*RESEARCH ARTICLE*

# FOURIER TRANSFORM ANALYSIS USING QUANTUM COMPUTATION METHOD

**Surya P[1], U. Pavan Kumar [2], Ch. Arun Prakash[1], Anjaiah Vasipalle[3] and D. Syambabu[1]**

1. Assistant Professor, Department of ECE, RISE Krishna Sai Prakasam Group of Institutions, Andhra Pradesh.
2. Associate Professor, Department of CSE - Data Science, Dayananda Sagar University, Bangalore, Karnataka.
3. Associate Professor, Department of ECE,PBR Visvodaya Institute of Technology and Science, Andhra Pradesh.

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| *Manuscript Info* | *Abstract* |
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A basic signal processing approach to transforming time-domain signals into frequency-domain forms is the Fourier Transform (FT). It is extensively utilized in several fields, such as spectrum analysis, audio and video processing, and 5G networks. Effective frequency analysis is essential for channel optimization, signal reliability, and noise reduction in 5G networks.The FFT is appropriate for real-time applications since it greatly increases processing speed. However, the Quantum Fourier Transform makes use of quantum entanglement and superposition, providing exciting new developments for secure data transfer and high-speed communication in the future. Furthermore, both the Inverse Fast Fourier Transform (IFFT) and the Inverse Quantum Fourier Transform are used to correctly recreate the signal.Implementation of Quantum's real valued computing of quantum-based signal processing is also covered in the survey.

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## Introduction:-

Global communication has been completely transformed by the quick deployment of 5G networks, which provide extremely fast data transfer, widespread device connectivity, and low latency. Applications such as augmented reality, self-driving automobiles, and smart infrastructure are supported by these developments. To guarantee dependable, high-quality transmission, 5G systems' massive data generation necessitates the use of extremely effective signal processing techniques. A key component in signal processing, the Fourier Transform (FT) [1-2] converts signals between the time and frequency domains and is necessary for signal analysis, filtering, and compression.By providing significantly quicker computations in real-time signal analysis, optimization, and secure communication, the Quantum Fourier Transform (QFT) improves on traditional Fourier approaches. The limitations of conventional methods are revolutionaryly addressed by QFT[3], which combines quantum computing with signal processing. This development has the potential to revolutionize high-bandwidth data processing, improve cryptographic security, and speed up problem-solving in both the industrial and scientific sectors. Gaining an understanding of QFT principles is essential as we move closer to a future driven by quantum technology [4-6].

**Corresponding Author:-**Surya P
**Address:-**Assistant Professor, Department of ECE,RISE Krishna Sai Prakasam Group of Institutions, Andhra Pradesh.

**Literature Review:-**

Fourier analysis is based on a number of mathematical ideas, such as discrete mathematics, quantum computation, and linear algebra. The basis for matrix representations, which are crucial to quantum computing, is provided by linear algebra. The concepts needed to comprehend quantum state transformations are introduced by quantum physics, and discrete mathematics is essential to the application of Fourier Transforms in digital systems.By using the exponential term's symmetry and periodicity, the FFT is an optimized technique that effectively calculates the Discrete Fourier Transform (DFT). This approach significantly reduces the computing barrier from $O(n2)$ to $O(nlogn)$ by breaking the work down into smaller subproblems, making large-scale frequency analysis feasible. n The DFT is split into contributions from even-indexed samples ($x2n$) and contributions from odd-indexed samples ($x2n+1$) in order to attain this efficiency [7-8]. The computation becomes recursive when these are substituted into the DFT equation and the exponential term's periodicity is used.

It is possible to comprehend the QFT circuit by looking at how it is implemented for different numbers of qubits. A Hadamard gate, which offers an equal superposition of computing base states, is used to accomplish the QFT for a single qubitAs seen in Fig. 1, the QFT circuit for two qubits includes a Hadamard gate on the first qubit after a controlled phase rotation that adds a phase factor based on the state of the first qubit. In order to guarantee that its frequency domain representation is correctly encoded before to measurement, the second qubit likewise undergoes a Hadamard transformation [9-10].The Inverse Fourier Transform (IFT) is a fundamental mathematical method for reconstructing a time domain signal using its frequency-domain representation. Because it enables the recovery of the original data from altered information, it is essential to signal processing.The IFFT is used in classical computing to effectively calculate the IFT. The IFFT reassembles frequency-domain components in the time domain. Applications including data compression, image reconstruction, and digital signal processing make extensive use of this method. However, little distortions or noise may occasionally arise in the reconstructed signal due to limitations in numerical precision [11-13]
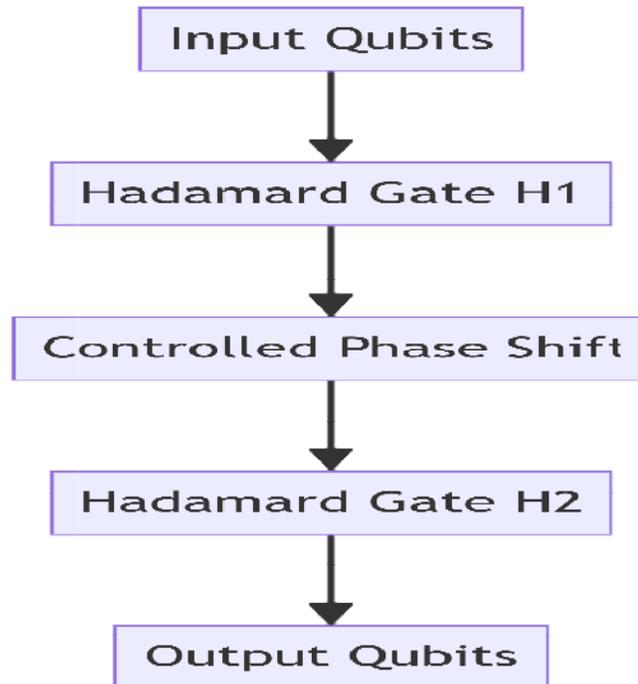


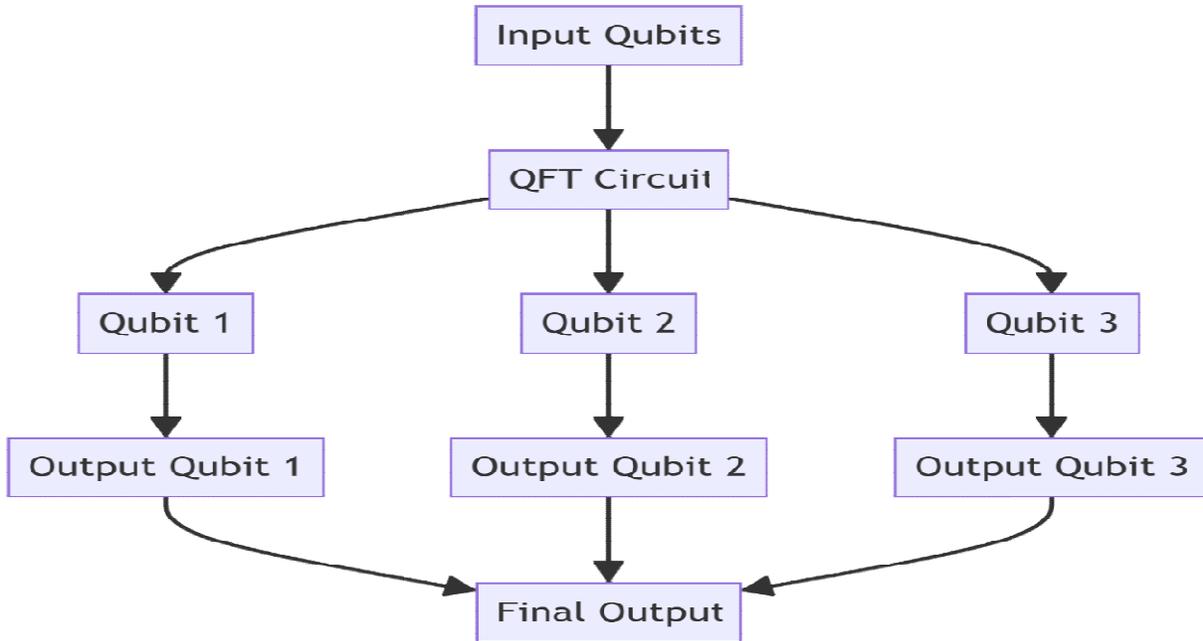**Fig1: Block representation for QFT through 2 qubits**

**Fig 2: Block representation for QFT through 3 qubits**

**Implementation Methods -**
Let's process N pictures, each with NxN pixels (for figure 3 shows N=8 method).
First, we examine the classical FFT's computational complexity for comparison.
Each sequence has an $O(N^2 \log_2 N^2)$ processing complexity, as stated in the introduction. Specifically, the FFT has a total calculation cost of $O(MN^2 \log_2 N^2)$ [14-15].

The classical data stored in the classical RAM must be encoded one by one in quantum states for the QFFT; this requires $O(MN^2)$ processes. To process the quantum sequence, the QFFT has a computational cost of $O(N^2 \log_2 N^2)$. As a result, $O(MN^2) + O(N^2 \log_2 N^2)$ is the entire complexity, including data encoding. Therefore, there isn't much of a benefit as long as we use the traditional RAM [16-17].
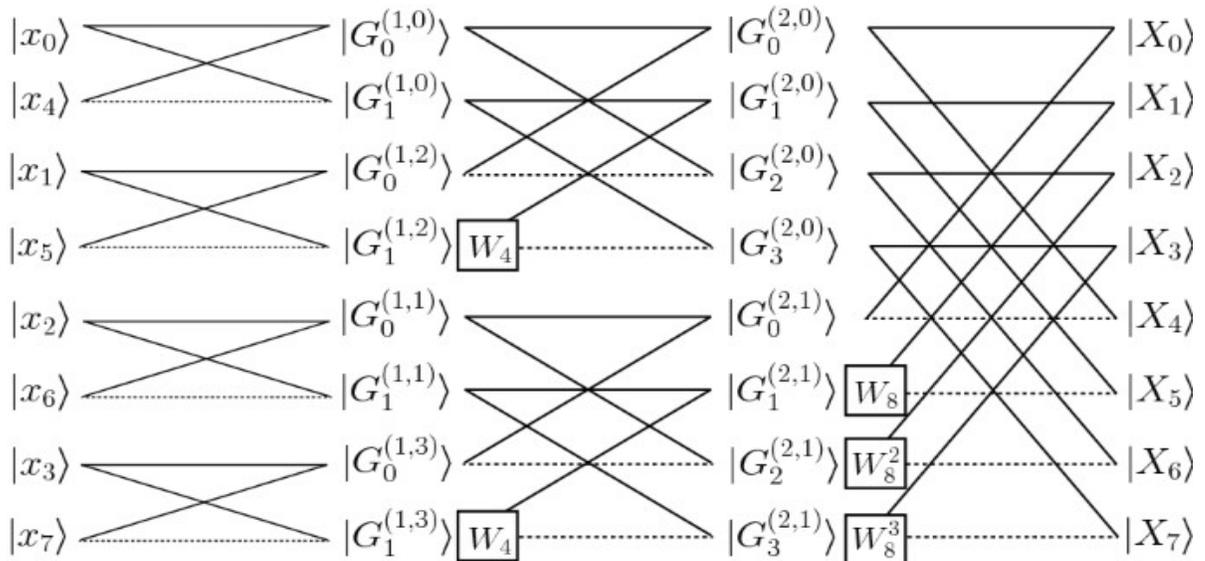


**Fig 3: N=8 point Computation of DIT-FFT method**

**Computational Complexity Of DFT, FFT and QFT:-**

| Method | Computational Order | Implementation |
|---|---|---|
| DFT | $O(N^2)$ | The direct computation of each frequency component results in quadratic complexity. |
| FFT | $O (NlogN$ | Redundancy was minimized by using recursive divide-and-conquer strategies. |
| QFT | $O(n^2)$ | Allows for possible exponential speedup by processing $n$ qubits, where $n = log(N)$. |

**Hardware Requirements for FFT Vs QFT:-**

| Aspects | FFT | QFT |
|---|---|---|
| Processing | CPUs, GPUs, TPUs | Qubits (Super Conducting) |
| Data | Binary Bits | Quatum States |
| Parllalism | Limitted by Hardware | Fully Parallel using Superposition |

**Applications of QFT:-**
QFT-based methods in modern signal and information processing.

**Mathematical Parallels Between QFT and Signal Processing:-**
Many of the tools in QFT have analogues in signal analysis, because both involve fields (functions of space and time) and use Fourier transforms, correlation functions, and spectral analysis [18].

**Quantum Signal Processing (QSP):-**
Although not QFT directly, Quantum Signal Processing borrows its name and some mathematical inspiration from field theory.It's a framework used in quantum computing to perform precise transformations of quantum states and many of these ideas stem from operator algebra and unitary evolution, both central in QFT [19].

**Applications in Imaging and Communications:-**
Treat electromagnetic signals as quantized fields (photons).Applied in optical communications, laser theory, and quantum imaging.Photon counting statistics → noise modeling and signal detection.Quantum Radar and Quantum Sensing:Uses entangled field states to improve signal-to-noise ratio (SNR).Described mathematically using QFT of the electromagnetic field.Medical Imaging (e.g., MRI, PET) [20].

**Emerging Research: QFT-Inspired Machine Learning:-**
There's growing research applying QFT frameworks to neural fields and signal representations:
Field Neural Networks (FNNs) and Neural Operators treat data as continuous fields.QFT-inspired regularization (e.g., renormalization-like coarse-graining) improves generalization and noise robustness in signal learning tasks [21].

**Future Directions in QFT Research:-**
Enhancements to uantum hardware include increased qubit count, coherence, gate fidelity, and error correction. Efficient QFT Circuits: Reducing gate count and depth by creating optimal and approximation QFT implementations. Combining QFT with traditional FFT for immediate practical use is known as hybrid integration. Expanded Uses: Utilizing QFT in advanced signal processing, chemistry, and quantum machine learning. Collaboration and education: promoting industrial relationships and multidisciplinary research. Topological Qubits: Investigating Majorana-based qubits for more stable QFT operations, such as Microsoft's method [22-23].

## Conclusion:-
In the context of signal processing and spectral analysis, this study evaluates the computing efficiency of many Fourier Transform methods, including DFT, FFT, and QFT. Although the fundamental framework is provided by the Discrete Fourier Transform (DFT), its O(N²) complexity makes it ineffective for processing vast amounts of data. With O (N log N) complexity, the Fast Fourier Transform (FFT) greatly increases efficiency, allowing for widespread application in traditional fields like image processing and telecommunications. A possible exponential speedup is provided by the Quantum Fourier Transform (QFT), which achieves O(n²) complexity, where n = $log$ ($N$) [24-25].

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