of these objects exist. These algebras belong to the category of Mock-Lie admissible algebras.

Hom-algebra structures were first introduced in the quasi-deformation of Lie algebras of vector fields. In fact, the discreteness of vector fields as twisted derivations played a key role in the birth of hom-Lie and quasi-hom-Lie structures, where the Jacobi identity is twisted[8]. One aspect of quasi-deformation was the development of q-derivations, compatible with order zero, later replaced by any σ-derivation. Among the first examples of q-quasi-deformations, the exchange of derivations for σ-derivations, which are excellent twists in trying to avoid division, had been proven in Witt and Virasoro algebras, e.g. [1, 4, 5].

Quadratic hom-Jacobi-Jordan algebras, defined as hom-Jacobi-Jordan algebras with symmetric, invariant, and nondegenerate bilinear forms, have introduced and studied in [7]. Representations and the cohomology theory of hom-Jacobi-Jordan algebras were further explored in [2].

For the present work, the central objective is the study of the connection between bimodules and matched pairs of hom-anticenter-symmetric algebras and hom-Mock-Lie algebras, and the construction of Manin triples of.

**Corresponding Author:- Gbêvèwou Damien Houndedj**
Address:- Gamal Abdel Nasser University of Conakry, Republic of Guinea, Department of Mathematics, BP 1147 Conakry, Guinea.

This paper is organized as follows: Section 2 defineshom-anticenter-symmetric algebras and their properties. Section 3 introduces the bimodules and matched pairs of hom-anticenter-symmetric algebras and studies their relationswith those of hom-Mock Lie algebras. In Section 4, we establish the Manin triples of the hom-anticenter symmetric algebras. Finally, Section 5 presents the concluding remarks.

**Main definitions and properties**

We give the basic definitions and properties of hom-anticenter-symmetric algebras.

Definition 2.1$A$ hom-anticenter-symmetric algebra is a triple $(\mathscr{H},\mu,\alpha)$, such that$\mathscr{H}$ is a vector space, $\mu:\mathscr{H}\otimes\mathscr{H}\rightarrow\mathscr{H}$ is a bilinear map, and $\alpha\in\mathrm{gl}(\mathscr{H})$ such that, for all $x,y,z\in\mathscr{H}$

$$\alpha(\mu(x,y))=\mu(\alpha(x),\alpha(y)),\#(2.1)$$

$$(x,y,z)_{-1,\alpha,\mu}=-(z,y,x)_{-1,\alpha,\mu},\#(2.2)$$

With$(x,y,z)_{-1,\alpha,\mu}:=\mu\big(\mu(x,y),\alpha(z)\big)+\mu\big(\alpha(x),\mu(y,z)\big)$, which is called $\alpha$-antiassociator associated to the product$\mu$.

In the sequel, as matter of notation simplification, we denote $(\mathscr{H},\mu,\alpha)$ by $(\mathscr{H},\alpha)$ or simplyby $\mathscr{H}$. The left $L$ and right $R$ representations of the bilinear product given on $\mathscr{H}$ are defined by$L_x(y)=xy$and$R_x(y)=yx$, respectively $\forall x,y\in\mathscr{H}$.

The adjoint representation ad $:=L+R$ of the sub-adjacent Mock Lie algebra $\mathcal{G}(\mathscr{H})$ of an anticenter-symmetric algebra $\mathscr{H}$is defined as follows:

$$\begin{aligned}\mathrm{ad}:\mathscr{H}&\longrightarrow\mathrm{gl}(\mathscr{H})\\ x&\longmapsto\mathrm{ad}_x:\mathscr{H}\longrightarrow\mathscr{H}\\ &\qquad y\longmapsto\mu(x,y)+\mu(y,x):=[x,y]\end{aligned}\#(2.3)$$

such that $\forall x,y\in\mathscr{H},\mathrm{ad}_x(y):=(L_x+R_x)(y)$.

Definition 2.2[7] A hom-Mock Lie algebra is a triple $\big(\mathrm{g},[,]_{\mathrm{g}},\alpha_{\mathrm{g}}\big)$ consisting of a vector space g, an algebra homomorphism $\alpha_{\mathrm{g}}:\mathrm{g}\rightarrow\mathrm{g}$, and a symmetric bilinear map $[\cdot,\cdot]_{\mathrm{g}}:\mathrm{g}\otimes\mathrm{g}\rightarrow\mathrm{g}$ satisfying the following relation, for all $x,y,z\in\mathrm{g}$ :

$$\big[\alpha_{\mathrm{g}}(x),[y,z]_{\mathrm{g}}\big]_{\mathrm{g}}+\big[\alpha_{\mathrm{g}}(y),[z,x]_{\mathrm{g}}\big]_{\mathrm{g}}+\big[\alpha_{\mathrm{g}}(z),[x,y]_{\mathrm{g}}\big]_{\mathrm{g}}=0.\#(2.4)$$

Definition 2.3 A representation of a hom-Mock Lie algebra ( $\mathrm{g},[,]_{\mathrm{g}},\alpha_{\mathrm{g}}$ ) on a vector space $V$ with respect to $\psi\in\mathrm{gl}(V)$ is a linear map $\rho:\mathrm{g}\rightarrow\mathrm{gl}(V)$ such that for all $x,y\in\mathrm{g}$, the following relations:

$$\rho\big(\alpha_{\mathrm{g}}(x)\big)\circ\psi=\psi\circ\rho(x),\#(2.5)$$

$$\rho\big([x,y]_{\mathrm{g}}\big)\circ\psi=-\rho\big(\alpha_{\mathrm{g}}(x)\big)\circ\rho(y)-\rho\big(\alpha_{\mathrm{g}}(y)\big)\circ\rho(x)\#(2.6)$$

are satisfied.

Proposition 2.4 Let $(\mathscr{H},\alpha)$ be a hom-anticenter-symmetric algebra. The following conditions are satisfied:

- The anticommutator$[x,y]:=xy+yx$ defines a hom-Mock Lie algebra $\mathcal{G}(\mathscr{H})_\alpha:=(\mathscr{H},[\,,],,\alpha)$.
- The couple $(\mathrm{ad}:=L+R,\alpha)$ gives a regular representation of the sub-adjacent hom-Mock Lie algebra $\mathcal{G}(\mathscr{H})_\alpha$.

Proof.Let $(\mathscr{H},\alpha,\mu)$ be ahom-anticenter-symmetric algebra. It is easy to see that the anticommutator$[,]_\mu$ associated to $\mu$ is bilinear and symmetric on $\mathscr{H}$. For all$x,y,z\in\mathscr{H}$, we have:

$$\begin{aligned}&\big[\alpha(x),[y,z]_\mu\big]_\mu+\big[\alpha(y),[z,x]_\mu\big]_\mu+\big[\alpha(z),[x,y]_\mu\big]_\mu=[\alpha(x),\mu(y,z)+\mu(z,y)]_\mu\\
&+[\alpha(y),\mu(z,x)+\mu(x,z)]_\mu+[\alpha(z),\mu(x,y)+\mu(y,x)]_\mu=[\alpha(x),\mu(y,z)]_\mu\\
&+[\alpha(y),\mu(z,x)]_\mu[\alpha(z),\mu(x,y)]_\mu+[\alpha(x),\mu(z,y)]_\mu-[\alpha(y),\mu(x,z)]_\mu+[\alpha(z),\mu(y,x)]_\mu\\
&=\mu(\alpha(x),\mu(y,z))+\mu(\alpha(y),\mu(z,x))+\mu(\alpha(z),\mu(x,y))+\mu(\alpha(x),\mu(z,y))\\
&+\mu(\alpha(y),\mu(x,z))+\mu(\alpha(z),\mu(y,z))+\mu(\mu(y,z),\alpha(x))+\mu(\mu(z,x),\alpha(y))\\
&+\mu(\mu(x,y),\alpha(z))+\mu(\mu(z,y),\alpha(x))+\mu(\mu(x,z),\alpha(y))+\mu(\mu(y,x),\alpha(z))\\
&=\{\mu(\alpha(x),\mu(y,z))+\mu(\mu(x,y),\alpha(z))\}+\{\mu(\alpha(y),\mu(z,x))+\mu(\mu(y,z),\alpha(x))\}\\
&+\{\mu(\alpha(z),\mu(x,y))+\mu(\mu(z,x),\alpha(y))\}+\{\mu(\alpha(x),\mu(z,y))+\mu(\mu(x,z),\alpha(y))\}\\
&+\{\mu(\alpha(y),\mu(x,z))+\mu(\mu(y,x),\alpha(z))\}+\{\mu(\alpha(z),\mu(y,x))+\mu(\mu(z,y),\alpha(x))\}\\
&=\{\mu(\alpha(x),\mu(y,z))+\mu(\mu(x,y),\alpha(z))\}+\{\mu(\alpha(z),\mu(y,x))+\mu(\mu(z,y),\alpha(x))\}\\
&+\{\mu(\alpha(y),\mu(z,x))+\mu(\mu(y,z),\alpha(x))\}+\{\mu(\alpha(x),\mu(z,y))+\mu(\mu(x,z),\alpha(y))\}\\
&+\{\mu(\alpha(z),\mu(x,y))+\mu(\mu(z,x),\alpha(y))\}+\{\mu(\alpha(y),\mu(x,z))+\mu(\mu(y,x),\alpha(z))\}\\
&=\big[\alpha(x),[y,z]_\mu\big]_\mu+\big[\alpha(y),[z,x]_\mu\big]_\mu+\big[\alpha(z),[x,y]_\mu\big]_\mu=0.\end{aligned}$$

Then, for all $x, y, z \in \mathscr{H}$, the following equality holds

$$\left[\alpha(x), [y,z]_\mu\right]_\mu + \left[\alpha(y), [z,x]_\mu\right]_\mu + \left[\alpha(z), [x,y]_\mu\right]_\mu = 0.$$

Hence, $\mathcal{G}(\mathscr{H})_\alpha$ is a hom-Mock Lie algebra.

Remark 2.5 The hom-Mock Lie algebra $\mathcal{G}(\mathscr{H})_\alpha := (\mathscr{H}, [\,,], , \alpha)$ defined in the Proposition 2.4 is called the sub-adjacent hom-Mock Lie algebra of $(\mathscr{H}, \alpha)$, and $(\mathscr{H}, \alpha)$ is the compatible hom-anticenter-symmetric algebra structure on the hom-Mock Lie algebra $\mathcal{G}(\mathscr{H})_\alpha$.

**Bimodule and matched pair of hom-anticenter-symmetric algebras**

Definition 3.1 Let a hom-anticenter-symmetric algebra $(\mathcal{H}, \alpha)$, a vector space $V$, two linear maps $l, r: \mathcal{H} \to \mathfrak{gl}(V)$, and $\varphi \in \mathfrak{gl}(V)$. The system $(l, r, V, \varphi)$ is called bimodule of the hom-anticenter-symmetric algebra $(\mathcal{H}, \alpha)$ if for all $x, y \in \mathcal{H}$ and $v \in V$,

$$\varphi \circ l_x = l_{\alpha(x)} \circ \varphi, \varphi \circ r_x = r_{\alpha(x)} \circ \varphi, \#(3.1)$$

$$l_{\alpha(x)} \circ l_y + l_{xy} \circ \varphi = -r_{yx} \circ \varphi - r_{\alpha(x)} \circ r_y, \#(3.2)$$

$$l_{\alpha(x)} \circ r_y + r_{\alpha(y)} \circ l_x = -l_{\alpha(y)} \circ r_x - r_{\alpha(x)} \circ l_y. \#(3.3)$$

For $\alpha = \mathbf{id}$, we obtain the classicalanticenter-symmetric algebra structure on the semi-direct vector space $\mathcal{H} \oplus V$ is given in [7].

Proposition 3.2 Let $(\mathcal{H}, \alpha)$ be a hom-anticenter-symmetric algebra, $V$ a vector space, $l, r: \mathcal{H} \to \mathfrak{gl}(V)$ two linear maps, and $\varphi \in \mathfrak{gl}(V)$. Then, $(\varphi, l, r, V)$ is a bimodule of the hom-anticenter-symmetric algebra $(\mathcal{H}, \alpha)$ if and only if $(\mathcal{H} \oplus V, *, \alpha \oplus \varphi)$ is a hom-anticenter-symmetric algebra, where $*$ and $\alpha \oplus \varphi$ are defined as follows: for all $x, y \in \mathcal{H}, u, v \in V$,

$$(x+u) * (y+v) = xy + l(x)v + r(y)u,$$

$$(\alpha \oplus \varphi)(x+u) = \alpha(x) + \varphi(u).$$

Proof.

For all $x, y, z \in \mathcal{H}$ and $u, v, w \in V$, using the relations (3.1), we have:

$$\begin{aligned}(\alpha \oplus \varphi)((x+u) * (y+v)) &= (\alpha \oplus \varphi)(xy + l_x v + r_y u) = \alpha(xy) + \varphi(l_x v) + \varphi(r_y u)\\ &= \alpha(x)\alpha(y) + (\varphi \circ l_x)v + (\varphi \circ r_y)u\\ &= \alpha(x)\alpha(y) + (l \circ \alpha(x))\varphi(v) + (r\alpha(y))\varphi(u)\\ &= (\alpha(x) + \varphi(u)) * (\alpha(y) + \varphi(v)).\end{aligned}$$

The antiassociator associated to the bilinear product $*$ gives:

$$\begin{aligned}&(x+u, y+v, z+w)_{-1,\alpha\oplus\varphi} = ((x+u) * (y+v)) * (\alpha \oplus \varphi)(z+w)\\ &+(\alpha \oplus \varphi)(x+u) * ((y+v) * (z+w)) = (xy + l_x v + r_y u) * (\alpha(z) + \varphi(w))\\ &+(\alpha(x) + \varphi(u)) * (yz + l_y w + r_z v) = (xy)\alpha(z) + l_{xy}\varphi(w) + (r \circ \alpha(z))(l_x v + r_y u) + \alpha(x)(yz)\\ &+r_{yz}\varphi(u) + \left((l \circ \alpha(x))(l_y w + r_z v)\right) = ((xy)\alpha(z) + \alpha(x)(yz))\\ &+(l_{xy} \circ \varphi + (l \circ \alpha(x)) \circ l_y)w + ((r \circ \alpha(z)) \circ l_x + l \circ \alpha(x) r_z)v + (r_\circ \circ \alpha(z) r_y + r_{yz})u = (x,y,z)_{-1,\alpha}\\ &+ (l_{xy} \circ \varphi + (l \circ \alpha(x)) \circ l_y)w + ((r \circ \alpha(z)) \circ l_x + (l \circ \alpha(x)) \circ r_z)v + ((r \circ \alpha(z)) \circ r_y + r_{yz})u.\end{aligned}$$

Also, we have:

$$\begin{aligned}(z+w, y+v, x+u)_{-1,\alpha\oplus\varphi} &= (z,y,x)_{-1,\alpha} + (l_{zy} \circ \varphi + (l \circ \alpha(z)) \circ l_y)u + ((r \circ \alpha(x)) \circ l_z\\ &+(l \circ \alpha(z)) \circ r_x)v + ((r \circ \alpha(x)) \circ r_y + r_{yx})w.\end{aligned}$$

Using the relations (3.2) and (3.3), we have:

$$\begin{aligned}&(x+u, y+v, z+w)_{-1,\alpha\oplus\varphi} + (z+w, y+v, x+u)_{-1,\alpha\oplus\varphi} = (x,y,z)_{-1,\alpha} + (z,y,x)_{-1,\alpha}\\ &+\Big(\big(l_{zy} \circ \varphi + (l \circ \alpha(z)) \circ l_y\big) + \big((r \circ \alpha(z)) \circ r_y + r_{yz}\big)\Big)u + \big(((r \circ \alpha(z)) \circ l_x + (l \circ \alpha(x)) \circ r_z)\\ &+((r \circ \alpha(z)) \circ l_x + (l \circ \alpha(x)) \circ r_z)\big)v + \Big(\big(l_{xy} \circ \varphi + (l \circ \alpha(x)) \circ l_y\big) + \big((r \circ \alpha(x)) \circ r_y + r_{yx}\big)\Big)w\\ &= 0.\end{aligned}$$

Hence, $(\varphi, l, r, V)$ is a bimodule of the hom-anticenter-symmetric algebra $(\mathcal{H}, \alpha)$ if and only if the equations (3.1), (3.2), and (3.3) are satisfied.

We denote the hom-anticenter-symmetric algebra $(\mathcal{H} \oplus V, *, \alpha \oplus \varphi)$ by $\mathcal{H} \ltimes_{l,r}^{-1,\alpha,\varphi} V$ or simply by $\mathcal{H} \ltimes^{-1} V$.

Example 3.3 Let $(\mathcal{H}, \alpha)$ be a hom-anticenter-symmetric algebra. $(L, R, \mathcal{H}, \alpha)$ is a bimodule of $(\mathcal{H}, \alpha)$.

Lemma 3.4 Let $(\varphi, l, r, V)$ be a bimodule of a hom-anticenter-symmetric algebra $(\mathcal{H}, \alpha)$. Then, we have:

- $(\rho = l + r, \varphi)$ is a representation of the sub-adjacent hom-Mock Lie algebra $\mathcal{G}(\mathcal{H})$ associated to $(\mathcal{H}, \alpha)$;
- For any representation $(\rho, \psi)$, with $\rho: \mathcal{G}(\mathcal{H}) \to \mathfrak{gl}(\mathcal{G}(\mathcal{H}))$, of the underlying hom-Mock Lie algebra $(\mathcal{G}(\mathcal{H}), \alpha)$ of the hom-anticenter-symmetric algebra $(\mathcal{H}, \alpha)$, the triple $(\rho, 0, \psi)$ is a bimodule of $(\mathcal{H}, \psi)$;
- The hom-anticenter-symmetric algebras $\mathcal{H} \ltimes_{l,r}^{-1,\alpha,\varphi} V$ and $\mathcal{H} \ltimes_{l+r,0}^{-1,\alpha,\varphi} V$ given by the bimodules $(l, r, \varphi)$ and $(l + r, 0, \varphi)$, respectively, have the same sub-adjacent hom-Mock Lie algebra given by the semidirect sum $\mathcal{G}(\mathcal{H}) \ltimes_{l+r}^{-1,\alpha,\varphi}$ V of the hom-Mock Lie algebra $(\mathcal{G}(\mathcal{H}), \alpha)$ and its representation $(l + r, \alpha, V)$ as: $[x + u, y + v] = [x, y] + (l + r)(x)v + (l + r)(y)u, \forall x, y \in \mathcal{H}, u, v \in V$.

Proof.

Let $(\varphi, l, r, V)$ be a bimodule of a hom-anticenter-symmetric algebra $(\mathcal{H}, \alpha)$. For all $x, y, z \in \mathcal{H}$and $u, v, w \in V$, we have :

- $(l + r)(\alpha(x)) \circ \varphi = l(\alpha(x)) \circ \varphi + r(\alpha(x)) \circ \varphi = \varphi \circ l_x + \varphi \circ r_x = \varphi \circ (l + r)(x)$.

$$(l + r)([x, y]) \circ \varphi = (l + r)(xy) \circ \varphi + (l + r)(yx) \circ \varphi = l(xy) \circ \varphi + r(xy) \circ \varphi + l(yx) \circ \varphi$$
$$+r(yx) \circ \varphi = (l(xy) + r(yx))\varphi + (l(yx) + r(xy))\varphi = -(l \circ \alpha(x)) \circ l(y) - (r \circ \alpha(x)) \circ r(y)$$
$$-(l \circ \alpha(y)) \circ l(x) - (r \circ \alpha(y)) \circ r(x) = -(l \circ \alpha(x)) \circ l(y) - (r \circ \alpha(x)) \circ r(y) - (l \circ \alpha(y)) \circ l(x)$$
$$-(r \circ \alpha(y)) \circ r(x) - (l \circ \alpha(x)) \circ r(y) - (r \circ \alpha(y)) \circ l(x) - (l \circ \alpha(y)) \circ r(x) - (r \circ \alpha(x)) \circ l(y)$$
$$=-((l + r) \circ \alpha(x)) \circ (l + r)(y) - ((l + r) \circ \alpha(y)) \circ (l + r)(x).$$

Hence, $(l + r, \varphi)$ is a representation of the sub-adjacent hom-Mock Lie algebra $\mathcal{G}(\mathcal{H})_\alpha$ of the hom-anticenter-symmetric algebra $(\mathcal{H}, \alpha)$.

- We know that$(\rho, \psi)$ is a representation of $\mathcal{G}(\mathcal{H})_\alpha$ and taking$r = 0$, then the relations (3.1) and (3.3) are verified, and, in addition, we have, for all $x, y \in \mathcal{H}$,

$$\rho([x, y]) \circ \psi = -\rho(\alpha(x)) \circ \rho(y) - \rho(\alpha(y)) \circ \rho(x)$$

which implies

$$\rho(xy) \circ \psi + \rho(\alpha(x)) \circ \rho(y) = -\rho(yx) \circ \psi - \rho(\alpha(y)) \circ \rho(x).$$

This is exactly the equation (3.2).

- The anticommutator of the bilinear product defined as $(x + u) * (y + v) := xy + l_x v + r_y u$ is given by $(x + u) * (y + v) + (y + v) * (x + u) = [x, y] + (l + r)(x)v + (l + r)(y)u = [x + u, y + v]$.
- 

Definition 3.5 Let $(\mathcal{H}, \alpha)$ be a hom-anticenter-symmetric algebra. The dual maps $l^*$, $r^*$ of the linear maps l, $r: \mathcal{H} \to \mathfrak{gl}(V)$ are defined, respectively, as:

$l^*: \mathcal{H} \longrightarrow \mathfrak{gl}(V^*), \langle l_x^* u^*, v\rangle := \langle u^*, l_x v\rangle;$ (3.4)

$r^*: \mathcal{H} \longrightarrow \mathfrak{gl}(V^*), \langle r_x^* u^*, v\rangle := \langle u^*, r_x v\rangle.$ (3.5)

for all $x \in \mathcal{H}, u^* \in V^*, v \in V$.

Proposition 3.6 Consider$(\mathcal{H}, \alpha)$ a hom-anticenter-symmetric algebra such that $\alpha^2 = \mathrm{id}$ and, $l, r: \mathcal{H} \to \mathfrak{gl}(V)$and $\varphi: V \to V$,three linear maps, with$V$ is a finite dimensional vector space. Then, the following statements are equivalent:

- $(l, r, \varphi, V)$is a bimodule of $\mathcal{H}$.
- $(r^*, l^*, \varphi^*, V^*)$is a bimodule of $\mathcal{H}$.

Proof.

Consider a hom-anticenter-symmetric algebra $(\mathcal{H}, \alpha)$ and two linear maps $l, r: \mathcal{H} \to \mathfrak{gl}(V)$, with their dual maps $l^*, r^*$ satisfying the relations (3.4) and (3.5).

- On the one hand, let's suppose that $(l, r, V)$ is a bimodule of the hom-anticenter-symmetric algebra $(\mathcal{H}, \alpha)$. For all $x, y \in \mathcal{H}, v \in V$, and $u^* \in V^*$, we have:

$$\langle(\varphi^* \circ r_x^*)u^*, v\rangle = \langle r_x^* u^*, \varphi(v)\rangle = \langle u^*, r_x \varphi(v)\rangle$$
$$= \langle u^*, \varphi \circ r_{\alpha(x)} v\rangle = \langle \varphi^*(u^*), r_{\alpha(x)} v\rangle$$
$$\langle(\varphi^* \circ l_x^*)u^*, v\rangle = \langle l_x^* u^*, \varphi(v)\rangle = \langle u^*, l_x \varphi(v)\rangle$$
$$= \langle u^*, \varphi \circ l_{\alpha(x)} v\rangle = \langle \varphi^*(u^*), l_{\alpha(x)} v\rangle$$

Then,

$$\varphi^* \circ l_x^* = l_{\alpha(x)}^* \circ \varphi^*, \varphi^* \circ r_x^* = r_{\alpha(x)}^* \circ \varphi^*.$$

Besides,

$$\langle -(r_{\alpha(x)}^* r_y^* - r_{xy}^* \circ \varphi^*)u^*, v\rangle = \langle u^*, (-r_y r_{\alpha(x)} - \varphi \circ r_{xy})v\rangle = \langle u^*, (-r_{\alpha^2(y)} r_{\alpha(x)} - r_{\alpha(xy)} \circ \varphi)v\rangle$$

$$= \left\langle u^*, \left(-r_{\alpha^2(y)} r_{\alpha(x)} - r_{\alpha(x)\alpha(y)} \circ \varphi\right) v \right\rangle$$
$$= \left\langle u^*, \left(l_{\alpha(yx)} \circ \varphi + l_{\alpha^2(y)} l_{\alpha(x)}\right) v \right\rangle = \left\langle u^*, \left(\varphi \circ l_{yx} + l_y l_{\alpha(x)}\right) v \right\rangle$$
$$\left\langle \left(-r^*_{\alpha(x)} r^*_y - r^*_{xy} \circ \varphi^*\right) u^*, v \right\rangle = \left\langle \left(l^*_{yx} \circ \varphi^* + l^*_{\alpha(x)} l^*_y\right) u^*, v \right\rangle.$$

It follows that

$$-r^*_{\alpha(x)} r^*_y - r^*_{xy} \circ \varphi^* = l^*_{yx} \circ \varphi^* + l^*_{\alpha(x)} l^*_y. \#(3.6)$$

Taking $\alpha^2 = \mathrm{id}_{\mathcal{H}}$, we obtain:

$$\left\langle \left(-r^*_{\alpha(x)} l^*_y - l^*_{\alpha(y)} r^*_x\right) u^*, v \right\rangle = \left\langle u^*, \left(-l_y r_{\alpha(x)} - r_x l_{\alpha(y)}\right) v \right\rangle = \left\langle u^*, \left(-l_{\alpha^2(y)} r_{\alpha(x)} - r_{\alpha^2(x)} l_{\alpha(y)}\right) v \right\rangle$$
$$= \left\langle u^*, \left(l_{\alpha^2(x)} r_{\alpha(y)} + r_{\alpha^2(y)} l_{\alpha(x)}\right) v \right\rangle = \left\langle u^*, \left(l_x r_{\alpha(y)} + r_y l_{\alpha(x)}\right) v \right\rangle$$
$$= \left\langle \left(r^*_{\alpha(y)} l^*_x + l^*_{\alpha(x)} r^*_y\right) u^*, v \right\rangle.$$

Then, using $\alpha^2 = \mathrm{id}_{\mathcal{H}}$, the relation

$$-r^*_{\alpha(x)} l^*_y - l^*_{\alpha(y)} r^*_x = r^*_{\alpha(y)} l^*_x + l^*_{\alpha(y)} r^*_x \#(3.7)$$

is satisfied.

Finally, from the relations (3.6) and (3.7), we conclude that the triple $(r^*, l^*, V^*)$ is a bimodule of the hom-anticenter-symmetric algebra $(\mathcal{H}, \alpha)$.

- On the other hand, suppose that $(r^*, l^*, V^*)$ is a bimodule of $\mathcal{H}$. Then, by a direct computation, we show that $(l, r, V)$ is a bimodule of $\mathcal{H}$.

Hence, it is clear that $(l, r, V)$ is a bimodule of the hom-anticenter-symmetric algebra $(\mathcal{H}, \alpha)$ if, and only if, the triple $(r^*, l^*, V^*)$, where $r^*, l^*$ are given by the relations (3.4) and (3.5), respectively, is a bimodule of the hom-anticenter-symmetric algebra $(\mathcal{H}, \alpha)$ with $\alpha^2 = \mathrm{id}_{\mathcal{H}}$.

Theorem 3.7 Consider $(\mathcal{H}, \cdot, \alpha_{\mathcal{H}})$ and $(\mathcal{B}, \circ, \alpha_{\mathcal{B}})$ two hom-anticenter symmetric algebras. Suppose there are linear maps $l_{\mathcal{H}}, r_{\mathcal{H}}: \mathcal{H} \to \mathbf{gl}(\mathcal{B})$ and $l_{\mathcal{B}}, r_{\mathcal{B}}: \mathcal{B} \to \mathbf{gl}(\mathcal{H})$ such that $(l_{\mathcal{H}}, r_{\mathcal{H}}, \alpha_{\mathcal{B}})$ and $(l_{\mathcal{B}}, r_{\mathcal{B}}, \alpha_{\mathcal{H}})$ are bimodules of the hom-anticenter-symmetric algebra $\mathcal{H}$ and $\mathcal{B}$, respectively, satisfying the following conditions for all $x, y \in \mathcal{H}$ and $a, b \in \mathcal{B}$:

$$\begin{gathered}(l_{\mathcal{B}}(a)x) \cdot (\alpha_{\mathcal{H}}(y)) + l_{\mathcal{B}}(r_{\mathcal{H}}(x)a)(\alpha_{\mathcal{H}}(y)) + l_{\mathcal{B}}(\alpha_{\mathcal{B}}(a))(x \cdot y) + r_{\mathcal{B}}(\alpha_{\mathcal{B}}(a))(y \cdot x) \\ +(\alpha_{\mathcal{H}}(y)) \cdot (r_{\mathcal{B}}(a)x) + r_{\mathcal{B}}(l_{\mathcal{H}}(x)a)(\alpha_{\mathcal{H}}(y)) = 0,\end{gathered} \#(3.8)$$

$$\begin{gathered}(r_{\mathcal{B}}(a)x) \cdot (\alpha_{\mathcal{H}}(y)) + l_{\mathcal{B}}(l_{\mathcal{H}}(x)a)(\alpha_{\mathcal{H}}(y)) + (\alpha_{\mathcal{H}}(x)) \cdot (l_{\mathcal{B}}(a)y) + r_{\mathcal{B}}(r_{\mathcal{H}}(y)a)(\alpha_{\mathcal{H}}(x)) \\ +(r_{\mathcal{B}}(a)y) \cdot (\alpha_{\mathcal{H}}(x)) + l_{\mathcal{B}}(l_{\mathcal{H}}(y)a)(\alpha_{\mathcal{H}}(x)) + (\alpha_{\mathcal{H}}(y)) \cdot (l_{\mathcal{B}}(a)x) + r_{\mathcal{B}}(r_{\mathcal{H}}(x)a)(\alpha_{\mathcal{H}}(y)) = 0,\end{gathered} \#(3.9)$$

$$\begin{gathered}(l_{\mathcal{B}}(x)a) \circ \left(\alpha_{\mathcal{B}}(b)\right) + l_{\mathcal{H}}(r_{\mathcal{B}}(a)x)\left(\alpha_{\mathcal{B}}(b)\right) + l_{\mathcal{H}}\left(\alpha_{\mathcal{H}}(x)\right)(a \circ b) + r_{\mathcal{H}}\left(\alpha_{\mathcal{H}}(x)\right)(b \circ a) \\ +(\alpha_{\mathcal{B}}(b)) \circ (r_{\mathcal{H}}(x)a) + r_{\mathcal{H}}(l_{\mathcal{B}}(a)x)(\alpha_{\mathcal{B}}(b)) = 0\end{gathered} \quad (3.10)$$

$$\begin{gathered}(r_{\mathcal{H}}(x)a) \circ (\alpha_{\mathcal{B}}(b)) + l_{\mathcal{H}}(l_{\mathcal{B}}(a)x)(\alpha_{\mathcal{B}}(b)) + (\alpha_{\mathcal{B}}(a)) \circ (l_{\mathcal{H}}(x)b) + r_{\mathcal{H}}(r_{\mathcal{B}}(b)x)(\alpha_{\mathcal{B}}(a)) \\ +(r_{\mathcal{H}}(x)b) \circ (\alpha_{\mathcal{B}}(a)) + l_{\mathcal{H}}(l_{\mathcal{B}}(b)x)(\alpha_{\mathcal{B}}(a)) + (\alpha_{\mathcal{B}}(b)) \circ (l_{\mathcal{H}}(x)a) + r_{\mathcal{H}}(r_{\mathcal{B}}(a)x)(\alpha_{\mathcal{B}}(b)) = 0.\end{gathered} \#(3.11)$$

Then, there exists a hom-anticenter-symmetric algebra structure on the vector space $\mathcal{H} \oplus \mathcal{B}$ given by:

$$(x + a) * (y + b) = (x \cdot y + l_{\mathcal{B}}(a)y + r_{\mathcal{B}}(b)x) + (a \circ b + l_{\mathcal{H}}(x)b + r_{\mathcal{H}}(y)a). \#(3.12)$$

Proof. We have for all $x, y, z \in \mathcal{H}$ and $a, b, c \in \mathcal{B}$:

$$(\alpha_{\mathcal{H}} + \alpha_{\mathcal{B}})((x + a) * (y + b)) = \alpha_{\mathcal{H}}(x \cdot y + l_{\mathcal{B}}(a)y + r_{\mathcal{B}}(b)x) + \alpha_{\mathcal{B}}(a \circ b + l_{\mathcal{H}}(x)b + r_{\mathcal{H}}(y)a)$$
$$= \alpha_{\mathcal{H}}(x \cdot y) + \alpha_{\mathcal{H}}(l_{\mathcal{B}}(a)y) + \alpha_{\mathcal{H}}(r_{\mathcal{B}}(b)x) + \alpha_{\mathcal{B}}(a \circ b) + \alpha_{\mathcal{B}}(l_{\mathcal{H}}(x)b) + \alpha_{\mathcal{B}}(r_{\mathcal{H}}(y)a)$$
$$= (\alpha_{\mathcal{H}}(x) \cdot \alpha_{\mathcal{H}}(y) + l_{\mathcal{B}}(\alpha_{\mathcal{B}}(a))\alpha_{\mathcal{H}}(y) + r_{\mathcal{B}}(\alpha_{\mathcal{B}}(b))\alpha_{\mathcal{H}}(x)) + (\alpha_{\mathcal{B}}(a) \circ \alpha_{\mathcal{B}}(a) + l_{\mathcal{H}}(\alpha_{\mathcal{H}}(x))\alpha_{\mathcal{B}}(b)$$
$$+ r_{\mathcal{H}}(\alpha_{\mathcal{H}}(y))\alpha_{\mathcal{B}}(a)) = \left((\alpha_{\mathcal{H}}(x) + (\alpha_{\mathcal{B}}(a)) * \left((\alpha_{\mathcal{H}}(y) + (\alpha_{\mathcal{B}}(b)\right);\right.$$

and

$$(x + a, y + b, z + c)_{-1, \alpha_{\mathcal{H}} \oplus \alpha_{\mathcal{B}}} = (x, y, z)_{-1, \alpha_{\mathcal{H}}} + (x, y, c)_{-1, \alpha_{\mathcal{H}} \oplus \alpha_{\mathcal{B}}} + (x, b, z)_{-1, \alpha_{\mathcal{H}} \oplus \alpha_{\mathcal{B}}} + (x, b, c)_{-1, \alpha_{\mathcal{H}} \oplus \alpha_{\mathcal{B}}}$$
$$+ (a, y, z)_{-1, \alpha_{\mathcal{H}} \oplus \alpha_{\mathcal{B}}} + (a, y, c)_{-1, \alpha_{\mathcal{H}} \oplus \alpha_{\mathcal{B}}} + (a, b, z)_{-1, \alpha_{\mathcal{H}} \oplus \alpha_{\mathcal{B}}} + (a, b, c)_{-1, \alpha_{\mathcal{B}}}.$$

Set

$$(x + a, y + b, z + c)_{-1, \alpha_{\mathcal{H}} \oplus \alpha_{\mathcal{B}}} = -(z + c, y + b, x + a)_{-1, \alpha_{\mathcal{H}} \oplus \alpha_{\mathcal{B}}}.$$

By a direct computation, we obtain the equations (3.8)-(3.11).

We denote the hom-anticenter -symmetric algebra $(\mathcal{H} \oplus \mathcal{B}, *, \alpha_{\mathcal{H}} \oplus \alpha_{\mathcal{B}})$ by $\left(\mathcal{H} \bowtie^{-1, l_{\mathcal{H}}, r_{\mathcal{H}}}_{l_{\mathcal{B}}, r_{\mathcal{B}}} \mathcal{B}, \alpha_{\mathcal{H}} \oplus \alpha_{\mathcal{B}}\right)$. The system $(l_{\mathcal{H}}, r_{\mathcal{H}}, l_{\mathcal{B}}, r_{\mathcal{B}}, \alpha_{\mathcal{H}}, \alpha_{\mathcal{B}}, \mathcal{H}, \mathcal{B})$, where $l_{\mathcal{H}}, r_{\mathcal{H}}, l_{\mathcal{B}}, r_{\mathcal{B}}, \alpha_{\mathcal{H}}$ and $\alpha_{\mathcal{B}}$ satisfy the conditions (3.8) –(3.11), is called a matched pair of the hom-anticenter -symmetric algebras $\mathcal{H}$ and $\mathcal{B}$.

Definition 3.8 A matched pair of hom-Mock Lie algebras $(\mathcal{G},\mathcal{H},\rho_{\mathcal{G}},\rho_{\mathcal{H}},\varphi_{\mathcal{G}},\varphi_{\mathcal{H}})$ consists of two hom-Mock Lie algebras $(\mathcal{G},[,]_{\mathcal{G}},\varphi_{\mathcal{G}})$ and $(\mathcal{H},[,]_{\mathcal{H}},\varphi_{\mathcal{H}})$, together with the associated hom-Mock Lie algebra representations $\rho_{\mathcal{G}}:\mathcal{G}\to gl(\mathcal{H})$ and $\rho_{\mathcal{H}}:\mathcal{H}\to gl(\mathcal{G})$ defined with respect to $\varphi_{\mathcal{H}}$ and $\varphi_{\mathcal{G}}$, respectively, satisfying the following relations

$$\rho_{\mathcal{H}}(\varphi_{\mathcal{H}}(a))[x,y]_{\mathcal{G}}=-[\rho_{\mathcal{H}}(a)(x),\varphi_{\mathcal{G}}(y)]_{\mathcal{G}}-[\varphi_{\mathcal{G}}(x),\rho_{\mathcal{H}}(a)(y)]_{\mathcal{G}},$$
$$\rho_{\mathcal{G}}(\varphi_{\mathcal{G}}(x))[a,b]_{\mathcal{H}}=-[\rho_{\mathcal{G}}(x)(a),\varphi_{\mathcal{H}}(b)]_{\mathcal{H}}-[\varphi_{\mathcal{H}}(a),\rho_{\mathcal{G}}(x)(b)]_{\mathcal{H}}$$

Corollary 3.9 Let $(\mathcal{H},\mathcal{B},l_{\mathcal{H}},r_{\mathcal{H}},l_{\mathcal{B}},r_{\mathcal{B}},\alpha_{\mathcal{H}},\alpha_{\mathcal{B}})$ be a matched pair of hom-anticenter-symmetric algebras $(\mathcal{H},\cdot,\alpha_{\mathcal{H}})$and $(\mathcal{B},\circ,\alpha_{\mathcal{B}})$. Then, $(\mathcal{G}(\mathcal{H}),\mathcal{G}(\mathcal{B}),l_{\mathcal{H}}+r_{\mathcal{H}},l_{\mathcal{B}}+r_{\mathcal{B}},\alpha_{\mathcal{H}},\alpha_{\mathcal{B}})$ is a matched pair of hom-Mock Lie algebras $\mathcal{G}(\mathcal{H})_{\alpha_{\mathcal{H}}}$and$\mathcal{G}(\mathcal{B})_{\alpha_{\mathcal{B}}}$.

Proof.

From Lemma 3.4, it is obvious to see that$(\rho_{\mathcal{G}(\mathcal{H})}:=l_{\mathcal{H}}+r_{\mathcal{H}},\alpha_{\mathcal{H}},\mathcal{B})$ and $(\rho_{\mathcal{G}(\mathcal{B})}:=l_{\mathcal{B}}+r_{\mathcal{B}},\alpha_{\mathcal{B}},\mathcal{H})$ are linear representations of the sub-adjacent hom-Mock Lie algebras $(\mathcal{G}(\mathcal{H}),\alpha_{\mathcal{H}})$and $(\mathcal{G}(\mathcal{B}),\alpha_{\mathcal{B}})$, respectively.Moreover, the linear maps $(\rho_{\mathcal{G}}=\rho_{\mathcal{G}(\mathcal{H})},\varphi_{\mathcal{G}}=\alpha_{\mathcal{H}})$ and $(\rho_{\mathcal{H}}=\rho_{\mathcal{G}(\mathcal{B})},\varphi_{\mathcal{H}}=\alpha_{\mathcal{B}})$ satisfy the relations (3.13) and (3.14).

Then, the hom-Jacobi identity (2.6 )condition associated to the underlying hom-Mock Lie algebra from the quadruple $(\mathcal{G}(\mathcal{H}),\mathcal{G}(\mathcal{B}),l_{\mathcal{H}}+r_{\mathcal{H}},l_{\mathcal{B}}+r_{\mathcal{B}},\alpha_{\mathcal{H}},\alpha_{\mathcal{B}})$ is equivalent to the relations (3.13) and (3.14).

Theorem 3.10 Consider $(\mathcal{H},\cdot,\alpha)$ and $(\mathcal{H}^*,\circ,\alpha^*)$ two hom-anticenter-symmetric algebras. $(\mathcal{H},\mathcal{H}^*,R^*_{\cdot},L^*_{\cdot},R^*_{\circ},L^*_{\circ},\alpha,\alpha^*)$is a matched pair of hom-anticenter-symmetric algebras $(\mathcal{H},\cdot,\alpha)$ and $(\mathcal{H}^*,\circ,\alpha^*)$ if, and only if,$(\mathcal{G}(\mathcal{H}),\mathcal{G}(\mathcal{H}^*),\mathbb{R}^*+L^*_{\cdot},R^*_{\circ}+L^*_{\circ},\alpha,\alpha^*)$ is a matched pair of the hom-Mock Lie algebras $\mathcal{G}(H)_{\alpha}$ and $\mathcal{G}(H^*)_{\alpha^*}$.

Proof.

From Theorem 3.7, taking $\mathcal{B}=\mathcal{H}^*,l_{\mathcal{H}}=R^*_{\cdot},r_{\mathcal{H}}=L^*_{\cdot},l_{\mathcal{B}}=R^*_{\circ},r_{\mathcal{B}}=L^*_0$, using Definition 3.8 by assuming that $\mathcal{G}(\mathcal{H})=\mathcal{G},\mathcal{G}(\mathcal{H}^*)=\mathcal{H},\rho_{\mathcal{G}}=R^*_{\cdot}+L^*_{\cdot},\rho_{\mathcal{H}}=R^*_{\circ}+L^*_{\circ},\varphi_{\mathcal{G}}=\alpha$, $\varphi_{\mathcal{H}}=\alpha^*$, and taking into account the relations (3.4) and (3.5), we establish the equivalence.

**Manin triple of hom-anticenter-symmetric algebras**

Definition 4.1 A Manin triple of hom-anticenter-symmetric algebras $(\mathcal{H},\cdot,\alpha)$ and $(\mathcal{B},*,\alpha')$ is a triple $(\mathcal{H}\oplus\mathcal{B},\mathcal{H},\mathcal{B})$, together with a nondegenerate symmetric bilinear form $S(\cdot,\cdot)$ on the hom-anticenter-symmetric algebra $(\mathcal{H}\oplus\mathcal{B},\star,\alpha\oplus\alpha')$ such that

- $S$is invariant, $i$. e. for all $x,y,z\in\mathcal{H}$ and $a,b,c\in\mathcal{B}$,
$$S((x+a)\star(y+b),(z+c))=S((x+a),(y+b)\star(z+c)),$$
$$S\Big((\alpha\oplus\alpha')(x+a),y+b\Big)=S(x+a,(\alpha\oplus\alpha')(y+b)).$$
- The hom-anticenter-symmetric algebras $\mathcal{H}$ and $\mathcal{B}$ are isotropic hom-anticenter-symmetric algebras of $A\oplus\mathcal{B}$.

In particular, consider$(\mathcal{H},\cdot,\alpha)$ is a hom-anticenter-symmetric algebra.Suppose, there exists a hom-anticenter-symmetric algebra structure on its dual space $\mathcal{H}^*$ denoted by $(\mathcal{H}^*,o,\alpha^*)$. Then, there is a hom-anticenter-symmetric algebra on the direct sum of the underlying vector space of $\mathcal{H}$ and its dual space $\mathcal{H}^*$ such that $(\mathcal{H}\oplus\mathcal{H}^*,\mathcal{H},\mathcal{H}^*)$ is the associated Manin triple with the invariant bilinear symmetric form given by $\mathfrak{B}_H(x+a,y+b)=<x,b>+<y,a>$ for all $x,y\in\mathcal{H}$ and $a,b\in\mathcal{H}^*$. It is called the standard Manin triple of the hom-anticenter-symmetric algebra $\mathcal{H}$; $<,>$ is a natural paring between algebra and its dual space.

Proposition 4.2 Let $(\mathcal{H},\cdot,\alpha)$ and $(\mathcal{H}^*,\circ,\alpha^*)$ be two hom-anticenter-symmetric algebras. Then, $(\mathcal{H},\mathcal{H}^*,R^*_{\cdot},L^*_{\cdot},R^*_{\circ},L^*_{\circ},\alpha,\alpha^*)$ is a matched pair of the hom-anticenter-symmetric algebras $(\mathcal{H},\cdot,\alpha)$ and $(\mathcal{H}^*,\circ,\alpha^*)$ if and only if $(\mathcal{H}\oplus\mathcal{H}^*,\mathcal{H},\mathcal{H}^*)$ is a standard Manin triple.

Proof.

Let us compute and compare the following relations: $S_{\mathcal{H}}((x+a)*(y+b),(z+c))$ and $S_{\mathcal{H}}((x+a),(y+b)*(z+c))\forall x,y,z\in\mathcal{H}$and $\forall a,b,c\in\mathcal{H}^*$. We have:

$$S_{\mathscr{H}}((x+a)*(y+b),(z+c)) = S_{\mathscr{H}}(xy + R_{\circ}^{*}(a)y + L_{\circ}^{*}(b)x + a\circ b + R_{\cdot}^{*}(x)b + L_{\cdot}^{*}(y)a, z+c)$$
$$= \langle xy + R_{\circ}^{*}(a)y + L_{\circ}^{*}(b)x, c\rangle + \langle z, a\circ b + R_{\cdot}^{*}(x)b + L_{\cdot}^{*}(y)a\rangle$$
$$= \langle xy, c\rangle + \langle R_{\circ}^{*}(a)y, c\rangle + \langle L_{\circ}^{*}(b)x, c\rangle + \langle z, a\circ b\rangle + \langle z, R_{\cdot}^{*}(x)b\rangle$$
$$+\langle z, L_{\cdot}^{*}(y)a\rangle = \langle xy, c\rangle + \langle y, R_a(c)\rangle + \langle x, L_b(c)\rangle + \langle z, a\circ b\rangle$$
$$+\langle R_x(z), b\rangle + \langle L_y(z), a\rangle$$
$$= \langle xy, c\rangle + \langle y, c\circ a\rangle + \langle x, b\circ c\rangle + \langle z, a\circ b\rangle + \langle zx, b\rangle + \langle yz, a\rangle.$$

$$S_{\mathscr{H}}((x+a),(y+b)*(z+c)) = S_{\mathscr{H}}(x+a, yz + R_{\circ}^{*}(b)z + L_{\circ}^{*}(c)y + b\circ c + R_{\cdot}^{*}(y)c + L_{\cdot}^{*}(z)b)$$
$$= \langle x, b\circ c + R_{\cdot}^{*}(y)c + L_{\cdot}^{*}(z)b\rangle + \langle yz + R_{\circ}^{*}(b)z + L_{\circ}^{*}(c)y, a\rangle$$
$$+\langle x, b\circ c\rangle + \langle x, R_{\cdot}^{*}(y)c\rangle + \langle x, L_{\cdot}^{*}(z)b\rangle + \langle yz, a\rangle + \langle R_{\circ}^{*}(b)z, a\rangle$$
$$+\langle L_{\circ}^{*}(c)y, a\rangle = \langle x, b\circ c\rangle + \langle R_y(x), c\rangle + \langle L_z(x), b\rangle$$
$$+\langle yz, a\rangle + \langle z, R_b(a)\rangle + \langle y, L_c(a)\rangle$$
$$= \langle x, b\circ c\rangle + \langle xy, c\rangle + \langle zx, b\rangle + \langle yz, a\rangle + \langle z, a\circ b\rangle + \langle y, c\circ a\rangle.$$

It follows that

$$S_{\mathscr{H}}\big((x+a)*(y+b),(z+c)\big) = S_{\mathscr{H}}\big((x+a),(y+b)*(z+c)\big). \#(4.1)$$

Hence, the invariance of the standard bilinear form on $\mathscr{H}\oplus\mathscr{H}^*$. Therefore, $(\mathscr{H}\oplus\mathscr{H}^*, \mathscr{H}, \mathscr{H}^*)$ is the standard Manin triple of the hom-anticenter-symmetric algebras $\mathscr{H}$and $\mathscr{H}^*$.

## Concluding Remarks

In this work, we have introduced the hom-anticenter-symmetric algebras. Their bimodules, dual bimodules and matched pairs are established. The relation of the bimodules and matched pairs of hom-anticenter-symmetric algebras between those of hom-Mock Lie algebras is studied. Finally, the Manin triple of hom-anticenter-symmetric algebras is constructed.

## References:-

[1] N. Aizawa, Sato H.: q-Deformation of the Virasoro algebra with central extension, Physics Letters B, Vol. 256, Issue 20, 185-190 (1991).
[2] J. Anitheou, S. Attan, K. Kangni, Deformations of relative Rota-Baxter operators on Hom-Jacobi-Jordan algebras, ArXiv:2308.04203 [math.RA] (2023), 21 pp.
[3] M. Camacho, Ivan Kaygorodov, Victor Lopatkin and Mohamed A. Salim, The variety of dual mock-Lie algebras, Communications in Mathematics 28 (2020) 161-178.
[4] Chaichian M., Ellinas D., Popowicz Z., Quantum conformal algebra with central extension, Phys. Lett. B 248, no. 1-2, 95-99, (1990).
[5] Chaichian, M., Isaev A. P., Lukierski J., Popowicz Z., Prešnajder P., q-Deformations of Virasoro algebra and conformal dimensions, Phys. Lett. B 262 (1), 32-38, (1991).
[6] C. E. Haliya and G. D. Houndedji, A double construction of quadratic anticentersymmetric Jacobi-Jordan algebras,Quasigroups and Related Systems 31 (2023), 25 - 52. https://doi.org/10.56415/qrs.v31.03
[7] C. E. Haliya and G. D. Houndedji, Hom-Jacobi-Jordan and Hom- antiassociative algebras with symmetric invariant nondegenerate bilinear forms, Quasigroups and Related Systems. 29 (2021), 61-88.
[8] J. Hartwig, D. Larsson and S. Silvestrov, Deformations of Lie algebras using $\sigma$-derivations, J. Algebra 295 (2006), 314-361.
[9] P. Zumanovich, Special and exceptional Mock-Lie algebras, Linear Algebra and its Applications (2017), 518, 79-96.