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RESEARCH ARTICLE

STATISTICAL PROPERTIES AND SIMULATION-BASED BAYESIAN INFERENCE FOR ODD CHEN–GAMMA DISTRIBUTION USING TWO PRIOR FUNCTIONS

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Abstract

In this study, a new two-parameters lifetime distribution called the Odd Chen Gamma distribution is proposed .The probability density function cumulative distribution function, reliability function, and some statistic al important properties of this distribution were derived.Bayesian analysis was carried out using two types of prior distributions for the unknown parameter: informative and non-informative prior.The estimator of the shape parameter was derived under both the Squared Error Loss and Weighted Loss functions.A simulation experiment was carried out to assess the efficiency of the Bayesian estimators by examining their mean squared error under different configurations of parameter values and varying sample sizes.

Introduction:-

The Odd Chen–Gamma (OC-Gamma) distribution is a newly proposed lifetime model that combines the flexibility of the classical Gamma distribution with the shape-enhancing capability of the Odd Chen transformation. This transformation allows the model to capture various hazard rate behaviors, making it suitable for reliability and survival analysis [1],[2],[3]. Despite its desirable properties and recent interest in Odd Chen-generated families, the Bayesian estimation of the OC-Gamma model has not yet been explored in the literature. This represents a significant gap, especially when compared to other related models like the Chen, Exponentiated Generalized Chen, and Odd Generalized Exponential Chen distributions, which have been widely analyzed under Bayesian frameworks [4],[5],[6]. Bayesian estimation methods have gained popularity in reliability studies due to their flexibility in incorporating prior knowledge and handling complex models. These methods have been successfully applied in contexts such as record values, stress-strength models, and system reliability under various lifetime distributions [7],[8],[9]. The Odd Chen transformation itself has given rise to several generalized distribution families including the Odd Chen–Exponential, Odd Chen–Fre´chet, and Odd Burr types which have been effectively utilized in fields such as environmental studies, engineering reliability, and industrial modeling [10],[11],[12],[13]. In this work, estimate the shape parameter α of the OC-Gamma distribution using Bayesian method, assuming a fixed scale parameter $\beta=2$. Our goals are to derive the some important functions for new distribution and some statistical properties, implement Bayesian estimation via appropriate loss functions, and evaluate the performance of the proposed estimators through simulation analysis[13],[14],[16].

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The Odd Chen–Gamma Distribution:-
probability density function (PDF):-
The probability density function p.d.f of the OC–Gamma distribution with shape parameter α>0 and scale parameter β>0 is defined on x > 0 as:

$$f(x;\alpha,\beta)=\frac{\alpha\beta[\frac{1}{\Gamma(\alpha)}\gamma(\alpha,\frac{x}{\beta})]^{\beta-1}\frac{x^{\alpha-1}e^{-\frac{x}{\beta}}}{\beta^{\alpha}\Gamma(\alpha)}}{[1-\frac{1}{\Gamma(\alpha)}\gamma(\alpha,\frac{x}{\beta})]^{\beta+1}}\left[e^{\left[\frac{\frac{1}{\Gamma(\alpha)}\gamma(\alpha,\frac{x}{\beta})}{1-\frac{1}{\Gamma(\alpha)}\gamma(\alpha,\frac{x}{\beta})}\right]^{\beta}}\right]e^{-\alpha\left[e^{\left[\frac{\frac{1}{\Gamma(\alpha)}\gamma(\alpha,\frac{x}{\beta})}{1-\frac{1}{\Gamma(\alpha)}\gamma(\alpha,\frac{x}{\beta})}\right]^{\beta}}-1\right]} \quad \ldots(1)$$

where:

- $\frac{1}{\Gamma(\alpha)}\gamma(\alpha,\frac{x}{\beta})$ is the lower incomplete gamma CDF, i.e., the CDF of a Gamma distribution.
- $\frac{x^{\alpha-1}e^{-\frac{x}{\beta}}}{\beta^{\alpha}\Gamma(\alpha)}$ is the standard Gamma p.d.f.

Cumulative Distribution Function (CDF):-
The cumulative distribution function CDF of the OC–Gamma distribution is given:-

$$F(x;\alpha,\beta)=1-e^{-\alpha\left[e^{\left[\frac{\frac{1}{\Gamma(\alpha)}\gamma\left(\alpha,\frac{x}{\beta}\right)}{1-\frac{1}{\Gamma(\alpha)}\gamma\left(\alpha,\frac{x}{\beta}\right)}\right]^{\beta}}-1\right]} \quad \ldots(2)$$

Reliability Function:-
The reliability function R(x) gives the probability that a random variable X exceeds a certain value x:

$$R(x;\alpha,\beta)=e^{-\alpha\left[e^{\left[\frac{\frac{1}{\Gamma(\alpha)}\gamma\left(\alpha,\frac{x}{\beta}\right)}{1-\frac{1}{\Gamma(\alpha)}\gamma\left(\alpha,\frac{x}{\beta}\right)}\right]^{\beta}}-1\right]} \quad \ldots(3)$$

where F(x) is the cumulative distribution function.

Hazard Function:-
For any random variable X which follows the Odd Chen–Gamma distribution, its hazard function is given as:

$$h(x;\alpha,\beta)=\frac{\alpha\beta[\frac{1}{\Gamma(\alpha)}\gamma(\alpha,\frac{x}{\beta})]^{\beta-1}\frac{x^{\alpha-1}e^{-\frac{x}{\beta}}}{\beta^{\alpha}\Gamma(\alpha)}}{[1-\frac{1}{\Gamma(\alpha)}\gamma(\alpha,\frac{x}{\beta})]^{\beta+1}}\left[e^{\left[\frac{\frac{1}{\Gamma(\alpha)}\gamma(\alpha,\frac{x}{\beta})}{1-\frac{1}{\Gamma(\alpha)}\gamma(\alpha,\frac{x}{\beta})}\right]^{\beta}}\right] \ldots(4)$$

Some statistical:-
Properties of the Odd Chen-Gamma Distribution:-
Quantile Function For the OC–Gamma distribution, the quantile function (inverse CDF) returns the value x such that:

$$F(x;\alpha,\beta)=u,\quad 0<u<1.$$

In other words,

$$Q(u)=F^{-1}(u)\Rightarrow F(Q(u);\alpha,\beta)=u$$

From our derivation, the closed form quantile function is

$$x=Q(u)=\beta\gamma^{-1}\left(\alpha,\Gamma(\alpha)\frac{\sqrt{\ln(1-\frac{1}{\alpha}\ln(1-u))}}{1+\sqrt{\ln(1-\frac{1}{\alpha}\ln(1-u))}}\right),\quad 0<u<1. \quad \ldots(5)$$

Moment Generating Function (MGF):-
Let X be a random variable that follows the OC–Gamma distribution, then, the moment generating function MGF of X as:

$$M_x(t) = E[e^{tx}] = \int_0^\infty e^{tx}\, f(x;\alpha,\beta)\, dx$$

$$M_x(t) = \alpha\beta e^{\alpha} \sum_{j=0}^{\infty} \frac{(-\alpha)^j}{j!} \int_0^\infty e^{\left[tG^{-1}\left(\frac{t}{1+t}\right)\right]}\, te^{(j+1)t^\beta} \quad dt \ldots (6)$$

Raw Moments:-
Let X denote a random variable that follows the OC–Gamma distribution. Then, the r^{th} order moment about the origin, denoted by μ'_r is given by:

$$\mu'_r = E[X^r] = \int_0^\infty X^r\, f(x;\alpha,\beta)\, dx$$

$$\mu'_r = E(x^r) = \alpha\beta e^{\alpha} \sum_{j=0}^{\infty} \frac{(-\alpha)^j}{j!} \int_0^\infty \left[G^{-1}\left(\frac{t}{1+t}\right)\right]^r te^{(j+1)t^\beta} \quad dt \qquad \ldots (7)$$

$$\mu'_1 = E(x^1) = \alpha\beta e^{\alpha} \sum_{j=0}^{\infty} \frac{(-\alpha)^j}{j!} \int_0^\infty \left[G^{-1}\left(\frac{t}{1+t}\right)\right]^1 te^{(j+1)t^\beta} \quad dt \qquad \ldots (8)$$

$$\mu'_2 = E(x^2) = \alpha\beta e^{\alpha} \sum_{j=0}^{\infty} \frac{(-\alpha)^j}{j!} \int_0^\infty \left[G^{-1}\left(\frac{t}{1+t}\right)\right]^2 te^{(j+1)t^\beta} \quad dt \qquad \ldots (9)$$

$$\mu'_3 = E(x^3) = \alpha\beta e^{\alpha} \sum_{j=0}^{\infty} \frac{(-\alpha)^j}{j!} \int_0^\infty \left[G^{-1}\left(\frac{t}{1+t}\right)\right]^3 te^{(j+1)t^\beta} \quad dt \qquad \ldots (10)$$

$$\mu'_4 = E(x^4) = \alpha\beta e^{\alpha} \sum_{j=0}^{\infty} \frac{(-\alpha)^j}{j!} \int_0^\infty \left[G^{-1}\left(\frac{t}{1+t}\right)\right]^4 te^{(j+1)t^\beta} \quad dt \qquad \ldots (11)$$

Central Moments:-
Let X denote a random variable that follows the Odd Chen–Gamma distribution. Then, the moments about the mean (i.e., the central moments), and the r^th order moment about the origin, denoted by μ_r and 〖μ^'〗 _r respectively, are given by:

$$\mu'_r = E[X^r], \mu_r = E[(X-\mu)^r]$$

$$\mu_2 = \alpha\beta e^{\alpha} \sum_{j=0}^{\infty} \frac{(-\alpha)^j}{j!} I_2(j) - \left[\alpha\beta e^{\alpha} \sum_{j=0}^{\infty} \frac{(-\alpha)^j}{j!} I_1(j)\right]^2 \qquad \ldots (12)$$

$$\mu_3 = \mu'_3 - 3\mu'_2\mu'_1 + 2(\mu'_1)^3$$

$$= \alpha\beta e^{\alpha} \sum_{j=0}^{\infty} \frac{(-\alpha)^j}{j!} I_3(j) - 3\left(\alpha\beta e^{\alpha} \sum_{j=0}^{\infty} \frac{(-\alpha)^j}{j!} I_2(j)\right)\left(\alpha\beta e^{\alpha} \sum_{j=0}^{\infty} \frac{(-\alpha)^j}{j!} I_1(j)\right) + 2\left(\alpha\beta e^{\alpha} \sum_{j=0}^{\infty} \frac{(-\alpha)^j}{j!} I_1(j)\right)^3 \qquad \ldots (13)$$

$$\mu_4 = \mu'_4 - 4\mu'_3\mu'_1 + 6\mu'_2(\mu'_1)^2 - 3(\mu'_1)^4$$

$$= \alpha\beta e^{\alpha} \sum_{j=0}^{\infty} \frac{(-\alpha)^j}{j!} I_4(j) - 4\left(\alpha\beta e^{\alpha} \sum_{j=0}^{\infty} \frac{(-\alpha)^j}{j!} I_3(j)\right)\left(\alpha\beta e^{\alpha} \sum_{j=0}^{\infty} \frac{(-\alpha)^j}{j!} I_1(j)\right) + 6\left(\alpha\beta e^{\alpha} \sum_{j=0}^{\infty} \frac{(-\alpha)^j}{j!} I_2(j)\right)\left(\alpha\beta e^{\alpha} \sum_{j=0}^{\infty} \frac{(-\alpha)^j}{j!} I_1(j)\right)^2 - 3\left(\alpha\beta e^{\alpha} \sum_{j=0}^{\infty} \frac{(-\alpha)^j}{j!} I_1(j)\right)^4 \qquad \ldots (14)$$

The mean and variance as:

$$\mu = \alpha\beta e^{\alpha} \sum_{j=0}^{\infty} \frac{(-\alpha)^j}{j!} \int_0^{\infty} \left[G^{-1}\left(\frac{t}{1+t}\right)\right]^1 \; te^{(j+1)t^{\beta}} \quad dt \qquad \dots(15)$$

$$\sigma^2 = \alpha\beta e^{\alpha} \sum_{j=0}^{\infty} \frac{(-\alpha)^j}{j!} \int_0^{\infty} \left[G^{-1}\left(\frac{t}{1+t}\right)\right]^2 \; te^{(j+1)t^{\beta}} \quad dt \qquad \dots(16)$$

Skewness and Kurtosis:-

The coefficient of skewness for the OC–Gamma distribution is given by:

$$S.K = \frac{\mu_3}{(\mu_2)^{3/2}}$$

$$= \frac{\alpha\beta e^{\alpha} \sum_{j=0}^{\infty} \frac{(-\alpha)^j}{j!} I_3(j) - 3\left(\alpha\beta e^{\alpha} \sum_{j=0}^{\infty} \frac{(-\alpha)^j}{j!} I_2(j)\right)\left(\alpha\beta e^{\alpha} \sum_{j=0}^{\infty} \frac{(-\alpha)^j}{j!} I_1(j)\right) + 2\left(\alpha\beta e^{\alpha} \sum_{j=0}^{\infty} \frac{(-\alpha)^j}{j!} I_1(j)\right)^3}{\left(\alpha\beta e^{\alpha} \sum_{j=0}^{\infty} \frac{(-\alpha)^j}{j!} I_2(j) - \left[\alpha\beta e^{\alpha} \sum_{j=0}^{\infty} \frac{(-\alpha)^j}{j!} I_1(j)\right]^2\right)^{3/2}} \qquad \dots(\mathbf{17})$$

Coefficient of Variation:-

The coefficient of variation for the OC–Gamma distribution is given by:

$$CV = \frac{\sigma}{\mu} = \frac{\sqrt{\alpha\beta e^{\alpha} \sum_{j=0}^{\infty} \frac{(-\alpha)^j}{j!} \int_0^{\infty} \left[G^{-1}\left(\frac{t}{1+t}\right)\right]^2 \; te^{(j+1)t^{\beta}} \quad dt}}{\alpha\beta e^{\alpha} \sum_{j=0}^{\infty} \frac{(-\alpha)^j}{j!} \int_0^{\infty} \left[G^{-1}\left(\frac{t}{1+t}\right)\right]^1 \; te^{(j+1)t^{\beta}} \quad dt} \qquad \dots(18)$$

where σ is standard deviation and μ is mean of the distribution

Bayesian Parameter Estimation:-

Consider a random x_1,x_2,…,x_ndrawn independently from the Odd Chen-Gamma distribution with probability density function f(x|α,β)), where α is a shape parameter and β is a scale parameter. In this Bayesian analysis, the scale parameter β is assumed to be known equal 2, while the shape parameter α is to be estimated based on the observed data. The posterior distribution of α is given by[17]:

$$p(\alpha|x) = \frac{L\left(\underline{x}|\alpha\right) q(\alpha)}{\int_0^{\infty} L\left(\underline{x}|\alpha\right) q(\alpha)\, d\alpha}$$

Where:

- $L\left(\underline{x}|\alpha\right) = \prod_{j=1}^{n} f(x_j|\alpha)$is the likelihood function.
- $q(\alpha)$is the prior distribution for the shape parameter α.

By fixing β, we simplify the estimation and focus the Bayesian analysis on the behavior of α alone.

Likelihood Function:-

The joint probability of the observed data, provided α, serves as the basis for the likelihood function:

$$L\left(\underline{x}|\alpha\right) = \prod_{j=1}^{n} f(x_j|\alpha)$$

$$L\left(\underline{x}|\alpha\right) = \alpha^n 2^{n-\alpha n} (\Gamma\alpha)^{-2n} \left(\prod_{j=1}^{n} x_j\right)^{\alpha-1} e^{-\alpha \sum_{j=1}^{n}\left(e^{u_j^2}-1\right)} e^{\sum_{j=1}^{n} u_j^2} \left(\prod_{j=1}^{n} \frac{\gamma\left(\alpha,\frac{x}{2}\right)}{\left[1-\frac{1}{\Gamma(\alpha)}\gamma\left(\alpha,\frac{x}{2}\right)\right]^3}\right) \dots(20)$$

Where:-

$$u = \frac{\frac{1}{\Gamma(\alpha)}\gamma\left(\alpha,\frac{x}{2}\right)}{1-\frac{1}{\Gamma(\alpha)}\gamma\left(\alpha,\frac{x}{2}\right)}$$

The posterior distribution:-
The posterior distribution under gamma prior[16]:
The gamma prior is defined as:

$$q_1(\alpha)=\frac{b^a}{\Gamma a}\alpha^{a-1}e^{-b\alpha}\quad ,\alpha>0\ ,a,b>0 \qquad \dots(21)$$

To find the posterior distribution under gamma prior we substitute the equation (20) and equation (21) we get:

$$p_1(\alpha|x)=\frac{\alpha^{n+a-1}2^{n-\alpha n}(\Gamma\alpha)^{-2n}\left(\prod_{j=1}^n x_j\right)^{\alpha-1}e^{-\alpha\left(b+\sum_{j=1}^n\left(e^{u_j^2}-1\right)\right)}e^{\sum_{j=1}^n u_j^2}\left(\prod_{j=1}^n\frac{\gamma\left(\alpha,\frac{x}{2}\right)}{\left[1-\frac{1}{\Gamma(\alpha)}\gamma\left(\alpha,\frac{x}{2}\right)\right]^3}\right)}{\int_0^\infty\alpha^{n+a-1}2^{n-\alpha n}(\Gamma\alpha)^{-2n}\left(\prod_{j=1}^n x_j\right)^{\alpha-1}e^{-\alpha\left(b+\sum_{j=1}^n\left(e^{u_j^2}-1\right)\right)}e^{\sum_{j=1}^n u_j^2}\left(\prod_{j=1}^n\frac{\gamma\left(\alpha,\frac{x}{2}\right)}{\left[1-\frac{1}{\Gamma(\alpha)}\gamma\left(\alpha,\frac{x}{2}\right)\right]^3}\right)d\alpha} \dots(22)$$

Squared Error Loss Function (SELF) [14]:
This loss function for this type defined as:

$$L(\hat{\alpha},\alpha)=(\hat{\alpha}-\alpha)^2\to\hat{\alpha}_{S_1}=E[\alpha|x]=\int_0^\infty\alpha\,p_1(\alpha|x)d\alpha$$

Then the Bayes estimator as:

$$\hat{\alpha}_{S_1}=\int_0^\infty\alpha\left(\frac{\alpha^{n+a-1}2^{n-\alpha n}(\Gamma\alpha)^{-2n}\left(\prod_{j=1}^n x_j\right)^{\alpha-1}e^{-\alpha\left(b+\sum_{j=1}^n\left(e^{u_j^2}-1\right)\right)}e^{\sum_{j=1}^n u_j^2}\left(\prod_{j=1}^n\frac{\gamma\left(\alpha,\frac{x}{2}\right)}{\left[1-\frac{1}{\Gamma(\alpha)}\gamma\left(\alpha,\frac{x}{2}\right)\right]^3}\right)}{\int_0^\infty\alpha^{n+a-1}2^{n-\alpha n}(\Gamma\alpha)^{-2n}\left(\prod_{j=1}^n x_j\right)^{\alpha-1}e^{-\alpha\left(b+\sum_{j=1}^n\left(e^{u_j^2}-1\right)\right)}e^{\sum_{j=1}^n u_j^2}\left(\prod_{j=1}^n\frac{\gamma\left(\alpha,\frac{x}{2}\right)}{\left[1-\frac{1}{\Gamma(\alpha)}\gamma\left(\alpha,\frac{x}{2}\right)\right]^3}\right)d\alpha}\right)d\alpha\dots(23)$$

Weighted Loss Function (WLF)[18]:
The weighted loss function defined as:

$$L(\hat{\alpha},\alpha)=\frac{(\hat{\alpha}-\alpha)^2}{\alpha}\to\hat{\alpha}_{W_1}=\frac{1}{E(\alpha^{-1}|x)}$$

Then the Bayes estimator under this loss function as:

$$\hat{\alpha}_{W_1}=\frac{1}{\int_0^\infty\alpha^{-1}\left(\frac{\alpha^{n+a-1}2^{n-\alpha n}(\Gamma\alpha)^{-2n}\left(\prod_{j=1}^n x_j\right)^{\alpha-1}e^{-\alpha\left(b+\sum_{j=1}^n\left(e^{u_j^2}-1\right)\right)}e^{\sum_{j=1}^n u_j^2}\left(\prod_{j=1}^n\frac{\gamma\left(\alpha,\frac{x}{2}\right)}{\left[1-\frac{1}{\Gamma(\alpha)}\gamma\left(\alpha,\frac{x}{2}\right)\right]^3}\right)}{\int_0^\infty\alpha^{n+a-1}2^{n-\alpha n}(\Gamma\alpha)^{-2n}\left(\prod_{j=1}^n x_j\right)^{\alpha-1}e^{-\alpha\left(b+\sum_{j=1}^n\left(e^{u_j^2}-1\right)\right)}e^{\sum_{j=1}^n u_j^2}\left(\prod_{j=1}^n\frac{\gamma\left(\alpha,\frac{x}{2}\right)}{\left[1-\frac{1}{\Gamma(\alpha)}\gamma\left(\alpha,\frac{x}{2}\right)\right]^3}\right)d\alpha}\right)d\alpha}\dots(24)$$

The posterior distribution under Hypothetical prior [15]:
The Hypothetical prior used here is:

$$q_2(\alpha)=k\alpha^2\quad ,\alpha>0\quad ,\quad k>0\ \textit{positive number}\dots(25)$$

To find the posterior distribution under this prior we substitute the equation (20) and equation (25) we get:

$$p_2(\alpha|x)=\frac{k\,\alpha^{2+n}2^{n-\alpha n}(\Gamma\alpha)^{-2n}\left(\prod_{j=1}^n x_j\right)^{\alpha-1}e^{-\alpha\left(\sum_{j=1}^n\left(e^{u_j^2}-1\right)\right)}e^{\sum_{j=1}^n u_j^2}\left(\prod_{j=1}^n\frac{\gamma\left(\alpha,\frac{x}{2}\right)}{\left[1-\frac{1}{\Gamma(\alpha)}\gamma\left(\alpha,\frac{x}{2}\right)\right]^3}\right)}{\int_0^\infty k\,\alpha^{2+n}\,2^{n-\alpha n}(\Gamma\alpha)^{-2n}\left(\prod_{j=1}^n x_j\right)^{\alpha-1}e^{-\alpha\left(\sum_{j=1}^n\left(e^{u_j^2}-1\right)\right)}e^{\sum_{j=1}^n u_j^2}\left(\prod_{j=1}^n\frac{\gamma\left(\alpha,\frac{x}{2}\right)}{\left[1-\frac{1}{\Gamma(\alpha)}\gamma\left(\alpha,\frac{x}{2}\right)\right]^3}\right)d\alpha}\dots(26)$$

Squared Error Loss Function (SELF):

$$\hat{\alpha}_{S_2} = \int_0^{\infty} \alpha \left(\frac{k\,\alpha^{2+n} 2^{n-\alpha n} (\Gamma\alpha)^{-2n} \left(\prod_{j=1}^{n} x_j\right)^{\alpha-1} e^{-\alpha\left(\sum_{j=1}^{n}\left(e^{u_j^2}-1\right)\right)} e^{\sum_{j=1}^{n} u_j^2} \left(\prod_{j=1}^{n} \frac{\gamma\left(\alpha,\frac{x}{2}\right)}{\left[1-\frac{1}{\Gamma(\alpha)}\gamma\left(\alpha,\frac{x}{2}\right)\right]^3}\right)}{\int_0^{\infty} k\,\alpha^{2+n} 2^{n-\alpha n} (\Gamma\alpha)^{-2n} \left(\prod_{j=1}^{n} x_j\right)^{\alpha-1} e^{-\alpha\left(\sum_{j=1}^{n}\left(e^{u_j^2}-1\right)\right)} e^{\sum_{j=1}^{n} u_j^2} \left(\prod_{j=1}^{n} \frac{\gamma\left(\alpha,\frac{x}{2}\right)}{\left[1-\frac{1}{\Gamma(\alpha)}\gamma\left(\alpha,\frac{x}{2}\right)\right]^3}\right) d\alpha} \right) d\alpha \ldots(27)$$

Weighted Loss Function (WLF):

$$\hat{\alpha}_{W_2} = \frac{1}{\int_0^{\infty} \alpha^{-1} \left(\frac{k\,\alpha^{2+n} 2^{n-\alpha n} (\Gamma\alpha)^{-2n} \left(\prod_{j=1}^{n} x_j\right)^{\alpha-1} e^{-\alpha\left(\sum_{j=1}^{n}\left(e^{u_j^2}-1\right)\right)} e^{\sum_{j=1}^{n} u_j^2} \left(\prod_{j=1}^{n} \frac{\gamma\left(\alpha,\frac{x}{2}\right)}{\left[1-\frac{1}{\Gamma(\alpha)}\gamma\left(\alpha,\frac{x}{2}\right)\right]^3}\right)}{\int_0^{\infty} k\,\alpha^{2+n} 2^{n-\alpha n} (\Gamma\alpha)^{-2n} \left(\prod_{j=1}^{n} x_j\right)^{\alpha-1} e^{-\alpha\left(\sum_{j=1}^{n}\left(e^{u_j^2}-1\right)\right)} e^{\sum_{j=1}^{n} u_j^2} \left(\prod_{j=1}^{n} \frac{\gamma\left(\alpha,\frac{x}{2}\right)}{\left[1-\frac{1}{\Gamma(\alpha)}\gamma\left(\alpha,\frac{x}{2}\right)\right]^3}\right) d\alpha} \right) d\alpha} \ldots(28)$$

Simulation procedure:-

A simulation was conducted according to the Monte-Carlo method using MATLAB 2024b. first a random set,{u_i:i=1…n}, of numbers were generated form a U(0,1) distribution. The values u_i were then used to get the sample x_i,i=1…n by solving the equation:-

$$F(x_i, \beta, \alpha) - u_i = 0$$

The sample was then used in the estimators to acquire the estimates for α for each estimation method.
This process was repeated 1000 times acquiring 1000 estimates $\hat{\alpha}_j, j = 1 \ldots 1000$ for each estimator.
Two types of priors were taken. Namely, gamma and non-informative. Each prior was taken with square error and weighted loss functions
The mean squared error for the values were then calculated from the formula:

$$MSE = \frac{\sum_{j=1}^{1000}\left(\hat{\alpha}_j - \alpha\right)^2}{1000}$$

The above value for each of the four estimators were then used to compare the performance between the two loss functions for each prior.The sample size was taken to be $n = 10,25,50,75,100$ with $\beta = 2$ and $\alpha = 0.5,1,2$

Table 1: MSE Values for $\alpha = 0.5$

n	**Gamma Prior**		
	S_1	W_1	**Best**
10	0.001947041	0.001730928	W_1
25	0.000985454	0.000924319	W_1
50	0.000720959	0.000693413	W_1
75	0.000637134	0.000619511	W_1
100	0.000638562	0.000625065	W_1
n	**Hypothetical prior**		
	S_2	W_2	**Best**
10	0.002949031	0.002570022	W_2
25	0.001206629	0.001129527	W_2
50	0.000812324	0.000781397	W_2
75	0.000693951	0.000674908	W_2
100	0.000681437	0.000667152	W_2

Table 2: MSE Values for $\alpha = 1$

n	Gamma Prior		
	S_1	W_1	**Best**
10	0.012963104	0.013469585	S_1
25	0.006042648	0.006199801	S_1
50	0.003494789	0.003565100	S_1
75	0.002801150	0.002845409	S_1
100	0.002498593	0.002531281	S_1
n	**Hypothetical prior**		
	S_2	W_2	**Best**
10	0.010692868	0.011024993	S_2
25	0.005225604	0.005356559	S_2
50	0.003105817	0.003169679	S_2
75	0.002551228	0.002592644	S_2
100	0.002311854	0.002342959	S_2

n	**Gamma Prior**		
	S_1	W_1	**Best**
10	0.003966372	0.003788024	W_1
25	0.001691149	0.001621639	W_1
50	0.001116442	0.001083357	W_1
75	0.000925629	0.000901193	W_1
100	0.000703765	0.000686215	W_1
n	**Hypothetical prior**		
	S_2	W_2	**Best**
10	0.005352269	0.004929726	W_2
25	0.002059418	0.001958847	W_2
50	0.001270677	0.001230471	W_2
75	0.001033513	0.001005926	W_2
100	0.000779563	0.000760285	W_2

Table 3: MSE Values for $\alpha = 2$

n

Conclusions:-

This research focused on estimating the shape parameter α of the Odd Chen–Gamma distribution using the Bayesian approach, with a fixed scale parameter $\beta = 2$. Two prior distributions were considered: the informative Gamma prior and a non-informative prior. Bayesian estimators were compared under the Squared Error and Weighted Loss functions.A simulation study was carried out using nine experiments with different sample sizes ($n = 10, 25, 50, 75, 100$), each repeated 1000 times. The results showed that the Weighted Loss function performed better when α was small (0.5 and 1), while the Squared Error Loss was more accurate for $\alpha = 2$in Hypothetical priorsalso improved estimation when prior knowledge was appropriate.These findings emphasize the importance of selecting the right loss function and prior distribution in Bayesian inference and suggest possible extensions to censored data or more advanced hierarchical models in future studies.

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