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RESEARCH ARTICLE

A STUDY OF CHAOS THEORY IN CRYPTOCURRENCY MARKETS

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Abstract

The growth of cryptocurrencies as financial instruments have raised questions about modelling their markets due to a high volatility and speculative nature. This study investigates the presence of deterministic chaos in the daily price movements of Bitcoin (BTC), Ethereum (ETH), and Solana (SOL), from April 2020 to February 2025. Employing a multi-faceted methodological approach, the study conducted Brock-Dechert-Scheinkman (BDS) tests for non-linearity, calculated the Largest Lyapunov Exponents (LLE) to gauge sensitivity to initial conditions, performed Recurrence Quantification Analysis (RQA) to detect deterministic structure, and derived Hurst Exponents to assess long-term memory. Post-filtering, significant non-linearity persisted in the BDS test. LLEs indicated dynamics at the "edge of chaos," but RQA and Hurst Exponents revealed strong deterministic structures. Collectively, these results reveal characteristics of a low-dimensional chaotic system, challenging the random walk hypothesis and implying a degree of short-term predictability with significant implications for quantitative finance and regulation.

Introduction:-

Cryptocurrency markets are characterized by a decentralised consensus system and stem from blockchain technologies which are shared immutable ledgers that facilitate the transparent dsitribution of information and recording of transactions. Cryptocurrencies that are not bitcoin are considered altcoins, which can be classified into three major categories: blockchain-native tokens like Ethereum and Solana; meme-coins that are speculative and use a blockchain-native token as a base; and stablecoins, that are cryptocurrencies tied to a stable asset like the US dollar or the Euro.

Secondly bubbles are often associated with technological innovation because of uncertainty about the technology. The cryptocurrency market being in a bubble could be due to their nature as a financially innovative tool. Third, their limited supply helps create a bubble as scarcity is an important aspect of bubble formation. Cryptocurrencies are also highly affected by external shocks including bans, taxes (Griffith & Clancey-Shang, 2023) and social media posts all off which have led to sharp changes (more than 10%) of cryptocurrency prices in the matter of hours. Sentiment about cryptocurrencies becoming legal tender or more widely expected can set off uncontrolled spikes in cryptocurrency prices. Studying chaos in cryptocurrency markets will help regulating cryptocurrencies through identifying volatility as deterministic or random.

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Chaos theory deals with complex systems that have behaviour which is highly sensitive to changes in starting conditions. Chaotic systems seem to be random as they can't be controlled but can be predicted through governing functions. Thus, chaos is deterministic as a group of chaotic motions can work together to create a predictable curve. An increase in the number of groups can lead to a higher accuracy in the chaotic model; hence, in large samples of chaotic elements, unsuspecting regularity is displayed. Chaos is non-linear and acts in fractal dimensions and follows a butterfly effect, with small changes in starting conditions leading to drastic differences in the results - which can magnify small, initial errors. Fractals are never-ending patterns that are infinitely complex and self-similar across different scales. They are created by repeating a simple process in an ongoing feedback loop. They are also dynamic systems that are driven by recursive phenomena and exist in familiar dimensions. Therefore, chaos theory is not the study of disorder but the study of the transitions between order and disorder.

Chaotic markets are much like helium balloons, and two points that were next to each other can end up far apart due to mixing, and they can't be unmixed, much like two molecules of helium within the balloon. Additionally, chaotic systems are often formed due to feedback. This is used as the first link between the cryptocurrency market and chaotic systems. There is a positive relationship between the price and the number of buy orders of a cryptocurrency (Kaur, Jain, & Sood, 2023) due to effects such as herding and the fear of missing out (FOMO) on cryptocurrency gains. Another connection between chaotic systems and the cryptocurrency market can be established by the speculative bubbles surrounding these markets. Speculative bubbles exist outside the lens of traditional economics (Brodie) and thus have to be governed by chaos to establish a logical appearance of events in such a bubble. Therefore, cryptocurrency markets many of which can be considered as speculative bubbles, are chaotic in nature.

To further prove chaos, especially in established, less volatile currencies, one can observe that they exhibit fractal fluctuations. Past price movement affects future prices, a phenomenon called long-term dependence and price or volatility movements look similar at varying time scales, both of which are characteristics of fractal fluctuations. These occur because traders have different time-horizons, reacting differently to the same information; information flow often leads to predictable price changes, adding to its fractal nature. To test cryptocurrencies' fractal-like nature, one can calculate their Hurst exponents to see if they have long-term memory and autocorrect their systems. A high fractal dimension indicates increased complexity and can be calculated by subtracting the Hurst exponent from 2. This can be used to show whether cryptocurrency markets follow long-term chaotic trends and to predict their complexity.

This study aims to identify whether cryptocurrency markets follow the efficient market hypothesis and see if they can be modelled through chaotic systems. Moreover, it seeks to test different models to see their effectiveness with existing data and establish chaotic trends. To the author's best knowledge, no research has been done to develop chaotic systems for the cryptocurrencies Bitcoin, Ethereum and Solana. The data includes the OHLC prices, trading volume and market capitalisation. The data starts from April 11th 2020 and ends on 9th February 2025, giving 1765 data points. The paper is structured as follows, Section 2 will showcase the methodology used to test for chaos - BDS Test, Lyapunov Exponents, Recurrence Quantification Analysis and Hurst Exponents; Section 3 processes and analyses the empirical data; Section 4 provides a conclusion to the study.

Methodology:-

The analysis will focus on three major cryptocurrencies: Bitcoin (BTC), the largest by market capitalization and a benchmark for the broader market; Ethereum (ETH), the second-largest, renowned for its smart contract functionality; and Solana (SOL), a newer blockchain platform recognized for its high-speed transactions and low fees. Data is sourced from Yahoo Finance and the daily closing price will be used to test for chaos using the following methodologies - BDS Test, Lyapunov Exponents, Recurrence Quantification Analysis and Hurst Exponents.

Brock-Dechert-Scheinkman Test:

The BDS Test is a method to identify dependency in any time series, it helps prove non-linearity, a key instrument in identifying chaos. It tests a null hypothesis of independent and identically distributed (i.i.d) against an unspecified alternative (Kuok Kun Chu, 2001). If the data rejects the null hypothesis it shows that the data exhibits a non-linear form of dependence. The BDS test is not a direct test for chaos but one for nonlinearity, provided for the removal of any form of linear dependence. In autoregressive (AR) and ARCH (Autoregressive Conditional Heteroscedasticity) models, the BDS test has low power, leading to failure in detecting dependence structures. The ARMA (Autoregressive Moving Average) or ARIMA (Autoregressive Integrated Moving Average) models can be used to remove linear components and leave behind residuals which are free of linear dependence. GARCH (Generalized Autoregressive Conditional Heteroscedasticity) models are used to capture time-varying volatility. By fitting a

GARCH model, the study removes the nonlinear dependence caused by changing volatility, leaving behind residuals that should ideally be free of both linear and nonlinear dependence.

With observations of a time series x_t , t = 1, . . . T , the m-history of the time series is

$$x_t^m = (x_t, x_{t+1}, x_{t+2}, \ldots, x_{t+m-1})$$

The sample correlation integral for a time series which is embedded in dimension m (which is equal to 1) and with an ε > 0 is

$$C_1 = \frac{2}{n(n-1)} \sum_{j=1}^{n-1} \sum_{k=j+1}^{n} I(|x_j - x_k| \leq \varepsilon)$$

where n = (T – m +1) and I(S) is a boolean indicator with a value of 1 if the statement is true and 0 if not. Hence, if ε is chosen to be higher than all x values, $C_1 = 1$ and if ε is lower than x values, $C_1 = 0$. In practice, ε is set in terms of standard deviations of the data.

The l^{th} order correlation integral finds the probability that l consecutive points in a time series are close to each other within a specified distance ε. It extends the basic correlation integral to higher dimensions by considering sequences of points.

Thus, the correlation integral C_l is found to be

$$C_l = \frac{2}{n(n-1)} \sum_{j=1}^{n-1} \sum_{k=j+1}^{n} \prod_{r=0}^{l-1} I(|x_{j+r} - x_{k+r}| \leq \varepsilon)$$

Mathematically the l^{th} order correlation integral is also defined as

$$C_l(\varepsilon) = \Pr(|x_j - x_k| < \varepsilon, |x_{j+1} - x_{k+1}| < \varepsilon, \ldots, |x_{j+m-1} - x_{k+m-1}| < \varepsilon)$$

The null hypothesis $\{x_t\}$ in the BDS test states that the increments of the time series are independent and identically distributed (i.i.d) with the probability being the product of individual probability with independent observations. Therefore the probabilities $E(C_l)$ and $E(C_m)$ are equal to C_1 and C_1^m respectively.

The standard deviation σ_n is calculated as

$$\sigma_n^2 = 4\left\{\beta^n + 2\sum_{j=1}^{n-1} \beta^{n-j} \times \alpha^{2j} + (n+1)^2\alpha^{2n} - n^2\beta\alpha^{2n-2}\right\}$$

where α and β are

$$\alpha = \frac{1}{n^2}\sum_{j=1}^{n}\sum_{k=1}^{n} I_{j,k}\,, \qquad \beta = \frac{1}{n^3}\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n} I_{i,j}I_{j,k}$$

The variance can be simplified to become

$$\sigma_n^2 = 4(\beta - \alpha^2)^2 \sum_{j=1}^{n-1} j^2\beta^{n-j-1}\alpha^{2(j-1)}, n \geq 2$$

This can be used to calculate the deviation for the dataset of Cryptocurrency prices, to see if it follows a non-linear dependent trend.

Lyapunov Exponents:-

Lyapunov exponents quantify the sensitive dependence on initial conditions in a dynamical system. They measure the exponential rate at which nearby trajectories diverge (or converge) in phase space. If two price trajectories start at an infinitesimally close level, their separation $\delta(t)$ will grow as $|\delta(t)| \approx e^{\lambda t}|\delta(0)|$ where λ is the Lyapunov exponent. Lyapunov exponents are an essential tool to predict chaos as the Largest Lyapunov Exponent (LLE) model can be used to determine it. A positive LLE indicates chaos indicating unpredictability and large divergences with small price differences (Raubitzek & Neubauer, 2021). On the other hand $\lambda \leq 0$ suggests convergence and bounded trajectories.

Therefore, a higher Lyapunov exponent indicates strong chaos, while $\lambda \to 0$ suggests stochastic dynamics. The significance of difference values of λ are explained below:

- $\lambda_{max} > 0 \to$ Chaotic dynamics which lead to divergence in trajectories. This example is mapped in Figure 1.
- $\lambda_{max} = 0 \to$ Stochastic dynamics which indicate at the system is at the edge of chaos.
- $\lambda_{max} < 0 \to$ The system has stable dynamics with converging trajectories (Soloviev & Bielinskyi, 2020).

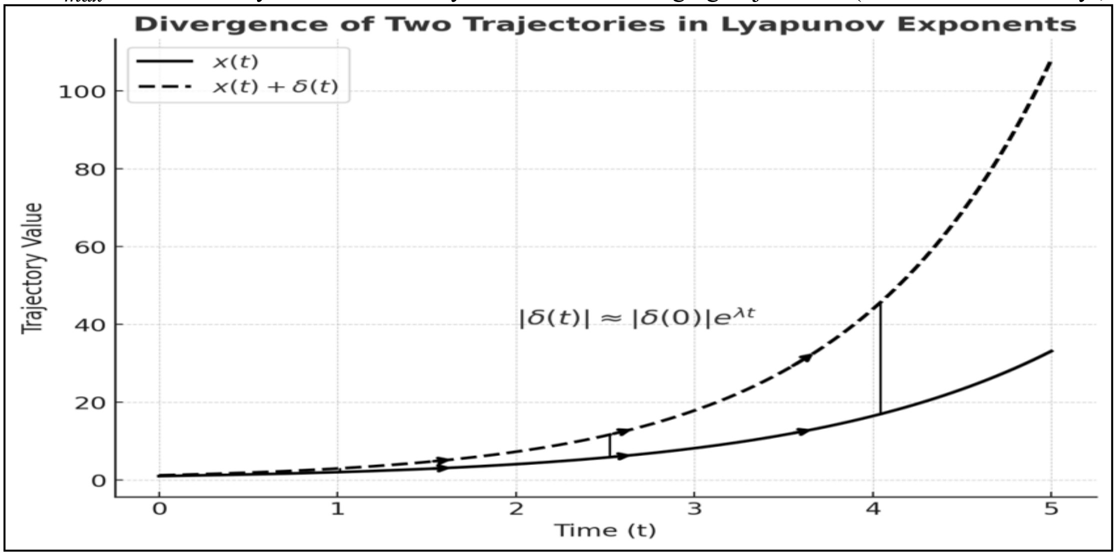

Figure 2.2.1: Divergence of two trajectories

For unknown systems that are dynamical in nature λ can be estimated using time-series data. This is true for the cryptocurrency market as well, and can be done through phase-space reconstruction to track how quickly trajectories separate. To computationally calculate the Largest Lyapunov Exponents (LLE) of the time series, the (Rosenstein, Collins, & De Luca) method is used. It initially constructs delay vectors $x_i = [\mathrm{p_i}, ; \mathrm{pi} + \tau, ; \ldots, ; \mathrm{p}_{\mathrm{i+(m-1)\tau}}]$ **with** τ being the time delay and m the embedding dimension. For each point x_i there is a point x_j which is its nearest neighbour ∀ i ≠ j. Therefore, the initial separation can be modelled as:

$$d(0) = x_i - x_j$$

Hence for any time t the distance $d(t) = \delta(t)$is:

$$d(t) = x_{i+t} - x_{j+t}$$

We also know that$d(t) = e^{\lambda t} d(0)$. With a small increase in t the equation is modified into:

$$d(t + \Delta t) = e^{\lambda \Delta t} d(t)$$

Taking the natural logarithm on both sides forms the equation:

$$\ln d(t + \Delta t) = \ln d(t) + \lambda \Delta t$$

$$\lambda = \frac{\ln d(t + \Delta t) - \ln d(t)}{\Delta t} = \frac{1}{\Delta t} ln \frac{d(t + \Delta t)}{d(t)}$$

The ensemble average for $ln \frac{d(t+\Delta t)}{d(t)}$ can be used to find λ_{max}:

$$\lambda_{max} = \frac{1}{\Delta t} \langle ln \frac{d(t + \Delta t)}{d(t)} \rangle$$

This can also be written as:

$$\lambda_{max} = \frac{1}{\Delta t}\langle ln\frac{x_{i+t+\Delta t} - x_{j+t+\Delta t}}{x_{i+t} - x_{j+t}}\rangle$$

The magnitude of λ indicates how fast predictability decays: for example, a positive Lyapunov exponent in a crypto price series would imply that forecasting errors grow exponentially over time, limiting long-term prediction accuracy. Thus a λ_{max} value above 0, determines chaos rather than random volatility in the markets for Bitcoin, Ethereum and Solana. The minimum time separation sets a lower bound on the time interval between pairs of points that are considered neighbours in the reconstructed phase space. It reduces the effect of autocorrection and smoothness of the time-series which can lead to points being closer than otherwise. The minimum time separation will henceforth be referred to as min_tsep.

Recurrence Quantification Analysis:
Recurrence Quantification Analysis is a tool which employs pattern recognition to analyse time-series and create recurrence plots (Unal, 2022). RP and RQA help showcase the structure of recurrence in a phase space, with RP being used to visualise when a trajectory in a phase space returns close to a previous state. This can dictate whether a dataset is chaotic or stochastic in nature. RPs can be analysed through visual means. To find the RP of an $m-$dimensional state space $\{x_i\}_{i=1}^{N}$, the recurrence matrix $RP_{i,j}$ is defined as:

$$RP_{i,j} = \Theta(T - \| x_i - x_j \|)$$

where Θ is the Heaviside step function and T is the threshold value. If the distance between vectors x_i and x_j is less than the threshold value, $RP_{i,j}$ is equal to 1. Plotting $RP_{i,j}$ as black and white dots for its respective values of 1 and 0, creates the recurrence plot. Adjacent points in RPs often form diagonals, indicating the recurrence of a vector in the state space. Vertical or horizontal lines indicate vectors being in the same state space over time. RPs with long diagonals indicate chaos or determinism while long vertical or horizontal lines, along with many isolated points suggest the presence of a stochastic system.

RQA goes beyond RP in that it quantifies the number and duration of recurrences which can be used to find out if a system is chaotic or not. There are multiple measures used in RQA including: the recurrence rate, determinism (the percentage of recurrence points which form a diagonal line with a minimum length), average length of diagonal lines and divergence (which is related to the length of the longest diagonal line). This paper will focus on determinism, recurrence rate and divergence. The recurrence rate is the density of points in an RP:

$$REC = \frac{1}{N^2}\sum_{i,j=1}^{N} R(i,j)$$

REC values between 5 and 15% indicate chaos while values below 5% suggest true randomness and values above 15% are indicative of periodicity. Determinism is calculated through the formula:

$$DET = \frac{\sum_{l=l_{min}}^{N} lP(l)}{\sum_{l=1}^{N} lP(l)}$$

where $P(l)$ is the frequency distribution of the lengths l of the diagonal lines. It measures the predictability of a changing system and a higher value of DET is indicative of a more chaotic system ($DET \gg 0.5$). Divergence is calculated as:

$$DIV = \frac{1}{L_{max}}$$

This can also be used as an estimator for λ_{max} the maximal Lyapunov Exponent as both indicators follow a common trend. A system is chaotic when $0.05 < \text{DIV} < 0.3$.

Hurst Exponents:
The Hurst Exponent H is a measure of the long-term memory of a time series. It uses the rescaled range analysis (R/S analysis) to determine the Hurst Exponent H (Raubitzek & Neubauer, 2021). It was originally introduced in hydrology (Hurst, Black, & Simaika, 1965) but later used for detecting fractal behaviour in finance. H is related to the

roughness and persistence of a time series. It is found through scaling R/S over different time intervals. For a time series length N the rescaled range is computed for sub-periods length n. The range of cumulative deviations from the mean R(n) is divided by the standard deviation S(n), and the expected rescaled range behaves in the following way:

$$\mathrm{E}\left[\frac{R(n)}{S(n)}\right] \propto \mathrm{n}^{\mathrm{H}}$$

$$E\left[\frac{R(n)}{S(n)}\right] = Cn^{H}$$

where C is a constant.

H can have the following values:

- $H = 0.5$ which suggests randomness, with no fractal structure or long-term dependency. A time-series will follow the Efficient Market Hypothesis (Macrosynergy, 2023).
- $0 \leq H < 0.5$ which indicates poor memory and anti-persistence. This also suggests mean-reversion where increments are negatively autocorrected. A past increase in price suggests a future fall in prices.
- $0.5 < H < 1$ which represents long memory or a persistent series. For these values of H the series will continue its previous upward or downward trend.

The Hurst Exponent can also be calculated through a detrended fluctuation analysis (DFA). It measures how fluctuations in log-transformed data evolve with time lag τ. DFA uses the variance of $\tau, Var(\tau)$ for a given time t through the following way:

$$Var(\tau) = \langle |log(t+\tau) - log(t)|^2 \rangle \sim \tau^{2H}$$

This study will use the DFA method to calculate the Hurst Exponent for the given dataset of cryptocurrencies. This is because it can filter out disturbances and white noise within the time-series.

Data Processing and Analysis:

This study focusses on 3 cryptocurrencies namely Bitcoin (BTC), Ethereum (ETH) and Solana (SOL), and has 1765 datapoints of daily OHLC prices starting from April 11th 2020 and ending on 9th February 2025. To ensure a fair test the daily closing price will be the only tested indicator over the 4 tests for chaos. This section will present the raw dataset (Table 3.1), containing the sample size, periods, mean, standard deviation and skew for BTC, ETH and SOL. The mean closing prices will be used to create data tables, 3.2 – Empirical Results of the BDS-test, 3.3 – Maximum Lyapunov Exponents, 3.4 – Recurrence Rate, Determinism and Divergence, and 3.5 – Hurst Exponent. This data will be analysed in Section 3, to determine whether the three currencies are follow chaotic market trends, and the extent to their chaos.

Tests were conducted through Python Code using the numpy arrays, nonlinearTseries, arch, panda and nolds packages. All data was processed on Microsoft Visual Studio Code:

Table 3.1 shows that the 3 cryptocurrencies used have positive returns in the selected time period and the ticker SOL has had the greatest average annual returns at 190.8%, followed by SOL (67%) and BTC (55.9%) has had the lowest, yet high, returns. Counterintuitively, ETH has the lowest volatility with a coefficient of variation of 51.72, this is expectedly followed by BTC (56.65) and then SOL (98.35). Therefore, the theory that a higher risk is always associated with a higher return is disproved due to ETH having a lower risk and higher returns than BTC. The asymmetric price cycles and speculative market behaviour leads to a positive skew for all 3 cryptocurrencies, with ETH having the lowest skew at 0.031.

Currency	**Mean**	**Average Return (%)**	**Standard Deviation**	**Variation Coefficient**	**Skewness**
BTC	39316.64	55.97	22271.37	56.65	0.812
ETH	2110.8	67.03	1091.72	51.72	0.031
SOL	71.91	190.8	70.73	98.35	0.834

Table 3.1 – Sample Data for 3 Currencies

The daily closing price data for BTC, ETH and SOL were selected to be analysed in the BDS-Test, Lyapunov Exponents, Recurrence Quantification Analysis and Hurst Exponent tests. The BDS test was used to detect non-linear dependencies within the time series, a tool to represent chaotic behaviour. The results of the test for the 3 cryptocurrencies over the given 1765 entries are displayed in table 3.2.

Cryptocurrency	ADF Statistic	ADF p-value	ARMA Order	ARMA AIC	LM-test (ARMA Res.) p-value
BTC	-13.179	0	(1,1)	-7163.53	0.1793
ETH	-12.539	0	(1,1)	-6241.76	0.0097
SOL	-8.894	0	(1,1)	-4516.54	0.0827

Table 3.2 – ARMA-GARCH Filtering

Cryptocurrency	GARCH AIC	Log-Likelihood	Volatility Persistence	Mean Return
BTC	-10006.8	5007.41	0.98	0.02930
ETH	-20387.2	10197.6	0.98	0.00213
SOL	-29583.0	14795.5	0.98	0.00072

Table 3.3 – ARMA-GARCH Filtering

The Augmented Dickey-Fuller (ADF) statistic shown in table 3.2 indicates the stationarity in data. The fact that the ADF is highly negative and the p-value of 0 rejects the null-hypothesis that there is a unit root. Thus, it indicates stationarity and that the three cryptocurrencies are suitable for further filtering through the ARMA-GARCH system. Optimal fit is shown because the ARMA Order is (1,1) when fitted to returns. There is more autocorrection in ETH than in BTC or SOL due to its lowed LM-test p-value, which suggests a higher degree of residual autocorrection.
The ARMA AIC (Akaike Information Criterion) is used to compare the quality of a statistical model, with a lower ARMA representing a higher quality, better-fitting model. Therefore BTC, which has the lowest ARMA AIC value, has the most efficient and best-fitting model.

The ARMA (1,1) relationship fits BTC returns better than it does ETH or SOL. The lower GARCH AIC on ETH and SOL show a better balance between complexity and model fit. These findings are supported by the log-likelihood which shows the likelihood of the observed data given the model parameters. All 3 cryptocurrencies have very persistent volatility, with a persistence of 0.98, thus this GARCH filtering is used to remove said predictable volatility to generate standardised residuals. This filtering will prevent spurious detection of non-linearity, associated with high levels of volatility, through the BDS test. Thus, any evidence of non-linearity on the BDS test data after ARMA-GARCH filtering will be indicative of chaos.

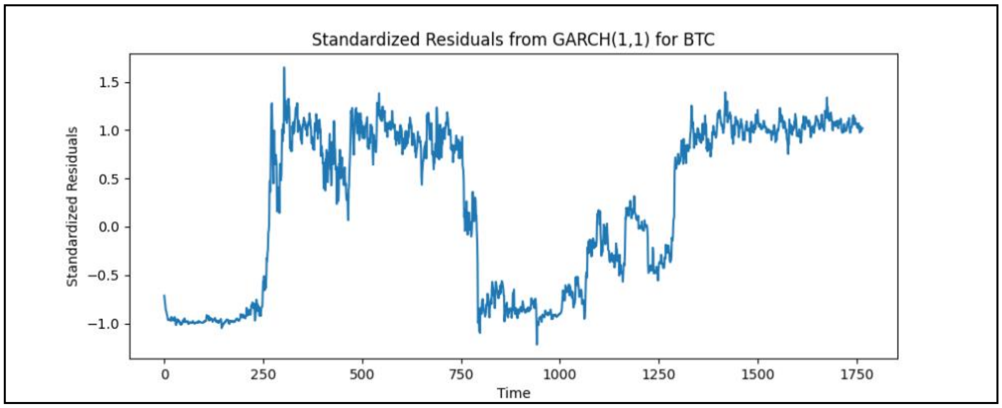

Figure 3.1 – BTC GARCH

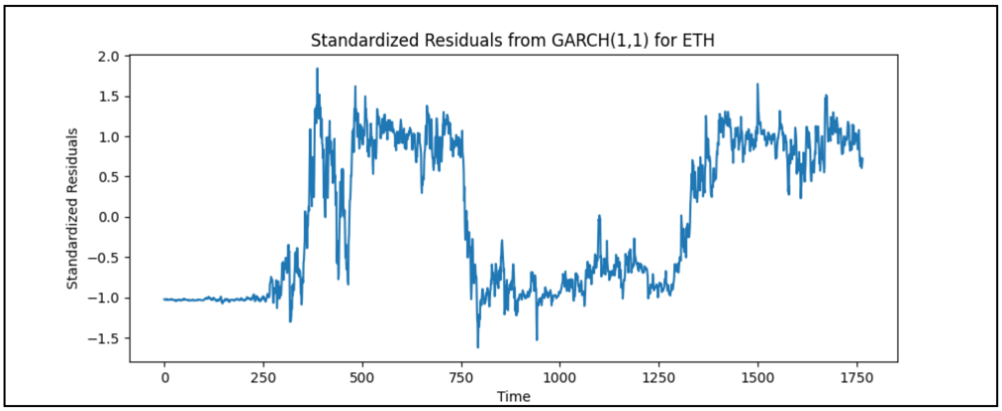

Figure 3.2 – ETH GARCH

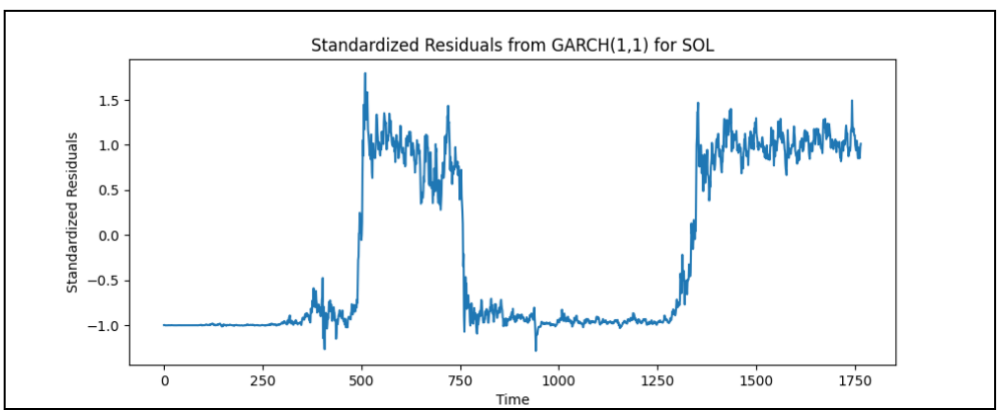

Figure 3.3 – SOL GARCH

The Figures 3.1 – 3.3 show the variation of GARCH residuals for the three cryptocurrencies, which indicate the difference between the actual returns and the model's fitted mean. The majority of the plots are centred around 0, indicating that the mean returns are captured well across the 3 plots. The sudden spikes and dips suggest that returns in those periods are significantly larger/smaller respectively, than predicted through the GARCH model. BTC's lower volatility of GARCH residuals also shows its increased maturity, liquidity and market strength, making it easier for the GARCH model to assess and eliminate its inherent volatility. The market for BTC though has a period of higher-than-average volatility between days 300 and 500, which disproportionately affects its log-likelihood and GARCH

AIC leading to worse fitting models. Table 3.4 shows the post BDS coefficients for the dataset for the embedding dimensions 2 through 5 before and after GARCH filtering.

	BTC		ETH		SOL	
Dimension	**Raw**	**Filtered**	**Raw**	**Filtered**	**Raw**	**Filtered**
2	8.2265	0.0224	7.5174	0.0013	9.5275	0.0191
3	10.4430	0.0413	9.2074	0.0076	13.1089	0.0763
4	10.8760	0.0783	9.3447	0.0152	14.3602	0.0841
5	10.7291	0.0881	9.0931	0.0147	14.7005	0.0896

Table 3.4 – BDS Test Results

The data shows a strong amount of non-linearity in the raw data, but the drastic decrease between the raw and filtered data indicates the extent of the volatility of all three cryptocurrencies. The amount of non-linearity decreases significantly when accounting for volatility but still exists at a smaller scale. Therefore, this study can conclude that much like the studies on other financial markets, the Dow Jones Industrial Average (Diaz, 2013) and the S&P 500 (Peters, 1991) the market for cryptocurrencies also follows chaotic trends. Since the BDS test can’t detect chaotic properties, further tests are carried out to assess the levels of non-linearity amongst BTC, ETH and SOL.

Tables 3.5 and 3.6 shows the results of the test for the Largest Lyapunov Exponent for BTC, ETH and SOL. It will also be used to test for chaos on the daily return and rate of return for BTC, ETH and SOL to see if the price changes are also chaotic in nature. As mentioned previously, a positive value for the Largest Lyapunov Exponent will indicate chaos in a time series. This calculation used the Rosenstein et al method, with an optimised time lag of 73 and a min_tsep of 544 for BTC. This suggests that there were 1090 trajectories made in the time series. This was repeated to get table 3.5, which shows the number of trajectories, the min_tsep and the time lag for the closing prices of BTC, ETH and SOL.

Cryptocurrency	**Trajectories**	**Min_tsep**	**Lag**
Bitcoin (BTC)	1090	544	73
Ethereum (ETH)	991	494	84
Solana (SOL)	1000	499	83

Table 3.5 – Metrics for the Rosenstein et al Method (price)

The processes used for Table 3.5 were repeated for the time series of the daily absolute returns and the daily rate of returns for BTC, ETH and SOL.

Cryptocurrency	**Price**	**Daily Returns**	**Rate of Returns**
Bitcoin (BTC)	0.00599829	0.036222402	0.034321986
Ethereum (ETH)	0.00585156	0.033160211	0.033418709
Solana (SOL)	0.00815429	0.039763529	0.030758403

Table 3.6 – Largest Lyapunov Exponents

Table 3.6 shows the LLEs for 9 time-series' are positive which would indicate chaos, but they are very close to 0 (less than 0.1 from 0) so stochastic dynamics are more evident when assessing the daily prices and returns for the cryptocurrencies. Distinguishing deterministic chaotic dynamics from stochastic noise is nontrivial in financial systems where noise and nonlinearities are inherently intertwined. Therefore, these time-series still retain a certain degree of chaos which can be determined by the size of the exponent (with larger LLEs representing a higher level of chaos). It is evident that the time-series for daily returns and rate of returns are more chaotic than the price time-series. The difference operators, used to find returns and rates, amplifies stochastic noise leading to the higher sensitivity for returns when compared to prices, as shown in the data. In addition, price is a cumulative function which develops over time, so longer term trends tend to be less volatile leading to higher predictability and a higher degree of determinism. This also shows that the price of cryptocurrencies tends to be volatile and not highly predictable in the short run. Further, financial markets are complex adaptive systems that are subject to the effects of exogenous factors, making it tougher to directly ascribe the dynamics observed to some low dimensional deterministic process. Hence positive LLEs provide suggestive evidence of nonlinear structure, but fall short of proving that chaos is present on their own. This test shows that the time-series are at the edge of chaos and randomness. Therefore, this test is used in tandem with the BDS-test, Hurst Exponents and the Recurrence Quantification Analysis (RQA).

RQA has been conducted through three indicators: recurrence rate, determinism and divergence all of which are used to suggest chaos; with the data for said tests provided in Table 3.7. This data is more helpful to establish whether randomness or chaos is more prevalent in the data for daily prices of BTC, ETH and SOL.

Cryptocurrency	**Recurrence Rate (%)**	**Determinism (%)**	**Divergence**
BTC	6.974	99.9954	0.002529
ETH	6.885	99.9987	0.002202
SOL	11.084	99.9919	0.003184

Table 3.7 – Recurrence Quantification Analysis

The RQA data shows a very clear chaotic trend amongst all 3 cryptocurrencies which is in line with the theory that financial markets are inherently chaotic in nature. The recurrence rates of BTC, ETH and SOL are all between 5 and 15%, which is the range for chaos, as previously established. Additionally, the high determinism shown of over 99% in all three cases, suggests strong deterministic behaviour which is typical of chaotic systems. Finally the low divergence indicates high levels of predictability, especially in short time periods. Thus, the cryptocurrency market for all 3 coins are shown to be chaotic. The daily recurrence rates for BTC, ETH and SOL are shown below in Figures 3.4, 3.5 and 3.6.

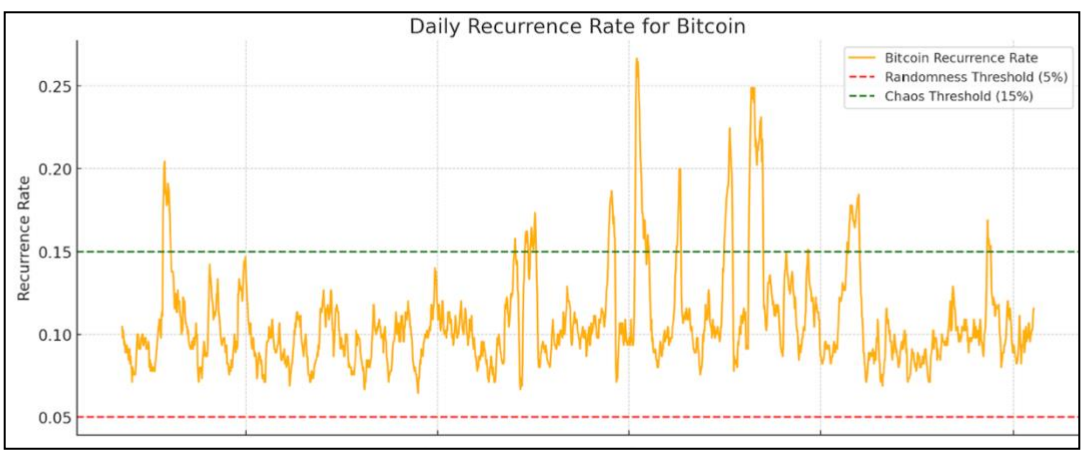

Figure 3.4 – Daily Recurrence Rate (BTC)

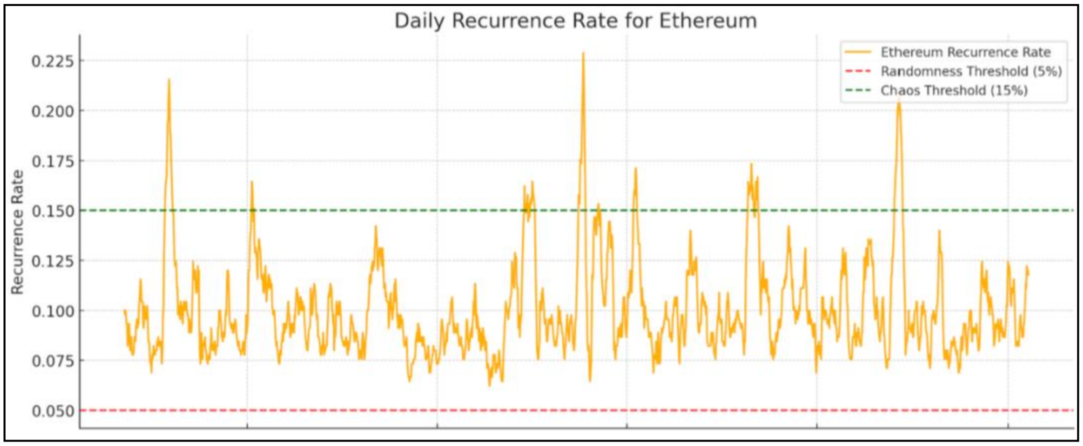

Figure 3.5 – Daily Recurrence Rate (ETH)

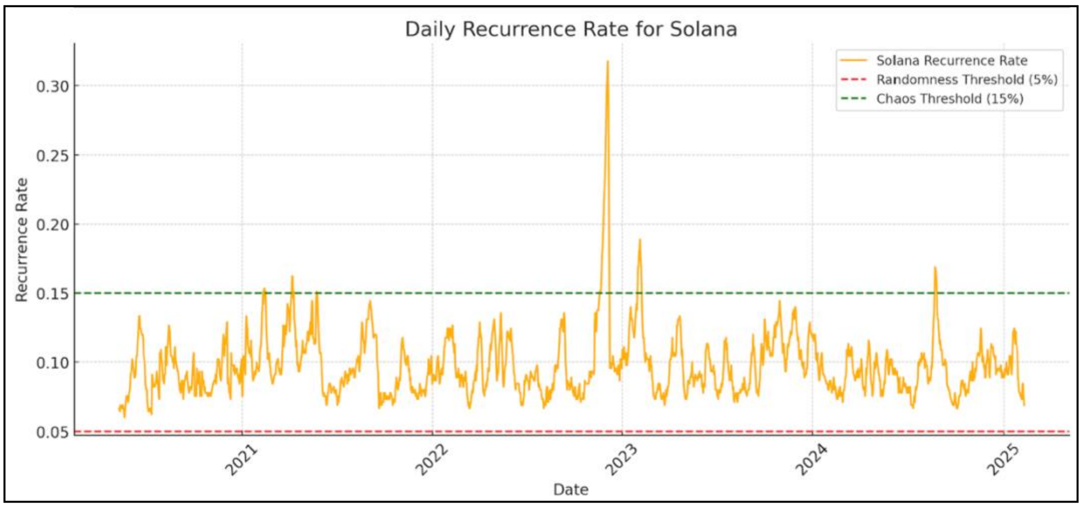

Figure 3.6 – Daily Recurrence Rate (SOL)

Figures 3.4-3.6 clearly show that the majority of recurrence rates are between the chaos and randomness thresholds of 5 and 15% respectively. There are occasional spikes in the recurrence rate entering the region of periodicity. This is due to the high volatility of cryptocurrency markets, where shocks are common. Thus, whenever market recovery occurs after a shock period-like behaviour is followed. Additionally, the prevalence of algorithmic trading systems doubles-down the periodicity as multiple systems synchronise and act similarly post said shocks. The maximum recurrence rate and the fluctuation of the recurrence rate are both positively related with the volatility of the cryptocurrency. Hence, BTC and ETH with lower volatility, have a maximum recurrence rate between 22.5 and 27.5%, while the much more volatile SOL has a maximum of over 30%.

Table 3.8 shows the Hurst exponent test for chaos on BTC, ETH and SOL – for both raw, scattered and ARMA-residual values. This helps showcase the difference in chaotic tendencies before and after normalising for linear dependency.

Cryptocurrency	Original Hurst Exponent	Hurst ARMA Residuals	Hurst Scrambled Series
Bitcoin (BTC)	0.5555	0.5658	0.5492
Ethereum (ETH)	0.5554	0.5735	0.5283
Solana (SOL)	0.5303	0.547	0.575

Table 3.8 – Hurst Exponents

All Hurst Exponent values are above 0.5, with BTC and ETH exhibiting a greater degree of persistence when compared to SOL's higher randomness in the original series. The data is then modified to account for ARMA residuals, removing any linear dependency, helping reduce SOL's randomness. Additionally, when the datapoints are scrambled, removing all temporal calculations and fractal properties – the more chaotic BTC and ETH end up coming closer to randomness while the more random SOL becomes more chaotic. Therefore, no matter the modifications the Hurst Exponent values for the three cryptocurrencies all indicate a degree of chaos and persistence. This opposes the notion that removing linear dependency will help reduce chaos. On the other hand, all values of the exponent are near 0.5, which suggest a low degree of overall chaos or persistence for the currencies. This could be because of some random movements which often occur in cryptocurrency markets, due to the multitude of factors that impact it, the irrationality of investors and the high degree of unpredictability associated with financial markets in the short term. Thus, cryptocurrencies showcase mixed dynamics of chaos and randomness with chaos taking the edge as the more dominant trait in our dataset. Finally, financial markets are never known to be highly predictable, but these values of the Hurst Exponent suggest that they are more often than not predictable, but when they are it is to a low degree, so trends are not strong nor sustained over long periods.

Conclusion:-

This study tests for chaos in 3 cryptocurrencies Bitcoin (BTC), Ethereum (ETH) and Solana (SOL) through 4 commonly used tests for chaos namely the BDS-test, DFA-analysis of Hurst Exponents, Largest Lyapunov Exponents and Recurrence Quantification Analysis. The research uses 1765 datapoints which indicate daily closing prices over a 5-year period, due to higher levels of accuracy being associated with long-series data while testing for chaos. The BDS test results for the raw data show high degrees of chaos, and post filtering non-linearity is maintained at lower levels. The test for LLE shows chaos amongst all indicators: the price, the absolute return and the rate of return. The almost 0 LLE for price shows that the data for the 3 cryptocurrencies are at the edge of chaos and randomness, but the degree of chaos amongst the return parameters are higher showing clearer non-linear trends. The recurrence quantification analysis tests for recurrence rate show chaotic trends with mean values between 5 and 15%, but the daily fluctuations are indicative of the high degree of volatility of the financial tokens. The almost 100% determinism and almost 0% divergence are much stronger displays of the chaotic tendencies exhibited by the cryptocurrencies. Finally, the DFA-analysis for the Hurst Exponents testing the original series, scrambled series, and ARMA residuals all indicate non-linear trends with values above 0.5.

These findings suggest that the movements of these three cryptocurrencies over this time period of 5 years are not in line with the random walk theory (Fama, 1965), which suggests that the prices for financial instruments (originally used for stocks) are random, so past movement cannot be used to predict a stocks future movement. It also shows that small initial differences can lead to vastly different outcomes. Through analysis of a larger time series the model's accuracy can be improved, leading to a clearer indication of whether these markets are truly chaotic, as many tests found the closing prices to be at the edge of chaos and randomness. Finally, the tests could be conducted on average OHLC (open-high-low-close) prices, rather than only on the closing price to improve the model's accuracy.

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