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### RESEARCH ARTICLE

## OPTIMAL DESIGN OF HYDROPOWER TUNNELS FOR COST EFFICIENCY IN ALBANIA

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### Abstract

Albania, with abundant water resources but currently exploiting only about 30% of its potential, plans the construction of 43 hydroelectric power plants (HPPs) with dam heights of 20–160 m, capacities of 2–170 MW, and design flows of 2–200 m<sup>3</sup>/s. These projects will involve approximately 100 km of tunnels with diameters of 5–6 m. Determining the optimal tunnel cross-section is critical to minimize total costs, which include both hydraulic friction losses and tunnel construction expenses. Hydraulic losses are influenced by flow, tunnel length, and friction factors, while construction costs depend on excavation, lining, and reinforcement requirements. The economic cross-section is achieved when the marginal cost of friction losses equals the marginal cost of tunnel construction. Sensitivity analysis shows that tunnel design is most affected by internal pressure, rock characteristics, energy price, and economic parameters such as bank interest and inflation. The study estimates that the economic tunnel diameter for Albania's HPPs ranges between 1.6 and 12.0 m, with corresponding water velocities of 1.0–1.8 m/s. The methodology presented offers a general and practical approach for the cost-effective design of hydropower tunnels, applicable to various types of tunnels and pipelines.

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### Introduction:-

In the new conditions of the world economy, where our country has entered the flow of the global economy, where competition is strong, the demand is that we produce goods at the lowest possible cost, in order to compete further in regional and world markets. Since Albania is rich in water resources and has utilized up to 30% of its large resources, there is a need to build hydroelectric power plants of various types, such as dam, diversion, and combined.

A study conducted by the Department of Hydraulic and Hydrotechnics on the potential for water use for the purpose of producing electricity yielded these results.

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There will be 43 Hydro Power Plants on the river main river in Albania.

The height of the hydroelectric dams will be  $H = 20 \div 160$  m

Installed capacity is  $N = 2 \div 170$  MW.

Design flow  $Q_{ll} = 2 \div 200$  m<sup>3</sup>/s

Net Head  $H_{ll} = 17 \div 114$  m

About 100 km of tunnels will have a diameter of  $D = 5 \div 6$  m.

Some major rivers in our country have these tunnel lengths.

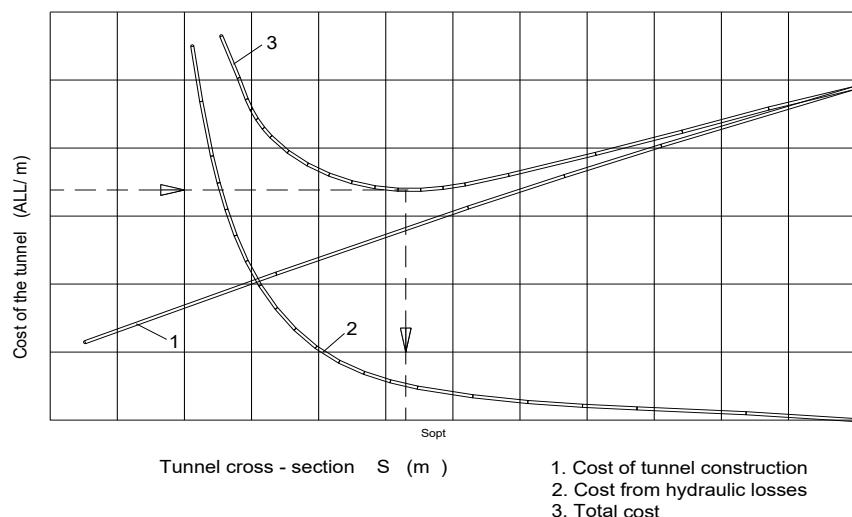
River name	Tunnel length
Devoll	1 $\div$ 9.3 km
Vjosa	0.5 $\div$ 1.6 km
Fani	2 $\div$ 11 km
Shkumbin	5 km
Osumi	1.3 $\div$ 7.5 km

Since several HPPs will be built in the future, we will try to provide a method for determining the optimal cross-section of pressure tunnels.

The optimal cross-sectional area of a tunnel depends on many factors, but the most important ones are:

- Cost of hydraulic friction losses.
- The cost of building the tunnel.

The optimal cross-sectional area is when the sum of the cost of friction losses and the construction cost is at a minimum.



### Cost of friction losses

$$\text{Hydraulic losses } h_w = \lambda \frac{l}{D_h} \frac{v^2}{2g} = \lambda \frac{l}{D_h} \frac{Q^2}{2g \cdot S^2}$$

$\lambda$  – coefficient of friction in the pipe.

$l$  – length of the tunnel.

$v$  – speed of water movement.

$D_h$  – tunnel diameter.

$S$  – the cross-sectional area of the tunnel.

$Q$  – the flow passing through the tunnel.

$$\text{The cost of friction losses } K_F = \eta \cdot \rho \cdot \lambda \frac{Q_d^3}{8 \cdot \alpha \cdot S^2} T \cdot p \cdot D \cdot 10^{-3} \text{ ( ALL/ml tunnel)}$$

$\eta$  – HPP efficiency .

$\rho$  – density of water.

$T$  – time of loss during use.

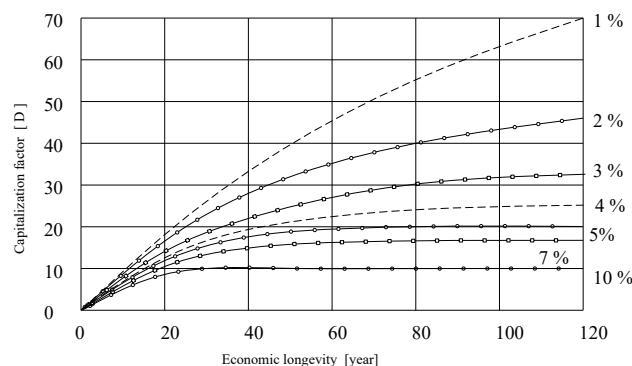
$p$  – average energy price.

$D$  – capitalization factor

$Q_d$  – water flow (calculated flow of the HPP or a turbine)

### Capitalization Factor

The capitalization factor is an economic indicator that shows the present value of money that comes from constant income during a year, for a number of “n” years. In our case, this income is the lost profit that comes from frictional losses.



The capitalization factor is affected by .

- The economic lifespan of the HPP .
- Bank interest.
- Inflation.

### Capitalization factor chart

$$D = \frac{(1+r)^n - 1}{r(1+r)^n}$$

- $r$  – real interest rate (%)  $r = \frac{i-q}{1+q}$  (%)

- $i$  – bank interest (%)

- $q$  – inflation (%)

- $n$  – economic life (years)

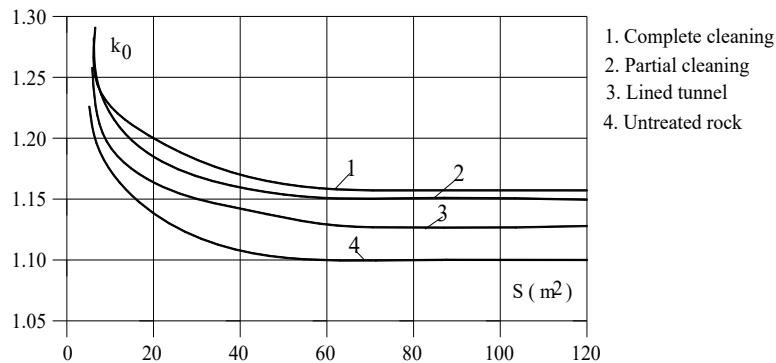
### Optimal Tunnel Section

From the tunnel excavation we have an over-excavation and the surface area left by the excavation is .

$$S_v = S_p \cdot k_0$$

$k_0$  – The over-excavation factor is a function of the tunnel treatment method.

- Unprocessed rock (the rock is left as it comes out of the excavation).
- Partial cleaning (only the path of the machinery is cleaned).
- Complete cleaning (the entire tunnel section is cleaned).
- Lined tunnel.



$$k_0 = 0.00129 \cdot S_p + \frac{0.43}{\ln S_p} + k_b$$

$k_b$  – is a constant that depends on the way the rock is treated.

By substituting the corresponding formula in place of  $k$ , we find the real cross-sectional area of the tunnel.

$$S_v = 0.0129 \cdot S_p^2 + \frac{0.43 \cdot S_p}{\ln S_p} + k_b \cdot S_p$$

$$\frac{dS_v}{dS_p} = 0.00258 \cdot S_p + \frac{0.43 \cdot (\ln S_p - 1)}{(\ln S_p)^2} + k_b$$

### Marginal cost of losses

Treatment method	Raw rock	Partial cleaning	Complete cleaning	Coated tunnel
$k_b$	0.99	1.04	1.05	1.02
$S'_v$	1.09	1.14	1.15	1.12

The marginal cost of friction loss expresses the absolute addition to the total cost when we increase the cross-sectional area of our tunnel by one unit. In our case, this unit is 1 additional  $m^2$  of cross-sectional area.

$$K_F = \eta \cdot \rho \cdot \lambda \frac{Q_d^3}{8 \cdot \alpha \cdot S^{2.5}} T \cdot p \cdot D \cdot 10^{-3} (\text{ALL/m})$$

In the formula, the coefficient of friction in pipes for the case of turbulent motion is found using Colebrook

$$\lambda = \frac{1}{\left(-2 \cdot \log \left(\frac{\Delta}{3.7 \cdot D_h}\right)\right)^2}$$

By accepting  $S = S_v$  and accepting as a constant all factors that do not depend on the tunnel cross-section, the cost formula for friction losses takes the form:

$$K_F = c \cdot \lambda \cdot S_v^{-2.5} (\text{ALL/m})$$

The constant  $c$  is:

$$c = \frac{1}{8 \cdot \alpha} \eta \cdot \rho \cdot Q_d^3 \cdot T \cdot p \cdot 10^{-3}$$

We find the marginal cost from losses by taking the derivative of the cost of losses with respect to the required section  $S_p$ .

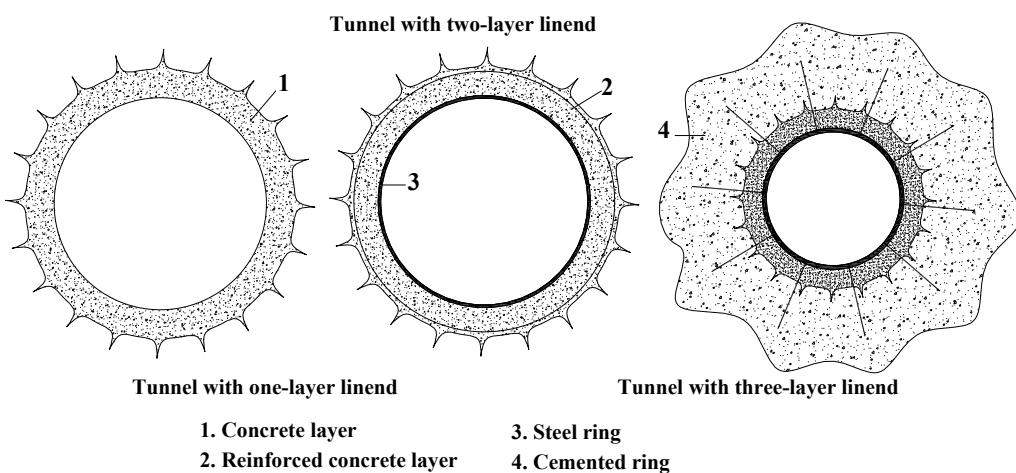
$$\frac{dK_F}{dS_p} = c \cdot \left( -2.5 \cdot S_v^{-3.5} \frac{dS_v}{dS_p} \lambda + S_v^{-2.5} \frac{d\lambda}{dS_p} \right)$$

From the actions we finally derive the marginal cost of friction losses.

$$\frac{dK_F}{dS_p} = c \cdot S_v^{-3.5} \frac{dS_v}{dS_p} \lambda + \left( 2.5 + \frac{2}{\ln 10 \cdot \sqrt{\lambda}} \right)$$

### Tunnel Construction Cost

Regarding the thickness of the tunnel lining, it depends mainly on the type of geological formation of the soil where the tunnel passes, the water pressure in the tunnel and the diameter of the tunnel.



To find the cost of the tunnel, we will first do its preliminary dimensioning based on the limit state of not allowing cracks.

### Tunnel data

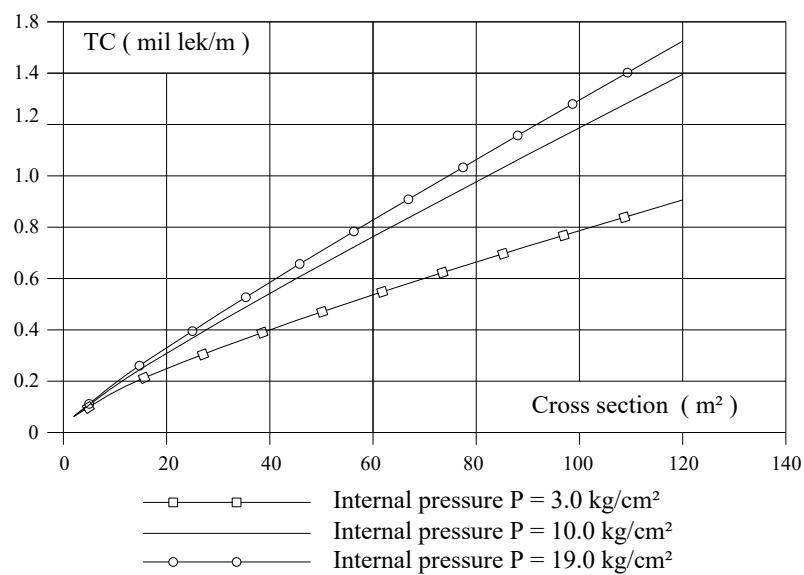
From the studies conducted on the use of water for the purpose of energy production, the following characteristics have emerged for the HPPs to be built:

- The height of the hydroelectric dams will be  $H = 20 \div 160$  m.
- There will be 43 HPPs in the tunnel trunks.
- Their power is  $N = 2 \div 170$  MW.
- Design flow  $Q \text{ l/s} = 2 \div 200$  m<sup>3</sup>/s
- Calculation drop  $H \text{ l} = 17 \div 114$  m

- About 100 km of tunnels will have a diameter of  $D = 5 \div 6$  m.
- Some major rivers in our country have these tunnel lengths.

We have obtained the prices of the items that are part of the cost of the tunnel from the Ministry of Construction, and they are as follows.

TITLE	Unit	PRICE ( ALL )
Surface excavations	$m^3$	1200
Underground excavations	$m^3$	2300
Metal reinforcements	tone	100000
CONCRETE	$m^3$	1000
Reinforcing steel	tone	76000
Reinforcing steel	tone	76000



#### Marginal cost of the tunnel

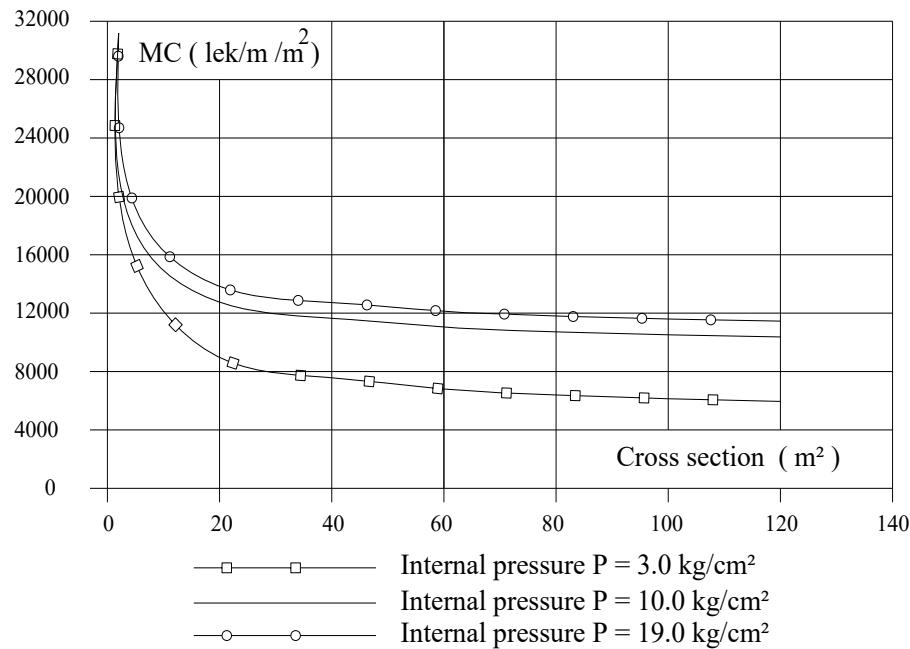
To find the optimal cross-section of the tunnel, we need to know the marginal cost of building it. We find this cost using the following formula.

$$MC_t = \frac{\Delta TC}{\Delta S} \quad \text{ALL/m}^2/m$$

$MC_t$  – Marginal cost of the tunnel.

$\Delta TC$  – Change in total cost.

$\Delta S$  – change in tunnel surface area.



### Optimum cross-section area

We find the optimum cross-section by equating the derivative of the sum of the marginal cost of losses with the marginal cost of construction over the real area to zero.

$$\frac{d(TC + K_F)}{dS_p} = 0$$

From further actions we obtain:

$$MC_t - c \cdot S_v^{-3.5} \frac{dS_v}{dS_p} \cdot \lambda \cdot \left( 2.5 + \frac{2}{\ln 10 \cdot \sqrt{\lambda}} \right) = 0$$

By replacing

$$S_v = S_p \cdot k_0 \text{ AND } \frac{dS_v}{dS_p} = S_v'$$

$$S_p = \left( \frac{\frac{1}{8\alpha} \eta \cdot \rho \cdot Q_d^3 \cdot T \cdot p \cdot 10^{-3} \cdot S_v' \cdot \lambda \cdot \left( 2.5 + \frac{2}{\ln 10 \cdot \sqrt{\lambda}} \right)}{MC_t \cdot k_0^{3.5}} \right)^{\frac{2}{7}} \text{ (m}^2\text{)}$$

### This formula is generally applicable to all types of tunnels and pipelines.

For the construction of the graph below, these values were accepted for calculating the optimum cross section. By analyzing the results of the hydropower exploitation schemes of Albania's rivers as well as the data of the Bank of Albania, we have accepted these data for further calculations.

$$\eta = 0.8$$

$$T = 3600 \text{ hours}$$

$$p = 8.0 \text{ ALL/kW}$$

$$N_{HPP} = 40 \text{ years}$$

$$L_t = 5000 \text{ m}$$

$$i = 7.5\%$$

$$\alpha = 0.282$$

$$q = 3.5\%$$

### Case studies

#### Skavica Hydropower Plant

- Limestone formation  $f = 7$
- The coefficient of elastic response of the rock  $K_o = 350 \text{ kg/cm}^3$
- The river flow is intended to be channeled through two tunnels  $Q = 2 \times 125 \text{ m}^3/\text{s}$
- Internal pressure  $P = 9.5 \text{ kg/cm}^2$

From the graph for determining the thickness of the reinforced concrete lining of the tunnel, we obtain the ratio  $\frac{h_v}{r_b} = 0.3$

Data	Values	
flow	$Q = 125 \text{ m}^3/\text{s}$	$Q = 250 \text{ m}^3/\text{s}$
Optimal preliminary section	$S_p = 74.51 \text{ m}^2$	$S_p = 134.58 \text{ m}^2$
$S_v'$	1.12	1.12
Lifespan	80 years	80 years
Over-Eks. coefficient	$k_o = 1.125$	$k_o = 1.125$
Marginal cost	$MC_t = 9849 \text{ ALL/m}^2/\text{m}$	$MC_t = 9715 \text{ ALL/m}^2/\text{m}$
Coefficient of friction	$\lambda = 0.0106$	$\lambda = 0.0104$
<sup>HEC</sup> working hours	$T = 3000 \text{ hours}$	$T = 3000 \text{ hours}$

$$S_p = \left( \frac{\frac{1}{8 \cdot 0.282} 0.9 \cdot 1000 \cdot 125^3 \cdot 3000 \cdot 8.0 \cdot 10^{-3} \cdot 1.12 \cdot 0.0106 \cdot \left( 2.5 + \frac{2}{\ln 10 \cdot \sqrt{0.0106}} \right)}{9849 \cdot 1.125^{3.5}} \right)^{\frac{2}{7}}$$

Since we have calculated the optimal area of the Skavica HPP section with the first method, we calculate this same area with the second method.

The second method that was used in the years of the centralized economy, as a basis for comparing the cost required to increase by one unit of tunnel surface (one  $\text{m}^2$ ), was the cost of purchasing or producing additional energy that would replace the energy lost due to friction losses.

This method calculates the optimal hydraulic reserve ( $R_{opt}$ ) using the following formula.

Method I	Method II
$S_p$ $= \left( \frac{\frac{1}{8 \cdot \alpha} \eta \cdot \rho \cdot Q_d^3 \cdot T \cdot p \cdot 10^{-3} \cdot S_v' \cdot \lambda \cdot \left( 2.5 + \frac{2}{\ln 10 \cdot \sqrt{0.0106}} \right)}{MC_t \cdot k_o^{3.5}} \right)^{\frac{2}{7}}$	$R_{opt}$ $= \sqrt{\frac{g \cdot \eta \cdot T \cdot Q_d^3 \cdot n^2 \cdot (5 + 2 \cdot y) \cdot \phi \cdot p_{z\bar{e}v}}{2 \cdot r \cdot MC_t \cdot \delta^2}}$
$D = 24.62$	$p_{value} = 6.4 \text{ ALL/KW}$
$MC_t = 9849 \text{ ALL/m}^2/\text{m}$	$MC_t = 9189 \text{ ALL/m}^2/\text{m}$

$Q_{ll} = 125 \text{ m}^2$	$Q_{ll} = 125 \text{ m}^2$
$S_{opt} = 74.51 \text{ m}^2$	$S_{opt} = 65.46 \text{ m}^2$
$D_{opt} = 9.75 \text{ m}$	$D_{opt} = 9.15 \text{ m}$
$Q_{ll} = 250 \text{ m}^2$	$Q_{ll} = 250 \text{ m}^2$
$S_{opt} = 134.58 \text{ m}^2$	$S_{opt} = 115.42 \text{ m}^2$
$D_{opt} = 13.10 \text{ m}$	$D_{opt} = 12.15 \text{ m}$

### Comparison of methods

The other method calculates  $S_{opt}$  with the following formula.

$$R_{opt} = \sqrt{\frac{g \cdot \eta \cdot T \cdot Q_{ll}^3 \cdot n^2 \cdot (5 + 2 \cdot y) \cdot \phi \cdot p_{zev}}{2 \cdot r \cdot MC_t \cdot \delta^2}}$$

$\eta$  – Turbine efficiency  $\eta = 0.8$

HEP operating hours

$Q_{ll}$  – Design flow of the HPP

$n$  – severity coefficient  $n = 0.013$

$y$  – power factor in the Chezi coefficient formula  $y = 1/6$

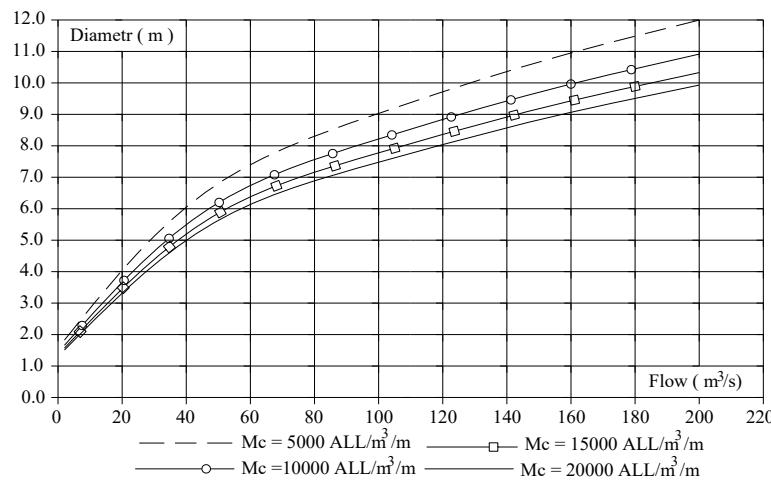
$\phi$  – Loss coefficient  $\phi = 1.15$

$p_{zev}$  – Price of replacement energy.  $p_{zev} = 6.4 \text{ ALL/kc}$

$r$  – Production costs  $r = 0.02$

$MC_t$  – marginal cost of the tunnel

$\delta$  – Geometric parameter.  $\delta = \sqrt{S}/R = 2 \cdot \sqrt{\pi}$



### Value based on fluctuation analysis

Since the parameters we have accepted are not standard, we must multiply the optimal surface area by a correction coefficient to change a parameter.

$$S_{p,k} = S_p \cdot k_D \cdot k_T \cdot k_p \cdot k_l$$

$S_{p,k}$  – Corrected surface

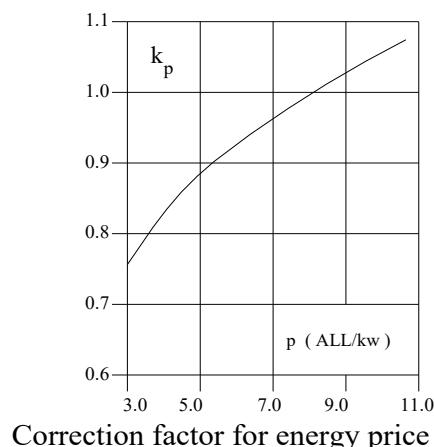
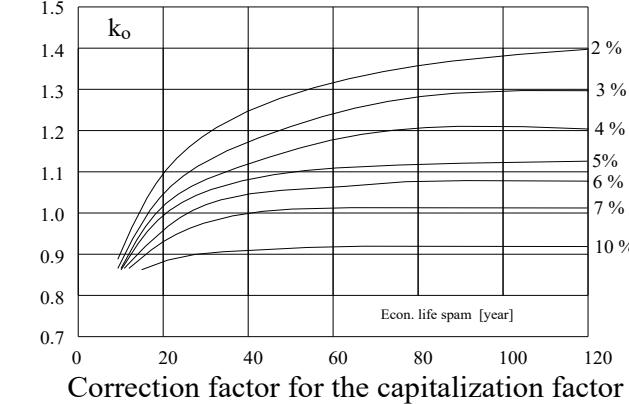
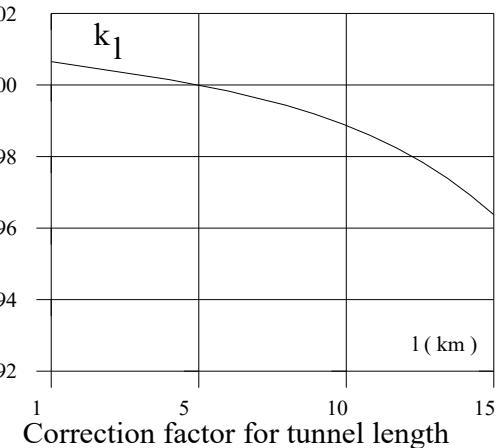
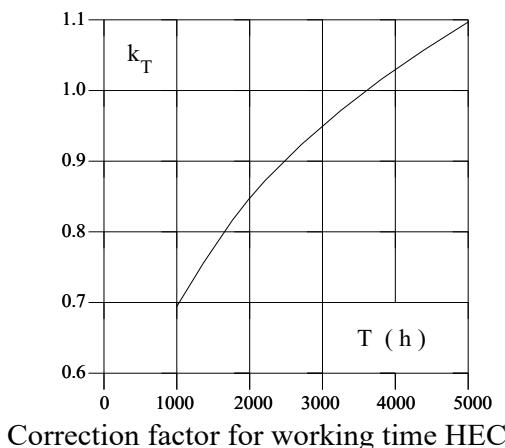
$S_p$  – The area previously obtained from the graph  $S_{opt} = f(Q)$ .

$k_D$  – Correction coefficient for the change in the capitalization factor.

$k_T$  – Correction coefficient for the change in the operating time of the HPP .

$k_p$  – Correction coefficient for the change in the price of energy.

$k_l$  – Correction coefficient for the change in tunnel length.



### Conclusions:-

To determine the economic area of the tunnel cross-section, the parameters of hydraulic losses resulting from friction and the total cost of tunnel construction are used.

From which study do we draw these conclusions?

#### The economic section of the hydrotechnical tunnel depends most significantly on these factors:

- Internal tunnel pressure.
- Rock characteristics.
- Energy price.
- Bank interest rate.
- Inflation rate.
- The optimal diameter of the economic section of the tunnel for the characteristic flows of the HPPs to be built  $Q = 2 \div 200 \text{ m}^3/\text{s}$  is  $D_{opt} = 1.6 \div 12.0 \text{ m}$  .
- The economic velocities for these tunnels with these flows are  $V_{ek} = 1.0 \div 1.8 \text{ m/s}$ .
- The change in the optimal section from a doubling energy price is 17.9%.

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