



RESEARCH ARTICLE

A NEW APPROACH TO MULTI-CRITERIA EVOLUTIONARY SEARCH AND ITS APPLICATIONS TO DECISION-MAKING FOR TUBULAR GAS HEATERS

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Manuscript Info

Manuscript History

Received: 17 July 2025

Final Accepted: 19 August 2025

Published: September 2025

Key words:-

Evolutionary algorithm, sequential selection, blocking function, advantage function, decision-making, tubular gas heaters, gas emissions, mathematical model

Abstract

There is a known evolutionary search method that uses a king The new approach consists of sequential selection, first using selection with a blocking function for conflicting criteria, and then selection with an advantage function for a new criterion. The results of solving a test problem with 30 variables and three criteria are presented. The results demonstrate the good performance of the new approach. The search for a solution with three criteria required almost 10 times fewer calculations than the number of calculations in an evolutionary search with two criteria and one selection. With its help, the problem of optimizing the emissions of a tubular gas heater on wood pellets was solved numerically using mathematical models of emissions based on experimental data. The mathematical models are presented in the form of three dimensionless complexes that describe the processes of burning wood pellets in a tubular gas heater. The result of optimization is the control parameters at which minimum gas emissions (CO and NOx) are ensured. The obtained solutions for optimizing heater emissions ensure compliance with the gas emission requirements for natural gas combustion.

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Introduction:-

Infrared tubular gas heaters and tubular gas heaters are well known, for example, [1-4], including the development of tubular pellet heaters [5-7]. When using pellets as fuel, gas emissions from pellet combustion are of great importance.

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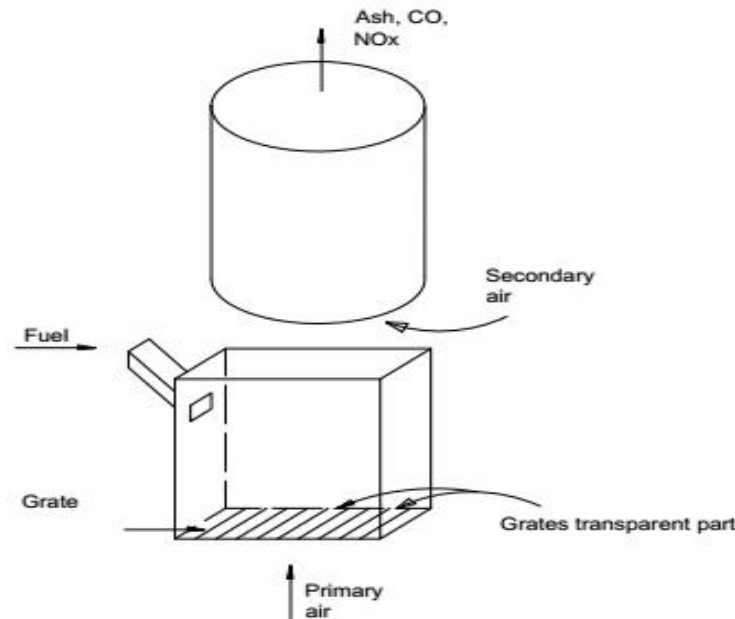


Fig. 1: – Tubular gas heater with pellets burner principle diagram

In [7], an experimental study of a tubular gas heater using wood pellets was conducted, primarily investigating emissions from pellet combustion. As a result of this study, a mathematical model of the tubular heater's operation was developed. This model is presented in the form of five dimensionless complexes (criteria) of the form:

$$F_4 = b_1 \cdot ((1 - F_1^2)^{b_2} / (1 - F_1 \cdot F_2)^{b_3}) \cdot (b_4 + (F_3 / F_2)^{b_5})$$

Where: $b_1 = 0.0256$, $b_2 = 5.945$, $b_3 = 63.4$, $b_4 = 1.95$, $b_5 = 0.48$.

$$F_5 = a_1 + a_2 \cdot (F_1)^{a_3} \cdot (F_2)^{a_4} \cdot (F_3)^{a_5}$$

Where: $a_1 = 1.096$; $a_2 = 31.33$; $a_3 = 3.2155$; $a_4 = -0.1776$; $a_5 = 0.7470$.

$$F_3 = d_1 \cdot (F_1)^{d_2} \cdot (F_2)^{d_3} \cdot (1 - F_1 \cdot F_2)^{d_4} \cdot (1 - F_1 \cdot F_2)^{d_5}$$

Where: $d_1 = 0.0116$, $d_2 = 1.465$, $d_3 = -1.029$, $d_4 = 6.34$, $d_5 = -0.14$.

F_1 - Dimensionless geometric parameter of the pellet burner of the heater, F_2 - Dimensionless parameter of the air flow supplied to the heater burner. Parameters F_1, F_2 - input parameters of the mathematical model, F_3, F_4, F_5 - output parameters of the mathematical model. $F_4 = (\alpha_{CO})^{0.5}$ - parameter of CO concentration, $F_5 = (\alpha_{NOx})^{0.5}$ - parameter of NOx concentration. F_3 - parameter characterizing the amount of ash in combustion products.

As the study showed, the influence of input parameters on output parameters is contradictory. It is necessary to find compromise solutions to ensure that gas emissions remain within acceptable limits. Ash emissions can be eliminated using various filters, such as cyclonic ash cleaners, while gas emissions should preferably be reduced by selecting the most suitable operating modes for heaters. Previously, solutions for optimizing emissions were found based on two criteria, but none were found based on all three minimization criteria. The purpose of this article is to present a new approach to solving multi-criteria optimization problems and apply it to solving the problem of optimizing emissions from a tubular gas heater based on three criteria.

Let us state the problem.

It is necessary to find a solution $x \in \Omega$ so that $xR_{S1}y$ and for all $y \in \Omega$ and so that $xR_{S1}y$ it is fulfilled $xR_{S2}y$ and $xR_{S3}y$. Let us create new binary relation R_{SSS} that takes into account limitation in the form of binary relation as follows:

$$xR_{SSS}y \equiv xR_{S1}y \wedge xR_{S2}y \wedge xR_{S3}y \quad (1)$$

Let us represent the original choice relation (1) in the form

$$xR_{SSS}y \equiv xR_{SS}y \wedge xR_{S3}y, \quad (2)$$

where

$$xR_{SS}y \equiv xR_{S1}y \wedge xR_{S2}y \quad (3)$$

It is this general case (3) of multicriteria optimization that is described in [8,9]. According to [8] the evolutionary search is represented in form

$$X_{jk} = S^{R_s}(G(X_{jk-1})), \quad k=1, 2, \dots, j=1, 2, \dots, N_B,$$

Where $S^{R_s}(X)$ - selection function as blocking function

$$S^{R_s}(X) = \{x \in X | \forall y \in [X \setminus S^{R_s}(X)], \bar{y}R_s x\}$$

and X_{jk} – the set of selection solutions according to the binary choice relation R_s and choice function as blocking function at the iterate step k for the branch j of evolutionary search, N_B – the number of branches.

A numerical example of an evolutionary search using the boxing function is given in [8].

To jointly take into account binary choice (1), we present the evolutionary search algorithm in the form

$$X_{jk} = S_{R_{SS}}(S^{R_{SS}}(G(X_{jk-1}))), \quad k=1, 2, \dots, j=1, 2, \dots, N_B,$$

It is this general case (2) following our previous studies it is considered here a problem of finding a solution $x_0 \in \Omega$ from elements $x = \{x^1, x^2, \dots, x^n\}$, so that $\forall x \in \Omega, x_0 R_{SSS} x$ where R_{SSS} - known binary relation that is a relation of non-strict preference. And the final choice takes the form of a advantage function

$$S_{R_{SSS}}(X) = \{x \in X | \forall y \in [X \setminus S(X)], xR_{SSS}y\}.$$

Each selection, if performed separately, ensures the convergence of evolutionary search to the corresponding optimal solution with probability 1. Such convergence is disrupted when successive selections are performed (one after another); in this case, the evolutionary search process can be considered as a search in the presence of interference. In this article, we will not theoretically analyze the convergence of evolutionary search with sequential selections, but will demonstrate the operation of the developed algorithm using the example of solving a well-known test problem with two criteria, to which we will add a third criterion.

$$\min(f_1(x^1), f_2(x^1, x^2, \dots, x^m)), \min(f_3(x))$$

$$f_1 = x^1, f_3(x) = \text{abs}(f_1(x) - 0.5)$$

$$g(x^2, \dots, x^m) = 1 + 9 \cdot \sum_{i=2}^m \frac{x^i}{(m-1)},$$

$$f_2(f_1, g) = 1 - \sqrt{\frac{f_1}{g}},$$

$$x^i \in [0, 1], \quad i = 1, \dots, m, \quad m=30.$$

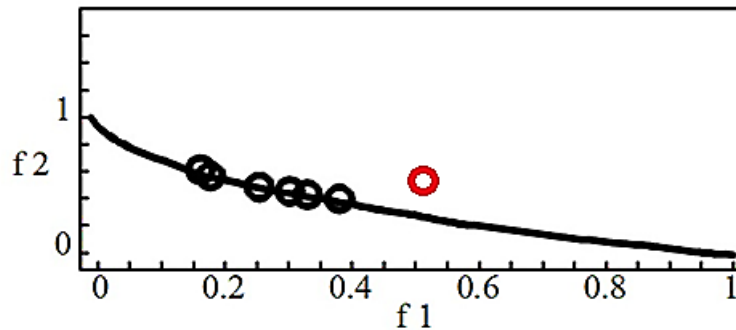
The following search parameters were used for the calculation: evolution branches – 3, solutions generated in each branch at one iteration step – 15, solutions after the first selection – 5, solutions in each branch at one iteration step after the second selection – 2. The table below shows the course of the evolutionary search.

Iteration number	$F_1 \rightarrow \min$	$F_2 \rightarrow \min$	$F_3 \rightarrow \min$
Iteration 1	0.6616573	0.6277951	0.1707633
	0.7865534	0.5891457	0.2895936
	0.3958411	0.7250972	0.09682715
Iteration 5	0.6095976	0.5535629	0.1095976
	0.6278895	0.5893612	0.1278895

	0.3737617	0.7063692	0.1262383
Iteration 12	0.5113736	0.5685698	0.01137364
	0.5102729	0.5717641	0.01027286
	0.3042426	0.6709272	0.1957574

The figure below shows the results of solving the problem of finding the minimum of functions f_1, f_2 .

The coordinates of the minimum found for correspond to: 0.5113736 and 0.5685698, which are, as can be seen, practically on the Pareto front. The number of iterations to find the minimization result shown in the table is 2700. At the same time, the number of iterations For example, in Fig. 1, the number of iterations is 115, which corresponds to 23,000 calculations of functions with the best approximation to the Pareto front, and for other algorithms, the results of which are also shown in the figure, this number exceeds 25,000. Using the developed algorithm, the problem of minimizing emissions for a tubular gas heater was solved according to three criteria, for which empirical dependencies of dimensionless parameters characterizing the process of pellet combustion in a tubular heater were used.



For tube heaters $F_1 \rightarrow \min$, $F_2 \rightarrow \min$, $F_3 \rightarrow \min$.

Iteration number	x_1	x_2	x_3	$F_1 \rightarrow \min$	$F_2 \rightarrow \min$	$F_3 \rightarrow \min$
Iteration number 1	0.449446	0.0765515	0.0407164	0.16564	1.32533	0.04071
	0.3901578	0.07043559	0.0376890	0.15100	1.23355	0.03768
	0.3939794	0.08	0.0326823	0.18604	1.22331	0.03268
Iteration number 5	0.3	0.08	0.0230011	0.170418	1.13677	0.02300
	0.3	0.07996977	0.0230114	0.17033	1.13678	0.023011
	0.3027829	0.07997749	0.0238489	0.17117	1.13839	0.023289
Iteration number 10	0.3	0.08	0.023001	0.170418	1.136771	0.023001
	0.3	0.08	0.023001	0.170418	1.136771	0.023001
	0.3	0.08	0.023001	0.170418	1.136771	0.023001

It is easy to see how the solution to the three-criteria optimization problem changes if the optimization condition for the third criterion is changed. Obviously, the solution where all criteria are required to be minimized is more appropriate.

For the found values 0.3 0.08 0.023001, the gas emissions obtained are

$$\alpha_{CO} = 0.0290; \alpha_{NO_x} = 1.292$$

$$Y_{CO} = 62.35; Y_{NO_x} = 218.93 \text{ mg} / \text{m}^3$$

At the same time, according to Ukrainian requirements, gas emissions must not exceed:

$$\alpha_{CO} \leq 130 \text{ mg} / \text{m}^3 \text{ and } \alpha_{NO_x} \leq 250 \text{ mg} / \text{m}^3.$$

Thus, the solution found fully meets the requirements for harmful gas emissions set forth in Ukraine for gas equipment that burns natural gas.

Conclusion:-

The task of making decisions to reduce harmful gas emissions when using wood pellets in tubular gas heaters is considered. The experimental results of the operation of a tubular gas heater on pellets were used to make decisions. The available experiments showed that the criterion for harmful CO concentration and the criterion for harmful NOx

concentration in the combustion products of pellet fuel have opposite effects. It is necessary to find compromise operating modes for tubular heaters that would be most preferable, taking into account the existing requirements for gas emissions. Unlike previous work on decision-making regarding emissions from tubular heaters, this work takes into account not two, but three criteria simultaneously. For this purpose, a new evolutionary search approach with sequential selections has been developed: first, selection based on two criteria using a blocking function, and then a second selection among those solutions that passed the first selection, based on the third criterion using a preference function. An algorithm for numerical search with the specified sequential selection of solutions has been developed and tested on a well-known test problem of minimizing two conflicting criteria with 30 variables. For this problem, a third criterion was added and a numerical search with two sequential selections was performed. The calculation results showed that the solutions obtained are found fairly quickly for all three criteria, and for the first two compromise criteria, the solutions are fairly close to the Pareto front, as can be seen from the figure presented. The developed algorithm was used to determine the most preferred solution for emissions from pellet-fired tubular gas heaters. The search resulted in finding a heater operating mode that provides optimal gas emission values: $Y_{CO} = 62.35$; $Y_{NO_x} = 218.93 \text{ mg} / \text{m}^3$. These values fully comply with existing requirements not only for pellet combustion products, but also gas emissions set forth in Ukraine for natural gas combustion.

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