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RESEARCH ARTICLE

BAYESIAN MIXTURE APPROACH TO INCOME INEQUALITY AND POVERTY INDICES OF LIBERIA: A STUDY USING HIES DATA OF 2016-2017

David Clarence Gray
1. Department of Mathematics and Statistics College of Science & Technology, University of Liberia.

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Abstract

This paper provides a generalized structure to evaluate income inequality and poverty, focusing on income distribution and applying a Bayesian approach to derive various poverty measures. A parametric model from income distribution and samples from the posterior distribution were used to formulate poverty indices. Explicitly, it scrutinizes the Foster Greer Thorbecke (FGT) poverty index which assesses poverty by incorporating its incidence, depth, and severity. We evaluate poverty using discrete and continuous FGT indices, applying a Bayesian mixture model with lognormal components in the liberal context. This approach offers robust poverty estimates by integrating prior knowledge and handling data uncertainties. The analysis reveals significant income inequality across different counties in Liberia, underlining disparities in income distribution. The posterior distributions for the FGT indices provide comprehensive insights into poverty levels, emphasizing the need for targeted policy to address income inequality.

Introduction:

This study adopts Bayesian methods to analyze Liberia's poverty and income inequality using the Household Income and Expenditure Survey data. The studyhighlights specific variables such as income in the past 12 months, income in the past 7 days, gender, age, household head, and education level. Meanwhile, there are some more traditional ways of measuring income inequality and poverty, such as the Gini coefficient, Lorenz curve, income quintiles/deciles, basic needs approach, and simple cross-sectional analysis that have so far provided a basic understanding of poverty and income inequality in Liberia. However, they often fail to measure the multi-dimensionality of poverty, the gap and severity of poverty, or even the dynamic changes in poverty and inequality over time. Often, these limitations can be addressed only by new methodologies like multi-dimensional poverty indices, longitudinal studies, and more advanced econometric techniques.

This study adopts Bayesian techniques to fill this gap through robust and nuanced estimates of poverty rates and income distribution across counties and different demographics in Liberia. The current study pre-processed the HIES data with a focus on key variables including household income, expenditures, and demographic details. These three poverty indices that measure income inequality and poverty are the FGT, Watts, and San. This study formulates a methodological framework for FGT. The discrete FGT of the poverty index is described as demonstrated in previous studies (Lubrano, 2022; Ali, & Thorbecke, 2000; Alkire, & Foster, 2007; Anand, 1977). The mixture of log normal is used to model the income distribution to have three posterior distributions: poverty head-count ratio or poverty rate,

Corresponding Author: -David Clarence Gray
Address: Department of Mathematics and Statistics College of Science & Technology, University of Liberia.

poverty gap index, and poverty severity (Fourrier-Nicolai & Lubrano, 2020; World Bank, 2004a; Weisbrod, & Julian, 2008). Posterior distributions for poverty rates and inequality measures can be computed using Markov Chain Monte Carlo (MCMC) methods.

There are several advantages to the Bayesian approach. First, statistical tests can be conducted immediately after inferences; second, nonparametric methods are distribution-free but could be sensitive to tail behavior and less effective with small sample sizes. According to Hasegawa and Ueda (2007) and Ravallion (1988), dynamic poverty research using Bayesian approaches is limited in modeling individual income; instead, it uses a stationary process and derives the distribution of poverty decomposition into total, chronic, and transitory poverty using panel data. Historically, the use of mixtures for modeling income distribution by employing lognormal densities is a rich one. For instance, Flachaire and Nunez (2007) used a mixture of normal densities for the log-transformed income distribution for the United Kingdom, while Lubrano and Ndoye (2016) formulated decomposing inequality indices using Bayesian inference for a mixture of lognormal. On the other hand, Anderson *et la* (2014) used a mixture of lognormal distributionto identify the poor. Alternative specifications were taken by other research, and Lubrano (2014) proposed Bayesian inference for a mixture of Pareto densities in high-wage formation modeling using US data. Income distribution comparisons between countries have been conducted by Chotikapanich and Griffiths (2008) and Lander *et al.* (2017) using a mixture of gamma densities to model Canada against Indonesia.

This paper explainsthat the poverty headcount ratio measures the proportion of the population in each country that lives below the poverty line. It only provides a general, rough idea of how widespread poverty is. The poverty gap index, on the other hand, measures the average shortfall of the poor from the poverty line, it goes beyond the headcount ratio and considers the depth of poverty.This measure is expressed as a proportion of the poverty line itself. This index informs us not only of the headcount of the poor but also about how poor they are. The poverty severity index increases the dimension by considering inequality among the poor. In simple terms, it calculates the square of the poverty gap for each individual. i.e., it gives more weight to those further away from the poverty line. This would help us understand the intensity and inequality of poverty within the poor population.

Our paper is organized as follows. Section 2 introduces the FGT indices for poverty measures and three poverty indices. In this section, Bayesian inference for mixtures of lognormal densities is discussed, and analytical formulas to determine the three FGT indices and income distribution from such a study model with a mixture of log-normal densities are derived. Section 3, meanwhile, elaborates on the trend in income inequality and poverty for each county of Liberia with statistics. Discussions and decisions for policymakers are drawn in Section 4, while Section 5 provides the conclusions.

MATERIALS AND METHODS

This section outlines the framework of the poverty indices, which are designed to summarize the lower end of an income distribution $g(x)$based on different axioms (see Iyengar *et al*., 1992). Within a Bayesian framework, the income distribution is modeled parametricallyas$g(x/\theta)$, and samples are drawn from the posterior distribution of θ. These posterior samples are used to construct a range of poverty indices, capturing different dimensions of poverty within the distribution.

A. Discrete FGT Poverty Index

Lubrano (2022) disclosed that the FGT poverty indices are a family of poverty measures that provide an assessment of poverty in a population by incorporating both the incidence and intensity of poverty. There are three cases of FGT, which this study will delve into in establishing a poverty index for various counties in Liberia. The FGT index is defined as

$$P_\alpha = \frac{1}{n}\sum_{i=1}^{q}\left(1-\frac{y_i}{u}\right)^\alpha, \quad (1)$$

where, P_α is the FGT index for a given parameter α, α is a positive integer that determines the sensitivity of the index to poverty depth and inequality among the poor, n is the number of individuals with with income y_i below the poverty line u, u is the poverty line, andy_i is the income of the i^{th} poor individual. When $\alpha = 0$there is a poverty headcount ratio (P_0). This refers to the measure of the incidence of poverty, which indicatesthat the proportion of the population is below the poverty line. A Liberian is considered poor if he/she has an income lower than the poverty line u. statistically, it is better to suppose that a person with an income equal to u is not poor. Let y be a vector of incomes in a population of size n. The number of poor is given by

$h = \sum_{i=1}^{n} \mathbf{1}\,(y_i < u)$

(2)

and the poverty rate is measured as

$$P_0 = \frac{h}{n}.$$

(example, Lubrano and Ndoye, 2016).

Where $\alpha = 1$, there is the poverty gap index (P_1). This refers to the depth of poverty measured by accounting for the average shortfall in the incomes of poor individuals relative to the poverty line. The poverty gap index is defined as

$P_1 = \frac{1}{n}\sum_{i=1}^{q}\left(1 - \frac{y_i}{u}\right).$

(3)

Where $\alpha = 2$, there is a squared poverty gap index (P_2). This index measures the severity of poverty by giving more weight to those further below the poverty line and is defined as

$P_2 = \frac{1}{n}\sum_{i=1}^{q}\left(1 - \frac{y_i}{u}\right)^2.$

(4)

B. Bayesian Approach to Poverty Index

The Bayesian mixture approach with lognormal components allows us to estimate the income distribution and derive the posterior distributions for the FGT indices, as demonstrated by Hasegawa & Ueda (2007). We incorporate prior knowledge and handle uncertainty in the Liberian HIES data. This approach provides robust and reliable estimates of poverty measures.

The sample income y_i is directly computed from the indices reviewed above. In the Bayesian approach, the observations of y_i are incorporated to infer the parameters θ for the complete income distribution. We assumed that a mixture of long-distance densities could be used to model the distribution. We model $g(y)$ using a mixture of long-distributions and express each poverty index in terms of the parameters of this mixture (e.g., Thuysbaert 2008). The studyassumes that a random variable X is lognormally distributed and is denoted as $\ln(X)$. Statistically, $X \sim Lognormal(\mu, \sigma^2)$; thus, $\ln(X) \sim norm(\mu, \sigma)$. This implies that $X = e^Y$, where Y is a normally distributed variable, $Y \sim Normal(\mu, \sigma)$.

1) Formulating the Income Distribution

Assume that each member follows a lognormal distribution, and a mixture of distributions acknowledges that the population consists of separate groups with distinct characteristics. Even though group membership is not directly observed and the formulation of a finite mixture with N members is

$g(y/\theta) = \sum_{i=1}^{N} \varphi_i\, g_\Lambda(y/\mu_i, \sigma_i^2).$

(5)

Here, $\varphi_i$is considered the weight that sums to 1, and μ_i, σ_i^2 are the parameters for each component. The parameter θ encompasses all these parameters. Mixtures possess beneficial properties due to their linearity, particularly allowing for straightforward expressions of the mean and cumulative distribution function (CDF), given as:

$$E[y|\theta] = \sum_{i=1}^{N} \varphi_i \int_0^\infty y\, g_\Lambda(y/\theta_i)dy$$

is the weighted average of the mean for each component and

$$G(y/\theta) = \sum_{i=1}^{N} \varphi_i\, G_\Lambda(y/\theta_i)$$

is the weighted average of the mean of each component CDF.

The probability density function (PDF) of the lognormal distribution is given as:

$g_\Lambda(y/\theta) = \frac{1}{y\sigma\sqrt{2\pi}} exp\left(-\frac{(\ln y - \mu)^2}{2\sigma^2}\right) \quad y > 0$

(6)

with

$G_\Lambda(y/\theta) = \int_0^y g_\Lambda\,(t/\theta)dt = \Phi\left(\frac{\ln y - \mu}{\sigma}\right),$

(7)

where Φ is the Gaussian CDF. The mean and variance are given as:

$$E[y/\theta] = \int_0^\infty y\, g_\Lambda(y/\theta)d\theta = e^{\mu+\frac{\sigma^2}{2}} \quad (8)$$

$$var[y/\theta] = \int_0^\infty y^2\, g_\Lambda(y/\theta)d\theta - (E[y/\theta])^2 = \left(e^{\sigma^2}-1\right)e^{2\mu+\sigma^2}. \quad (9)$$

This require additional indicators such as partial moments because our focus is on the integral between zero and u rather than from zero to infinity to get the distribution of poverty indices (see, e.g. Fourrier-Nicola *et al.* 2021). The first partial moment are:

$$\int_0^u y\, g_\Lambda(y/\theta)d\theta = e^{\mu+\frac{\sigma^2}{2}}\Phi\left(\frac{\ln(u)-\mu-\sigma^2}{\sigma}\right) \quad (10)$$

and

$$\int_0^u y^2\, g_\Lambda(y/\theta)d\theta = e^{2\mu+\sigma^2}\Phi\left(\frac{\ln(u)-\mu-2\sigma^2}{\sigma}\right) \quad (11)$$

Ravenzwaaij *et al.* (2018) described Markov Chain Monte Carlo (MCMC) as a class of algorithms used to sample from a probability distribution when direct sampling is difficult. These methods are particularly useful in Bayesian Statistics for estimating the posterior distribution of the model parameters. The study obtained m drawings from the MCMC output using a Gibbs sampler for the equations obtain. We refer to each of these draws $\varphi_i^{(k)}$ for the weights and $\mu_i^{(k)}, \sigma_i^{(k)}$ for the parameters of the lognormal members.

C. Continuous FGT Poverty Indices

Utilizing the sums of discrete elements, poverty indices were formulated under the assumption that the income distribution is discrete. Now, considering a continuous distribution for y, we must present a general expression for these indices. Hence, Foster *et al.* (1984) describe the general class of poverty indices as follows:

$$P(u,\alpha) = \int_0^u \left(1-\frac{y}{u}\right)^\alpha g(y)dy \quad (12)$$

Here, $g(y)$ is a mixture of lognormal distributions for each common value of α. Therefore, we seek analytical solutions for equation (12). Where $\alpha = 0$, poverty head-count ratio or poverty rate (P_0) is

$$P(u,0) = \int_0^u g\,(y)dy.$$

The expression for the cumulative distribution of a mixture of lognormal distributions is presented, and the integral calculation has a straightforward solution with:

$$P\left(u/\theta^{(k)}\right) = \sum_{i=1}^N \varphi_i^{(k)}\, \Phi\left(\frac{\ln(u)-\mu_i^{(k)}}{\sigma_i^{(k)}}\right) \quad (13)$$

Where $\alpha = 1$, the poverty gap index can be formulated as:

$$\int_0^u \left(1-\frac{y}{u}\right) g(y)dy = G(u) - \frac{1}{u}\int_0^u y\, g(y)dy,$$

with (7) and (10), we formulated the poverty gap index as:

$$P\left(u/\theta^{(k)},1\right) = \sum_{i=1}^N \varphi_i^{(k)}\left[\Phi\left(\frac{\ln(u)-\mu_i^{(k)}}{\sigma_i^{(k)}}\right) - \frac{1}{u}e^{\mu_i^{(k)}+\sigma^{2(k)}/2}\Phi\left(\frac{\ln(u)-\mu_i^{(k)}-\sigma_i^{2(k)}}{\sigma_i^{(k)}}\right)\right] \quad (14)$$

Where $\alpha = 2$, we evaluate

$$\int_0^u g\,(y)dy - \frac{2}{u}\int_0^u y\, g(y)dy + \frac{1}{u^2}\int_0^u y^2\, g(y)dy.$$

Using (7),(10),(11), we get

$$\Phi\left(\frac{\ln(u)-\mu-2\sigma^2}{\sigma}\right) - \frac{2}{u}e^{\mu+\sigma^2/2}\Phi\left(\frac{\ln(u)-\mu-\sigma^2}{\sigma}\right) + \frac{1}{u^2}e^{2\mu+2\sigma^2}\Phi\left(\frac{\ln(u)-\mu-2\sigma^2}{\sigma}\right).$$

Now, for the mixture of lognormal we have

$$P\left(u \mid \theta^{(k)}, 2\right) = \sum_{i=1}^{N} \varphi_i^{(k)} \left[\Phi\left(\frac{\ln(u)-\mu_i^{(k)}}{\sigma_i^{(k)}}\right) - \frac{1}{u} e^{\mu_i^{(k)}+\frac{\sigma_i^{2(k)}}{2}} \Phi\left(\frac{\ln(u)-\mu_i^{(k)}-\sigma_i^{2(k)}}{\sigma_i^{(k)}}\right) + \frac{1}{u^2} e^{2\mu_i^{(k)}+2\sigma_i^{2(k)}} \Phi\left(\frac{\ln(u)-\mu_i^{(k)}-2\sigma_i^{2(k)}}{\sigma_i^{(k)}}\right)\right] \quad (15)$$

RESULTS

D. Income Inequality Distribution

` Using the Liberia HIES Data of 2016-2017 on the income distribution, described in Section 2 for the partial moment, Table 1 provides proof of the fact that member 1 indicates the average expected value of incomes μ below the partial moment 1 (11.93), which means that, on average, this income distribution has moderately lower incomes. In contrast, partial moment 2 (142.38) indicates that the difference is quite high for the variance below μ, meaning great variability or inequality in the lower incomes within this distribution. Member 2 shows that the expected value is lower than the partial moment (4.83), indicating that the lower part of this distribution has relatively low incomes, whereaspartial moment 2 (23.31) shows that the variance is also low, implying less dispersion in this group of incomes.

Member 3 shows that the expected value is moderate, suggesting this group has slightly higher incomes in the lower portion of its distribution than with the second case, whereas partial moment 2 (8.946) indicates that the variance is relatively high, which implies more variability among the lower incomes of this group. Member 4 shows that the large, expected value means the lower tail of this distribution has much larger incomes. This might be because the distribution is moving rightward. Also, partial moment 2 (965.44) indicates a high variance, which makes a deviation or inequity among the lower incomes in this distribution, hence making it very likely to represent a mix of very high and very low incomes. Member 5 shows a lower expected value, which would mean that the incomein this part of the distribution are generally lower, whereas partial moment 2 (29.89) compared with the variance shows moderate variance, which indicates partial variability but is not as pronounced as in the first cases.

Table 1. Statistics from Income Distribution

Member				Partial moment 1	Partial moment 2
1	1.96109995	1.0179882	0.30108915	11.932311	142.38002
2	1.06057621	1.01387101	0.02442022	4.8285365	23.3147648
3	1.69116368	1.69116368	0.57975387	8.9465387	8.9465387
4	3.00634672	0.92730157	0.56743127	31.071494	965.437
5	1.23373731	0.96434238	0.16239509	5.466920	29.8872164

1) Identifying Areas with High Inequality

The graph of the posterior density describes the income inequality distribution across counties in Liberia, As shown in Figure 1,counties with a sharp drop in the histogram are characterized by a long tail to the right in their income distribution, but an extended stretch in the KDE may exhibit significant income inequality, with a few high-income individuals or households, such as Montserrado and Margibi. In contrast, counties with very narrow peaks and limited spread tend to have a more homogeneous income distribution, where most people earn similar amounts, such as River Gee and Grand Kru. Multiple peaks in the distribution (bi-modal or multi-modal) suggest the presence of distinct subgroups within the county, potentially reflecting economic disparities between different sectors or communities, for example, Nimba and Lofa.

Additionally, Figure 1 demonstrates that the Bayesian mixture model's distributions, represented by $E[\sigma/y]$ and $E[\mu/y]$, indicate that the parameters of the log-normal distribution are estimated with relatively tight clustering around the mean and variance. This suggests that the model effectively captures the central tendency and dispersion of the income distribution. Partial moment 1, which shows less variability, reflects the central tendency, while partial moment 2, which accounts for greater dispersion, highlights the influence of income variability and the presence of outliers. This implies that the Bayesian mixture model is robust in capturing the essential characteristics of income distribution, particularly in the presence of skewness.

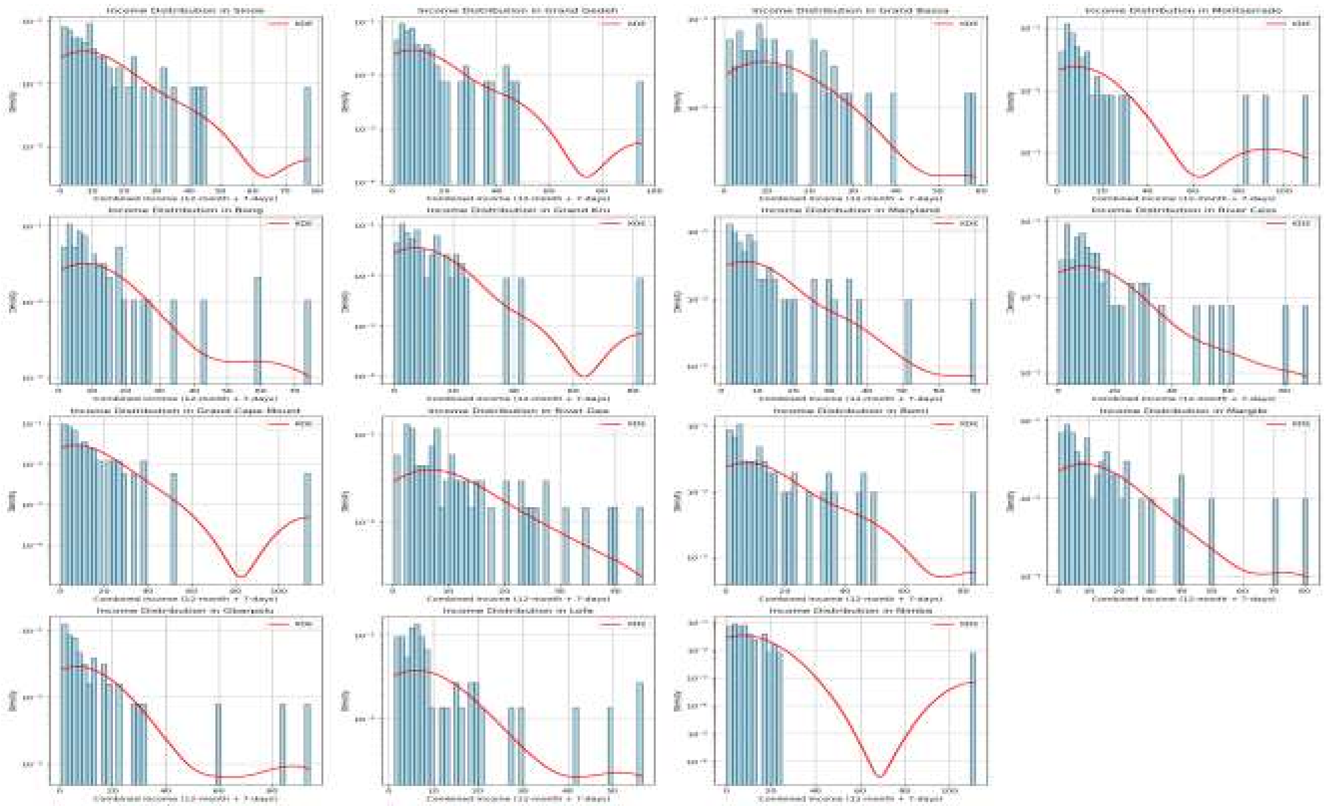

Fig 1: Posterior Distributions across Counties in Liberia

Figure 2 illustrates a typical income inequality distribution with a significant skew to the right and a concentration of income around the central range. The presence of a heavy tail suggests the existence of very high income, highlighting the disparity in income distribution. The alignment between the empirical and theoretical PDF further supports the use of a lognormal distribution for the HIES income data model.

Fig-2: Income Distribution (HIES 2016-2017)

2) Empirical comparison of Income Inequality of counties in Liberia

Using the MCMC methods, we perform complex statistical analysis and draw inferences about income distribution parameters. These results represent the output from the Gibbs sampling process for different counties in Liberia displayed on Table 2 shows that counties like Montserrado and River Cess might require targeted interventions to address significant income inequality. Policymakers need to focus on such inequality. Counties like River Gee and Grand Kru, with more homogeneous income levels, might have different economic dynamics, potentially requiring different policy approaches. In contrast, high-income areas, such as River Cess and Montserrado appear to be relatively wealthier counties, which could attract different types of investment or require specific economic planning to maintain growth while addressing inequality.

Table2. Income Inequality of Countries, Liberia HIE Data of 2016-201

County			φ	Inequality Classification
Sinoe	12.1780	12.905	72	Moderate Inequality
Grand Gedeh	13.4316	15.0346	70	High Inequality
Grand Bassa	13.8224	11.8921	59	Moderate Inequality
Montserrado	13.9565	20.7472	55	High Inequality
Bong	12.4706	13.9863	64	Moderate Inequality
Grand Kru	10.9353	12.05354	68	Moderate Inequality
Maryland	11.1471	12.0875	73	Moderate Inequality
River Cess	15.3583	17.1680	74	High Inequality
Grand Cape Mount	11.7808	15.6027	74	High Inequality
River Gee	11.0918	9.9123	76	Low Inequality
Bomi	13.518	15.4922	62	High Inequality
Margibi	13.902	15.5354	62	High Inequality
Gbarpolu	11.3242	16.4201	68	High Inequality
Lofa	10.4882	11.9387	66	Moderate Inequality
Nimba	10.0115	14.6767	57	Moderate Inequality

Table 3 provides a rich and detailed view of how income inequality varies across different demographic groups. By examining the patterns in the table, you can identify specific groups that may be more vulnerable to income disparities and understand the broader trends in income inequality across age, gender, and education levels. This information is crucial for targeted policy interventions and for understanding the dynamics of economic inequality in a population.

E. Education Level Trends

Primary education trends to associate with high-income inequality in certain age and gender groups, such as males aged 18-25and 46-55. Thissuggests that individuals within these groups who have primary education may experience varied economic outcomes, potentially due to different career paths or access to further training and education. In contrast, other groups such as females aged 46-55 exhibit lower income inequality among those with primary education, indicating more uniform distributionof income within these groups. Additionally, higher education tends to correlate with higher income inequality particularly among males aged 46-55. This could be because higher education opens a broader range of income opportunities, leading to greater disparity among individuals.

F. Gender Trends

Males exhibit higher income inequality compared to females within the same age and education levels. This might reflect broader career paths, higher risk-reward jobs, or more significant income disparities in male-dominated industries. In contrast, females exhibit lower income inequality, particularly among younger and middle-aged groups with lower education levels. This pattern may indicate more uniform earnings within female-dominated sectors or roles, or it could point to systemic constraints that limit income variability among women.

G. Age Trends

At the young age group (18-25), males with primary education show very high-income inequality, reflecting early entry into the workforce, where income can vary widely depending on job type, family support, or early career opportunities. On the other hand, females in this age group show lower income inequality, suggesting more consistent income patterns, due to uniform early career paths or societal expectations. Among middle-aged individuals (46–55 and 56–65), males with either primary or higher education also experience high income inequality. This indicates that by midlife, income differences become more pronounced, driven by variations in career progression, job stability, and accumulated wealth over time.In the oldest age group (65+), income inequality is relatively moderate for both genders, suggesting that incomes may stabilize or become more uniform in retirement or late career stages. These results were further illustrated in Figure 3.

Table 3: Income Inequality Varies Across Different Demographic Groups

Education Level		No Education	Primary	Junior High	Senior High	Higher	All
Age Group	Gender						
18-25	Female	9.080848	14.364023	22.049028	10.382025	11.287669	13.857251
	Male	5.279513	10.057390	24.156584	19.149943	8.551005	13.338212
26-35	Female	5.002545	25.485213	5.628362	15.485829	10.554755	11.813522
	Male	22.665827	7.220386	23.754342	9.964835	16.781306	15.993406
36-45	Female	10.746424	10.847674	9.960165	20.053665	16.043064	13.695331
	Male	11.915148	15.905670	8.359577	10.286856	8.688118	10.827710
46-55	Female	7.117601	5.919758	6.134735	9.839371	7.098581	7.480353
	Male	6.134987	29.358485	8.203449	18.160614	28.870731	17.957172
56-65	Female	8.779654	12.199158	17.223273	13.043443	27.463103	15.674939
	Male	9.511939	6.392643	11.301967	3.813917	10.688210	8.651233
65+	Female	11.922550	7.239397	16.757919	21.795876	20.457272	15.400328
	Male	9.518333	10.276876	11.518857	16.394305	11.057625	15.400328
All		9.877691	13.189877	13.193601	14.641694	15.011452	13.152554

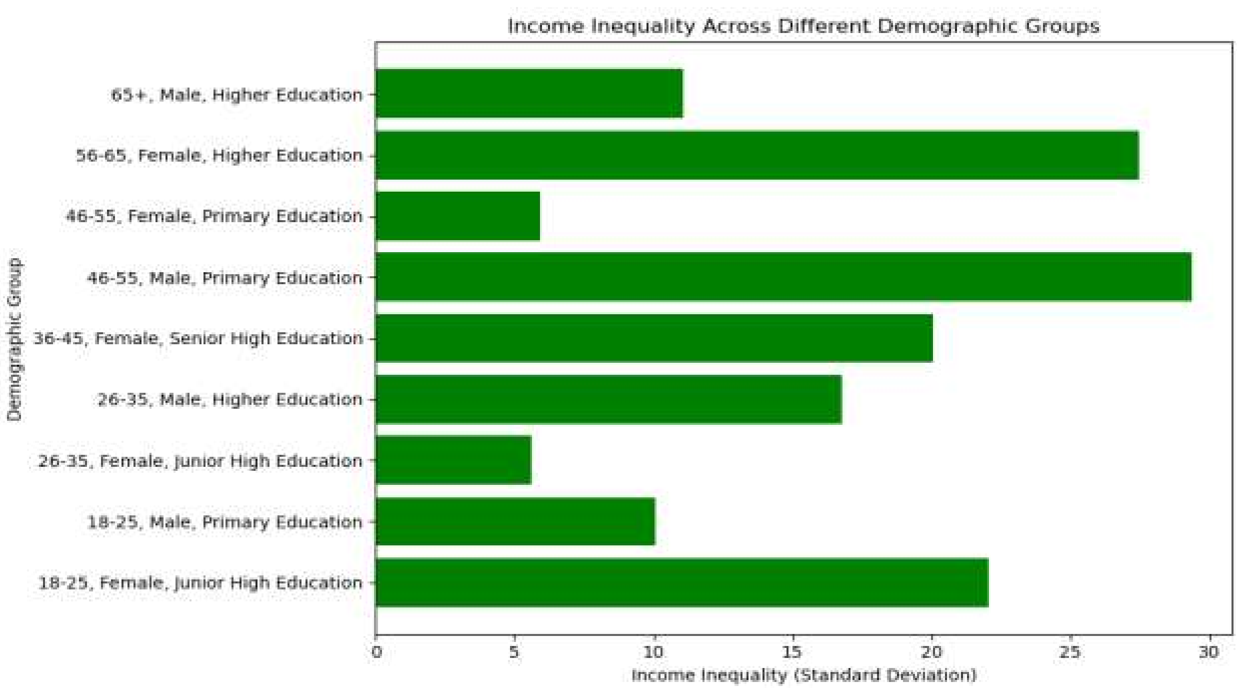

Fig 3: Comparing Income Inequality by Demographic Groups

H. Poverty Indices Measurement across Counties in Liberia

The posterior distributions of the three FGT indices, which include the poverty headcount ratio (PHR), poverty gap index (PGI), and poverty severity (PS), provide a deep view of poverty by incorporating uncertainty and variability into the estimates. The table below shows similar results for PHR, PGI, and PS because the income distribution of the population is similar across counties (HIES data, 2016-2017).

Table 4. Poverty Indices for Liberia (2016-2017 HIES Data)

County	Poverty Headcount Ratio	Poverty Gap Index	Poverty Severity
Sinoe	0.1121	0.1126	0.1132
Grand Gedeh	0.0536	0.0463	0.0429
Grand Bassa	0.0543	0.0546	0.0547
Montserrado	0.0441	0.0397	0.0392
Bong	0.0655	0.0630	0.0608
Grand Kru	0.1065	0.1052	0.1042
Maryland	0.0577	0.0397	0.0562
River Cess	0.05	0.0496	0.0493
Grand Cape Mount	0.1109	0.1097	0.1086
River Gee	0.1074	0.1081	0.1069
Bomi	0.0980	0.1042	0.1075
Margibi	0.1059	0.1042	0.1012
Gbarpolu	0.1266	0.1251	0.1236
Lofa	0.0713	0.0673	0.0625
Nimba	0.1006	0.0989	0.0962

The summary of poverty indices for different counties in Liberia estimated from the 2016-2017 Household Income and Expenditure Survey is provided in Table 4 above, using $2.15 as a poverty line. The indices include the Poverty Head Count Ratio, Poverty Gap Index, and Poverty Severity, pointing to greater county disparities. Sinoe is one of the counties that has significantly high poverty levels, with the highest percentage of people living below the poverty line. Not only does it have a high level of poverty, but it is also deep and severe across all the measures used. In contrast, Grand Gedeh has lower poverty levels, with gaps and severity that are lower than in Sinoe. The headcount ratio of poverty in Montserrado, including the capital Monrovia, is the lowest, meaning that the economic conditions are relatively better, as the levels of gap and severity are low. The county of Bong has moderate levels of poverty manifested by substantial gap and severity, though not the highest in Liberia. Grand Kru is experiencing high poverty like Sinoe, where a significant percentage of the population is deep and severe in their poverty levels." Maryland has moderate performance in the poverty headcount index, where less severe poverty is reflected from the gap and severity indices. River Cess is showing a relatively low scale of poverty where the indices are congruent. Grand Cape Mount, together with River Gee and Bomi, presents one of the highest poverty headcount ratios, being in deep and severe poverty. Margibi also shows significantly higher levels of poverty, characterized by its gap and severity. The highest poverty headcount ratio is in Gbarpolu, where the poverty is very deep and severe. Moderately and significantly, the poverty levels show in Lofa and Nimba, with gaps and severity being less than in more affected counties like Gbarpolu and Sinoe.

1) Measuring Poverty across Different Demographic Groups in Liberia

Tables 5 and 6 highlight the variations in the poverty gap indices (PGI) and Poverty Headcount Ratio (PHR) across different demographic groups, classified by age and education level. The results show that the younger age group (18 – 25 years) with no education has a PHR of 37.97%, indicating a moderate level of poverty. Grandchildren with senior high education experience the highest level of poverty in this age group at 52.28%. The percentages of household heads among those with higher education in this age group is 77.5%, which is counterintuitive and may suggest other factors, such as unemployment influencing poverty in this demographic. Spouses with no education have the highest PGR at 68.9%, demonstrating the strong link between lack of education and poverty.Primary education has a PGR of 35.22%, showing a moderate level of poverty,while senior high education has aPGR of 78.36%, showing high level of poverty gap.

Further, the middle age group (26-35 years) as grandchildren with no education has the highest PHR (82.38%) indicating a higher vulnerability in this demographic, primary education at PHR (59.19%) indicating high poverty and senior high education at PHR (38.95%). Household heads across all levels of education are relatively high, with those lacking education at PHR (30.8%). Sons/daughters and spouses with junior high education show a high PHR especially sons/daughters with a PHR of 93.45%, and so on with the others.

The study concludes by statingthat poverty is significantly influenced by education levels across all age groups. Those with no education or only primary education exhibit higher PHRs and PGI across almost all demographic groups, suggesting a strong correlation between lack of education in age groups and higher poverty levels. Young grandchildren and household heads with low education levels are particularly vulnerable, while elderly household heads and spouses also face high levels of poverty, especially when they have little or no education.

Fig 4 visually demonstrates the distribution of poverty across the countries, highlighting the level of poverty across counties in Liberia.

Table 5: Poverty Gap Varies across Different Demographic Groups

Education Level		Higher	Junior High	No Education	Primary	Senior High	All
Age Group	Household Categories	PGI	PGI	PGI	PGI	PGI	
18-25	GRANDCHILD	0.30698	0.000	0.37903	0.31684	0.52285	0.41261
	HEAD	0.77522	0.70089	0.91054	0.000	0.68027	0.75356
	SON/DAUGHTER	0.000	0.000	0.81042	0.71517	0.000	0.76280
	SPOUSE	0.000	0.000	0.68999	0.35224	0.78364	0.57362
26-35	GRANDCHILD	0.000	0.13346	0.82382	0.59193	0.38957	0.50596
	HEAD	0.87713	0.82960	0.30828	0.000	0.000	0.67167
	SON/DAUGHTER	0.000	0.119771	0.093459	0.082972	0.585483	0.293434
	SPOUSE	0.129412	0.000	0.236231	0.000	0.000	0.200625
36-45	GRANDCHILD	0.000	0.000	0.580197	0.475289	0.000	0.527743
	HEAD	0.000	0.335591	0.000	0.414567	0.490819	0.394142
	SON/DAUGHTER	0.000	0.000	0.000	0.000	0.882860	0.882860
	SPOUSE	0.000	0.000	0.000	0.844440	0.000	0.200625
46-55	GRANDCHILD	0.009688	0.433439	0.624855	0.926213	0.654508	0.561152
	HEAD	0.953721	0.949297	0.000	0.000	0.594166	0.772838
	SON/DAUGHTER	0.000	0.442015	0.000	0.842134	0.069859	0.549036
	SPOUSE	0.681359	0.473221	0.681359	0.473221	0.488410	0.566976
56-65	GRANDCHILD	0.513655	0.000	0.097469	0.159870	0.524593	0.361848
	HEAD	0.000	0.159867	0.000	0.627643	0.417606	0.444662
	SON/DAUGHTER	0.000	0.085001	0.432148	0.399499	0.000	0.305549
	SPOUSE	0.157632	0.325685	0.503427	0.174112	0.000	0.250081
65+	GRANDCHILD	0.224206	0.000	0.301831	0.000	0.000	0.250081
	HEAD	0.639256	0.178367	0.766964	0.000	0.649463	0.574661
	SON/DAUGHTER	0.447378	0.000	0.000	0.000	0.443879	0.445628
	SPOUSE	0.651674	0.727696	0.000	0.256140	0.159621	0.502031
All		0.493889	0.433150	0.529682	0.509249	0.499672	0.496058

Table 6: Poverty Ratio Varies across Different Demographic Groups

Education Level		Higher	Junior High	No Education	Primary	Senior High	All
Age Group	Household Head	PHR	PHR	PHR	PHR	PHR	
18-25	GRANDCHILD		0.000	0.324683	0.742632	0.604672	0.534946
	HEAD	0.854614	0.767934	0.009240	0.000	0.359271	0.551798
	SON/DAUGHTER	0.000	0.000	0.229219	0.174909	0.000	0.202064
	SPOUSE	0.000	0.000	0.846459	0.177812	0.030453	0.415799
26-35	GRANDCHILD	0.000	0.747079	0.569012	0.311860	0.335254	0.457532
	HEAD	0.703495	0.645785	0.715561	0.000	0.000	0.688280
	SON/DAUGHTER	0.000	0.980700	0.181631	0.833038	0.622029	0.647885
	SPOUSE	0.677263	0.000	0.643462	0.000	0.000	0.654729
36-45	GRANDCHILD	0.000	0.000	0.660586	0.743835	0.000	0.754811
	HEAD	0.498140	0.269479	0.000	0.000	0.726427	0.555119
	SON/DAUGHTER	0.000	0.000	0.000	0.000	0.456015	0.456015
	SPOUSE	0.000	0.000	0.000	0.657459	0.456015	0.456015
46-55	GRANDCHILD	0.186458	0.229442	0.660586	0.743835	0.676411	0.547676
	HEAD	0.498140	0.269479	0.000	0.000	0.726427	0.555119
	SON/DAUGHTER	0.000	0.483409	0.000	0.566125	0.214881	0.457635
	SPOUSE	0.730442	0.580447	0.471752	0.343606	0.204866	0.466222
56-65	GRANDCHILD	0.653850	0.000	0.194342	0.286915	0.456130	0.449017
	HEAD	0.000	0.092596	0.000	0.388687	0.749254	0.617379
	SON/DAUGHTER	0.000	0.985032	0.881585	0.617658	0.000	0.828092
	SPOUSE	0.862310	0.045615	0.471752	0.343606	0.000	0.435326
65+	GRANDCHILD	0.343474	0.000	0.343474	0.000	0,000	0.409815
	HEAD	0.406252	0.646264	0.878870	0.000	0.749254	0.617379
	SON/DAUGHTER	0.428224	0.000	0.000	0.000	0.406120	0.417172
	SPOUSE	0.652875	0.297842	0.000	0.000055	0.000	0.400912
All		0.539677	0.509890	0.568226	0.447104	0.541296	0.521601

Figure 5 shows that the poverty gap index (PGI) measures the intensity of poverty. It indicates the average shortfall of the poor relative to the poverty line, expressed as a percentage of the poverty line itself. The PGI indicates that the poor are significantly below the poverty line (U=$2.15). Gbarpolu and Grand Cape Mount are among the counties that exhibit a high Poverty Gap Index (more than 10%), which implies higher poverty, since an average poor person is at a large distance below the poverty line. These counties are marked on the map with a darker shade, giving an indication of poverty intensity, while several other counties, including Sinoe, Margibi, Bomi, and Nimba, fall in the moderate category. These regions of the country have a Poverty Gap Index of between 5% and 10%. The poverty level, though quite high, is relatively less intense compared to those counties that fall in the category of high Poverty Gap Index. The other counties in the low Poverty Gap Index group include Montserrado and Grand Gedeh. These regions have alower depth of poverty, signifying that though people are poor, the ’depth’ of the average poor is not very high from the poverty line (U=$2.15).

The Montserrado County, which has the capital city Monrovia, has a low Poverty Gap Index, which was quite expected since all urban areas have greater access to material assets, infrastructure, and social services than their rural counterparts do. These benefits are likely to explain the relatively lower intensity of poverty in Montserrado. At the same time, poverty intensity is much higher in the more rural and remote counties, such as Gbarpolu and Grand Cape Mount due to the lack of infrastructure, services, and economic activities.

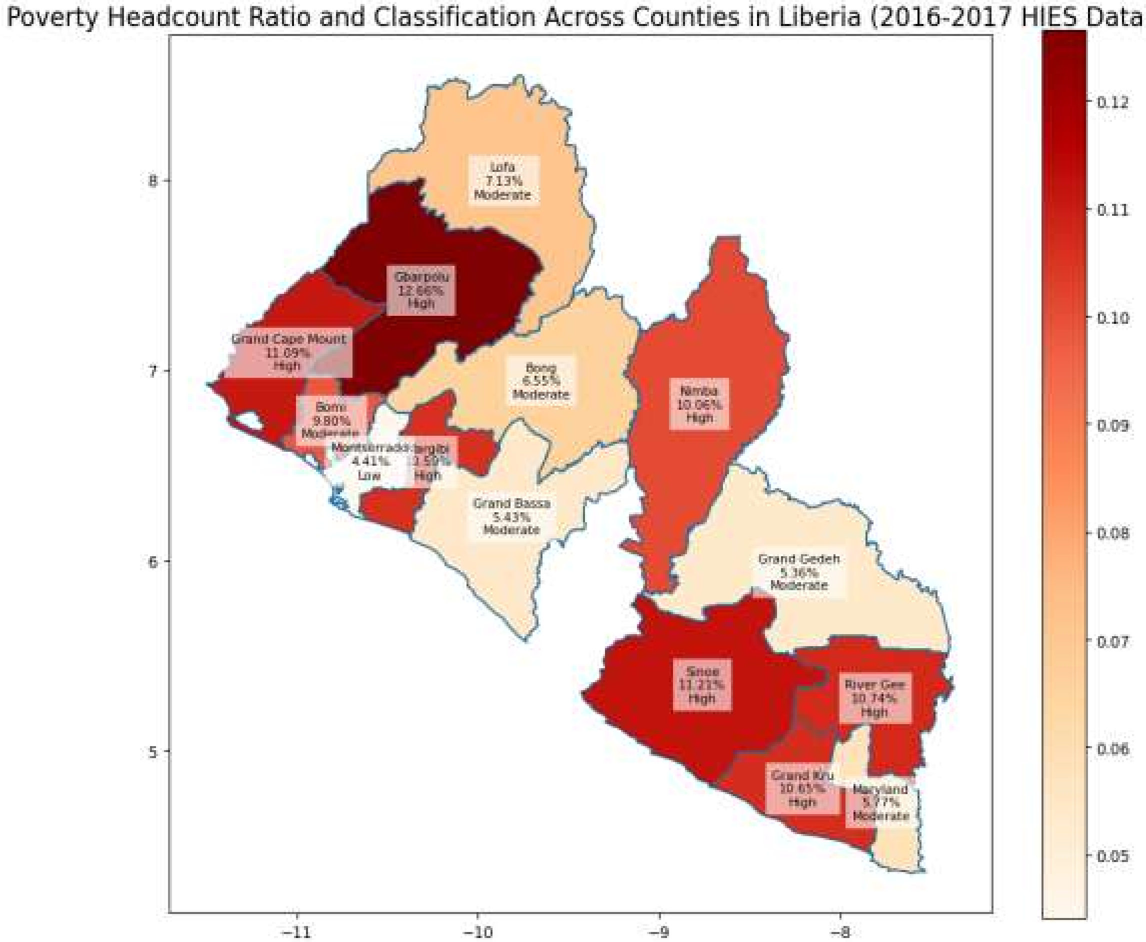

Fig 5 Classification of Poverty per County in Liberia (HIES 2016-2017)

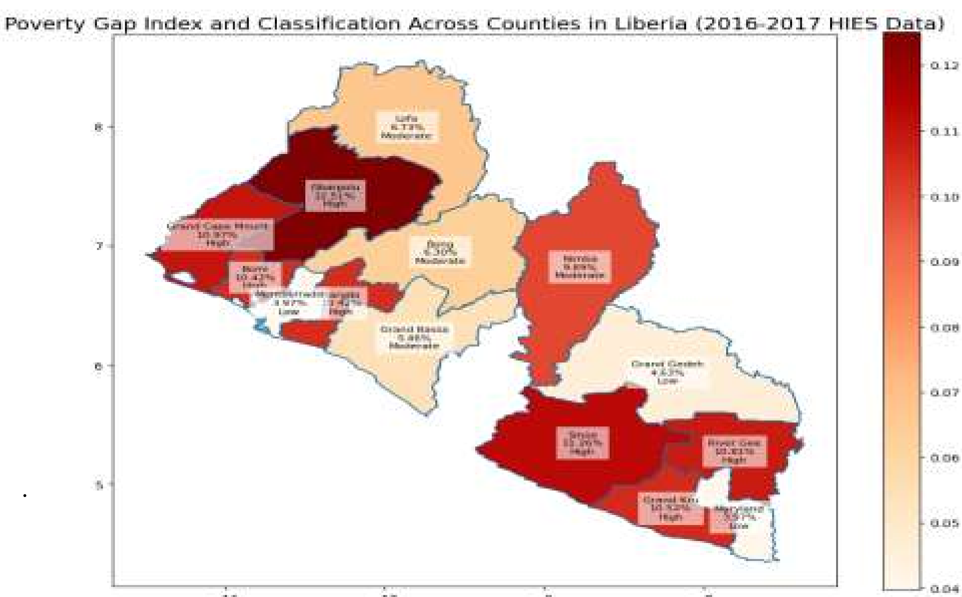

Fig 6: Classification of Poverty Gap per County in Liberia (HIES 2016-2017)

Discussion:-

The index provided in the poverty framework here influences access to FGT poverty measures in a population by attaching the incidences and intensity of the poverty measure. Indices play a special role in summarizing distribution across lower incomes, being sensitive to different dimensions of poverty, depth, and severity. We derive the posterior distributions for the poverty indices by exploiting a parametric model for income distribution within a Bayesian framework, which yields a more robust estimation of the poverty metrics in Liberia.

The Bayesian approach of mixtures of lognormal distributions allows for flexible modeling of the income distribution, capturing nuances and variability within the data. The latter feature of this method is particularly valuable because it can accommodate previous knowledge and be much more effective in dealing with uncertainty than traditional methods. Markov chain Monte Carlo methods yield samples from the posterior distribution of the model parameters, which are, in turn, used for inference. Researchers from other countries may consider adopting these Bayesian frameworks in analyzing poverty and inequality (Eckernkemper and Gribisch, 2021; Chotikapanich& Griffiths, 2000; Griffiths *et al*., 2005; Chotikapanich and Griffiths, 2005; Kakamu, 2016; Kakamu& Nishino, 2019). In this direction, a lot of mileage has been contributed to this domain. The following critical insights can be captured based on the analysis of the poverty metrics (poverty ratio, poverty gap, and poverty severity). The data show strong heterogeneities in poverty incidence according to age, level of education, and household head status. Gbarpolu, Sinoe, and Grand Cape Mount counties are those with the highest incidence of poverty; deeply and severely poor people constitute a great share of their populations. In contrast, Montserrado, comprising the capital of Monrovia, has the lowest incidence of poverty, where the general situation is better and severe poverty is significantly low. Education appears to be critical in reducing poverty, because a high level of education is directly, constantly related to a low index of poverty.

The status of the household head is also among some very important factors related to the poverty measures, with categories, like those of grandchildren and spouses, often having higher levels of poverty than the category of the household head does. Overall, the findings underline the fact that there is an urgency to devise a targeted poverty reduction strategy for the most vulnerable regions and groups. The reduction of poverty and promotion of economic equity in the country can best be achieved through minimizing educational disparities and focusing policy implementations on support provision to the households that have non-head members.

Conclusion

Here is our fresh take on measuring poverty in Liberia! The method shared here provides a new way to understand poverty in Liberia, using FGT poverty indices in a Bayesian framework for detailed insights into poverty levels. Modeling income distribution with various lognormal distributions gives a better picture of economic gaps.The results show significant differences in income across Liberia's counties, with wealth concentrated in specific areas. The FGT indices' posterior distributions reveal the extent and seriousness of poverty, pointing out where interventions are most necessary. Though the method is strong and insightful, the study must note the limitationssuch as data quality, assumptions made and demands on computing power. Future studies could address these by using long-term data and trying different models for a more precise representation of income distribution. Overall, this study aims to help tackle poverty effectively.

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