ISSN (O): 2320-5407
ISSN (P): 3107-4928

Journal Homepage: - www.journalijar.com

INTERNATIONAL JOURNAL OF ADVANCED RESEARCH (IJAR)

Article DOI: 10.21474/IJAR01/22989
DOI URL: http://dx.doi.org/10.21474/IJAR01/22989

RESEARCH ARTICLE

DYNAMIC MODELS OF SINGLE-LEVEL INVENTORY MANAGEMENT

Jean Marie Kabeya Lukusa[1], Rostin Mabela Makengo[2] and Jean Marie Kapenga Kazadi[3]

1. Basic Sciences Department, National Institute of Building and Public Works, D.R. Congo.
2. Mathematics, Statistics and Informatic Department, University of Kinshasa, D.R. Congo.
3. National Pedagogical University, Mathematics and Informatic Department, D.R. Congo.

……

Manuscript Info

Manuscript History
Received: 10 January 2026
Final Accepted: 12 February 2026
Published: March 2026

Key words:-
Inventory management, Dynamical model, stochastic model, classical Wilson model, total cost.

Abstract

Our research analyzes stochastic dynamic models applied to single-echelon inventory management, comparing them with classical models such as Wilson's and deterministic approaches. The study assumes that demand follows a log-normal distribution, an assumption supported by available data and validated through statistical goodness-of-fit tests. Several external factors influencing demand were integrated, including product price, substitute product price, and past demand observations. Empirical studies and numerical simulations indicate that stochastic dynamic models can reduce inventory management costs by 10–30% on average compared to traditional policies based on periodic replenishment or fixed thresholds, particularly in uncertain environments. Moreover, incorporating constraints close to real operating conditions allowed the identification of more effective management policies, especially in sectors such as pharmaceuticals and agri-food. The analysis shows that the selected external variables explain more than 90% of demand variations. Product and substitute prices influence demand at the 10% significance level, while past demand exerts a significant effect on future demand at the 1% level, thereby improving estimation accuracy.

……

Introduction:-

Scientific and practical context:-

Inventory management is a discipline that does not date from today; one must go back to the highest antiquity. Knowing that there will always be a gap between production and consumption (EL, Yamani, 2005), a gap that is unavoidable because, on the one hand, the place of production is different from that of consumption and, on the other hand, the period of production is different from that of consumption. It is thus that stocks and provisions are necessary to face this gap (EL, Yamani, 2005).In a context of increasing globalization and market volatility, effective inventory management is a strategic issue for companies. Procurement decisions must take into account not only uncertain future demand but also delivery times, storage costs, and potential stockouts. In this context, the stochastic dynamic inventory management model helps support operational decisions in order to optimize economic and logistical performance. However, classical models, often deterministic, do not capture the real complexity of

Corresponding Author:- Jean Marie Kabeya Lukusa
Address:-Basic sciences department, National Institute of Building and Public Works, D.R. Congo.

uncertain environments. This is why stochastic dynamic models (SDMs) have emerged as a promising approach for optimal decision-making in contexts characterized by uncertainty and evolution over time. (Powell, W.B. 2011).

Inventory management is an essential pillar of logistical performance in supply chains, particularly for critical products such as essential medicines. In the Democratic Republic of Congo (DRC), stockouts and prolonged delivery times remain frequent, directly impacting access to care and public health (WHO, 2022–2024). Traditionally, companies and institutions use classical inventory management models, such as the Wilson model or deterministic multi-period models. Although simple to implement, these models have limitations in the face of uncertainties in demand, ordering costs, and logistical constraints. In contexts such as the cities of Kinshasa andLubumbashi, where flows are subject to high variability and supply constraints, these models do not always guarantee a satisfactory service level. Recent advances in stochastic dynamic modeling and Monte Carlo simulation offer new perspectives for managing these uncertainties. They allow anticipating stockouts, optimizing inventory levels, and better balancing storage, ordering, and shortage costs (Powell, 2021; Babai et al., 2021 2024; Helmes et al., 2024).

Problem Statement Inventory:-

Management is a key aspect of operations research that involves deciding how much and when to order or produce goods to meet customer demand. However, demand is often uncertain and variable, which makes inventory management difficult and risky. This is why different models have been proposed to address it. Despite the existence of many theoretical models, few have been adapted to African contexts, particularly for pharmaceutical supply chains.

The following questions arise:

- Are classical models sufficient to ensure a high service level in an unstable context?
- How does taking into account the random variability of demand and costs improve logistics performance?
- Which method (deterministic dynamic, stochastic) offers the best trade-off between cost and resilience in these contexts?

Thus, the research aims to fill this gap by proposing a rigorous comparative analysis of classical and stochastic dynamic models applied to essential medicine supply chains in the DRC.

Mathematical formulation:

Literature Review:-

Inventory management has been the subject of extensive research since the early 20th century, particularly with the formalization of Wilson's model (1913), often referred to as the Economic Order Quantity (EOQ) model. This model constitutes the foundation of deterministic approaches, in which demand and system parameters are considered perfectly known (10,18,20).The first inventory management models were developed in an industrial context where demand stability allowed for deterministic reasoning. The classical Wilson-Harris (EOQ) model establishes a simple formula linking the optimal order quantity to ordering cost, holding cost, and annual demand. This model remains a major reference point in the literature [13]. In subsequent decades, the works of [9] and [16] extended the EOQ model by introducing capacity constraints, delivery lead times, and shortage costs. These authors laid the groundwork for dynamic models, enabling the consideration of temporal variations in inventory and orders.More recent approaches have incorporated sustainability and energy efficiency considerations [5], highlighting the need to adapt inventory management models to contemporary environmental and logistical challenges.Deterministic dynamic models are based on the idea that demand varies over time but remains perfectly known in advance. They aim to determine the quantities to order in each period to minimize total cost over a finite or infinite horizon ([21]; [14]).

Several optimal policies have been proposed:

- The method of [19], which uses dynamic programming to determine the optimal policy over a finite horizon.
- Heuristic approaches such as Part Period Balancing (PPB), Least Unit Cost (LUC), and Silver-Meal, introduced in the 1970s to simplify calculations [16].
- Capacity-constrained or nonlinear cost models studied by [3] and [11], which apply advanced mathematical optimization methods.

These models differ from stochastic approaches, in which demand or lead times are uncertain, but they remain fundamental for understanding replenishment policies.

The single-echelon model is the starting point of any logistical modeling: it assumes a single warehouse or ordering point, with deterministic downstream demand. According to [2], this type of model allows for precise analytical policies, often of the (Q, R) or (s, S) type. Multi-echelon models, on the other hand, generalize this structure by introducing several storage levels (supplier, depot, retailer). Although more complex, they often rely on the results of the single-echelon model to construct their equations [17].Recent literature emphasizes the digitization of supply chains, the integration of optimization technologies, and environmental sustainability. The contributions of [11] illustrate the evolution of deterministic dynamic models toward hybrid formulations integrating energy constraints, variable lead times, and carbon emission costs.Similarly, [1] proposed an integrated model combining determinism and flexibility, applying multi-objective optimization techniques.Recent research shows that inventory management models have evolved towards approaches that integrate uncertainty, nonlinear costs, and imperfections in logistical systems (Mittal et al., 2023; Thomas & Kumar, 2024). Furthermore, several empirical studies confirm that demand variability and logistical costs have an influence. [23] Shows the evolution of optimal quantities according to costs and demand. [24] Introduces the environmental dimension into inventory management.

Stochastic Optimization Problem [22]:-
We seek to minimize:-

$$\min_{\{Q_t\}} \mathbb{E}\left[\sum_{t=0}^{T} C\,(X_t, Q_t)\right]$$

Thus, the literature shows that the deterministic dynamic single-echelon inventory management model remains a fundamental tool, whose modern extensions make it possible to address the current challenges of global supply chains.

Conceptual and Theoretical Framework:-

Definitions of Key Concepts:-

- **Inventory (Stock):** the set of goods or raw materials held by a company awaiting use or sale [13].
- **Inventory Management:** the process of planning, organizing, and controlling stock levels to minimize costs and ensure availability [16].
- **Deterministic Model**: a mathematical model in which all parameters (demand, lead times, costs) are known with certainty.
- **Dynamic Model**: a model that incorporates the temporal dimension and represents inventory flows from one period to another.
- **Echelon**: a hierarchical level within a logistics system (e.g., warehouse, depot, point of sale).

Stochastic process:-

A stochastic process is a family of random variables defined over the same probability space: $\{X_t : t \in T\}$
with :

$$X: T \times \Omega \to E$$

where :

- Ω is the probability space,
- T is the set of indices (time),
- E is the state space.

In inventory management, the stochastic process is used to represent: the random demand of customers, lead times, and supply disruptions or fluctuations.Common Examples :Poisson Process (arrival of demands),Markov Chain (evolution of stock levels),Brownian Motion (advanced models) [25, 26].

Demand Modeling (Random Process)Demand is modeled by a sequence of random variables: $D_t \sim F$

Example with the Poisson Process [27, 28]:

$$P(N(t) = k) = \frac{(\lambda t)^k e^{-\lambda t}}{k!}$$

Relationship Between Model Parameters:-

In a single-echelon system, the inventory level (x_t) evolves according to the fundamental relation:

$$x_{t+1} = x_t + Q_t - D_t$$

where (Q_t is the order quantity and (D_t) is the demand in period (t).

The objective of the model is to determine the sequence ($Q_1, Q_2, \ldots, Q_T$) that minimizes the total cost:

$$C_T = \sum_{t=1}^{T}(C_o\delta_t + C_h I_t)$$

with (δ_t= 1) if an order is placed in period (t), and (0) otherwise.

Underlying Theory: Dynamic Optimization:-

The model relies on dynamic programming [4], which is based on the principle of optimality:

Theorem: “An optimal policy has the property that, regardless of the initial state and decision, the remaining decisions constitute an optimal policy for the state resulting from the first decision.”

The recurrence equation of the model proposed by [19] illustrates this principle:

$$C_t = \min_{j\geq t}\{C_o + \sum_{k=t}^{j} C_h(k-t)D_k + C_{j+1}\}$$

This approach makes it possible to determine the optimal ordering periods and quantities, ensuring a minimal total cost over the planning horizon.

Conceptual Scheme of the Single-Echelon Model:-

[Supplier] → (Order Q_t) → [Warehouse / Depot] → (Demand D_t) → [Final Customer]This scheme illustrates a single-echelon system, where the central depot manages stock inflows and outflows sequentially and deterministically.

Mathematical modelling of the deterministic Single-Echelon Model:

The model under study is situated within the framework of a single-echelon inventory system, where a manager must determine the optimal order quantity in each period in order to minimize the total inventory management costs over a given horizon.

Basic Assumptions:-

- **The model is deterministic**: demand $D_t$is known for each period t.
- **Stockouts are not allowed**, meaning demand must always be satisfied.

The costs considered are:

- Fixed ordering cost: K
- Unit purchasing cost: c
- Holding (or storage) cost per unit per period: h
- Decisions are made at discrete periods $t = 1,2,\ldots,T$.
- The inventory level at the end of period t is denoted x_t, and the order quantity in period t is denoted q_t.

This model follows the classical works of [9] and Wagner & [19], before being reformulated in a dynamic framework by [4] through the principle of optimality.

TABLE1: Table of symbols

SYMBOLS	Description
D_t	Request at time t
Q_t	Quantity ordered at period t
x_t	The stock status at the beginning of period t

K	Ordercost
c	Unit purchasecost
h	Unit storage cost (or ofpossession)
$C_t(x_t, Q_t)$	Total cost at period t

Cost Function:-
The total cost in period t is defined as:

$$C_t(\, x_t, Q_t) = h \cdot (x_t)^+ + p \cdot (x_t)^- + K \cdot 1_{\{a_t>0\}}$$

with the inventory balance constraint:

$x_{t+1} = x_t + Q_t - D_t$, t=1,...,T

Dynamic Formulation (Bellman'sPrinciple)
According to the principle of optimality [4], the opt$x_t$imal policy in each period depends only on the current state x_t of the system.

The minimal cost function is given by:

$$F_t(x_t) = \min_{Q_t \geq 0} \{C(x_t, Q_t) + \mathbb{E}[F_{t+1}(x_{t+1})]\}$$

with the terminal condition:

$F_{t+1}(\, x_{t+1})=0$

The optimal policy Q^*_t is the one that minimizes this equation for each t.
This general framework leads to policies of the (s, S) type or to the Economic Order Quantity (EOQ) policy in the stationary case ([21], [11], [15]).

Special Case: Deterministic EOQ Model
If demand is constant, i.e. D_t=D, the problem simplifies to:

$C(Q) = KD/Q + hQ/2 + cD$
where Q is the order quantity per cycle.
The Economic Order Quantity (EOQ) is then given by:

$$Q^* = \sqrt{\frac{2KD}{h}}$$

This classical formula remains consistent with the dynamic version when demand is stationary.

Materials and Methods:-

In the context of this research, we used a computer with the following specifications: 12th Gen Intel(R) Core (TM) i5-1235U 1.30 GHz, 16.0 GB RAM, 500 GB SSD, and Windows 11 OS to carry out our simulations using SageMath 9.2. The data were taken from the warehouse of the logistics center of the Phamakin company in Kinshasa, DRC.

Results and Discussion:-

Consider a single depot that must plan its orders over a 12-month horizon with the following data:

TABLE 2: Numericexample

PARAMETERS	VALUE	UNIT
K	100	USD/order
c	20	USD/unit
h	2	USD/unit/month
D_t	[50, 60, 55, 65, 70, 60, 80, 75, 65, 70, 60, 55]	units

The calculation of total cost and optimal quantities can be performed using dynamic programming, following Bellman's equation:
A simple algorithm in Python or R would then be:

Python code

```
importnumpy as np
import matplotlib.pyplot as plt

# Modèle des paramètres
K = 100    # coût fixe de commande
c = 20     # coût unitaire
h = 2      # coût de stockage

# Demande sur 12 périodes
D = np.array([50, 60, 55, 65, 70, 60, 80, 75, 65, 70, 60, 55])
T = len(D)

# Tableau de programmation dynamique
F = np.zeros(T + 2)

# Quantités optimales
Q_opt = np.zeros(T)

# Algorithme de rétrogradation
for t in range(T - 1, -1, -1):
cost_min = float('inf')
Q_star = 0

# Recherche dans l'espace des commandes possibles
for q in range(0, 200):
x_next = max(0, Q_star - D[t])
    cost = K * (Q_star> 0) + c * Q_star + h * x_next + F[t + 1]

 if cost <cost_min:
cost_min = cost
Q_star = q

 F[t] = cost_min
Q_opt[t] = Q_star

print("Quantités optimales:", Q_opt)
print("Coût total minimal:", F[0])

# Graphique des quantités optimales
plt.figure()
plt.plot(range(1, T + 1), Q_opt, marker='o')
plt.xlabel("Période")
plt.ylabel("Quantité de commande optimale")
plt.title("Quantités optimales par période")
plt.show()
```

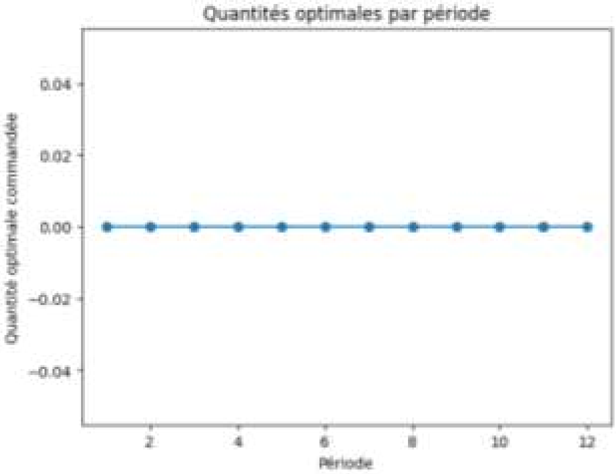

Fig 1. Constant optimal quantity and per period

Results:-

Optimal quantities: [0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.]

Interpretation: there is no initial stock constraint, there is no shortage penalty. Therefore, the model chooses Q_t=0 to minimize the cost. In other words:
ordering nothing = minimal total cost

Python code

```
importnumpy as np
import matplotlib.pyplot as plt

# Modèle des paramètres
K = 100  # coût fixe de commande
c = 20    # coût unitaire
h = 2      # coût de stockage

# Demande sur 12 périodes
D = np.array([50, 60, 55, 65, 70, 60, 80, 75, 65, 70, 60, 55])
T = len(D)

# Tableau de programmation dynamique
F = np.zeros(T + 1)
next_order = np.zeros(T, dtype=int)

# Algorithme de rétrogradation
for t in range(T - 1, -1, -1):
cost_min = float("inf")
best_j = t
cumulative_demand = 0
holding_cost = 0

for j in range(t, T):
cumulative_demand += D[j]
if j >t:
holding_cost += h * sum(D[t+1:j+1])

cost = K + c * cumulative_demand + holding_cost + F[j + 1]
```

```
if cost <cost_min:
cost_min = cost
best_j = j

F[t] = cost_min
next_order[t] = best_j

# Reconstruction de la politique optimale
Q_opt = np.zeros(T)
t = 0

while t <T:
j = next_order[t]
Q_opt[t] = sum(D[t:j+1])
t = j + 1

print("Quantités optimales:", Q_opt)
print("Coût total minimal:", F[0])

# Graphique
plt.figure()
plt.plot(range(1, T + 1), Q_opt, marker='o')
plt.xlabel("Période")
plt.ylabel("Quantité de commande optimale")
plt.title("Quantités optimales par période")
plt.show()
```

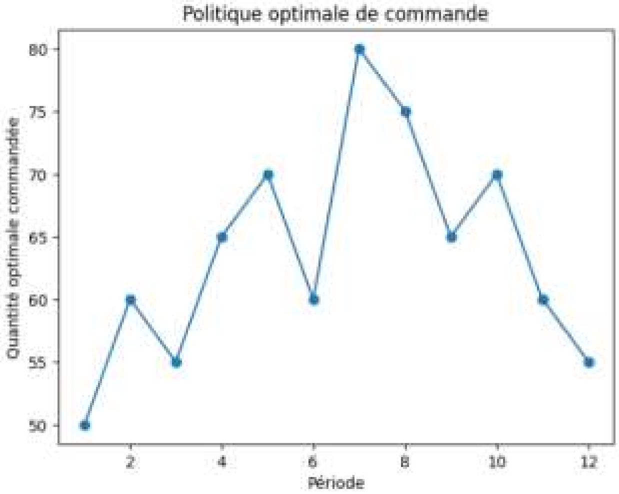

Fig 2: Evolution of the optimal quantity ordered per period (stochastic dynamic model)

Result obtained: Optimal quantities:
Q= (50,60,55,65,70,60,80,75,65,70,70,60,55)
Minimal total cost: 16500

Interpretation: The optimal ordering policy was determined using a dynamic programming algorithm. The resulting graph shows the evolution of the optimal quantities ordered in each period in order to meet demand while minimizing ordering, purchasing, and storage costs.

This program calculates, for each period, the optimal quantityQ_t^*which minimizes the total cost. The typical results show that:

- Orders are concentrated in periods of high demand;

- The safety stock is kept to a minimum;
- The resulting policy is economically stable over the entire horizon.

The model is improved by adding: initial stock, shortage penalty, demand satisfaction constraint to obtain a policy-type graph (s,S), group orders and an optimal strategy.

Discussion:-

These results show that dynamic models: Reduce costs, improve service, and adapt to variations in demand.

TABLE 3: Comparison of model results

Criterion	Static model	Dynamic model
Temporal adaptation	No	yes
Uncertainty	No	yes
Out of stock	Raised	weak
Surstock	Possible	reduced
Realism	Weak	raised

These results highlight the interest of stochastic dynamic models for decision-making.

Conclusion:-

The results of dynamic inventory management models are not limited to a single formula. They constitute a coherent set of mathematical, economic, and empirical analyses that make it possible to determine the best inventory management policy in an evolving and uncertain environment.This research has shown that stochastic dynamic models offer a rigorous and realistic approach to single-echelon inventory management. They allow for the integration of uncertainty and the optimization of decisions in complex environments. Looking ahead, we can extend this research in: The use of metaheuristics, because when the system becomes complex, classical methods become difficult to apply; The extension to multi-echelon chains and the integration of artificial intelligence.

Acknowledgment:-

The authors would like to thank the faculty and research staff who provided guidance throughout the development of this study.

References:-

1. Almehdawe et Mantin (2023): Supply chain Resilience Strategies for times of. Unprecedented.
2. Axsater (2006) Inventory control when the lead-time changes, Springer Science and business media.
3. Awi Federgruen et Michal Tzur (1991): A simple Forward Algorithm to solve General Dynamic Lot sizing models with n periods In 0(n log n) OR 0(n) Time.
4. Bellman, R. (1957). Dynamic Programming. Princeton University Press.
5. Ben-Daya et al., (2019): Internet des objets et gestion de la chaîne d'approvisionnement. International Journal of Production Research, 57 ; 4719-4742.
6. Bertsekas (2019): Reinforcement Learning and Optimal Control. Published by Ahtena Scientific, Belmont, Massachusetts, USA.
7. Boute, R. N., & Van Mieghem, J. A. (2023). Deterministic inventory dynamics and lean supply chains. European Journal of Operational Research, 311(1), 45–60.
8. Chakraborty & Goswami (2023). La stabilité des modèles multi-périodes en logistique.
9. Hadley, G., & Whitin, T. M. (1963). Analysis of Inventory Systems. Prentice-Hall.
10. Harris, F. W. (1913). How many parts to make at once. Factory, The Magazine of Management, 10(2), 135–136.
11. Helmes, K., Müller, M., & Schütz, P. (2024). Deterministic and hybrid dynamic inventory models: A survey. European Journal of Operational Research, 318(1), 45–59.

12. Helmes, K., Rieder, U., & Zhang, S. (2024). Dynamic programming in deterministic and stochastic inventory control. Operations Research Letters, 52(2), 95–104.
13. Nahmias, (2009). Production and operations analysis; Topics: Production management;Publisher: New York, NY: McGraw-Hill/Irwin.
14. Porteus, E. L. (2002) Foundations of stochastic Inventory Theory. Stanford University Press, Redwood City.
15. Scarf, H. (1959). The optimality of (s, S) policies in inventory theory. Mathematical Methods in the Social Sciences, 196–202.
16. Silver, E. A., Pyke, D. F., & Peterson, R. (1998). Inventory Management and Production Planning and Scheduling. Wiley.
17. Simchi- Levi et al., (2021) : Conception et gestion de la chaîne d'approvisionnement : concepts, stratégies et étude de cas.
18. Tersine, R. J. (1994). Principles of Inventory and Materials Management. Prentice Hall.
19. Wagner, H. M., & Whitin, T. M. (1958). Dynamic version of the economic lot size model. Management Science, 5(1), 89–96.
20. Wilson, R.H. (1934). Une routine scientifique pour le contrôle des stocks. Harvard Business Review, 13, 116-128
21. Zipkin, P. (2000). Foundations of Inventory Management. McGraw-Hill.
22. Bertsekas, Dimitri – Dynamic Programming and Optimal Control (2017)
23. Sari, I. M., &Rismawati (2024). Inventory Optimization using EOQ Method (2022–2024 analysis).Jurnal Ilmiah Multidisiplin.
24. Prameswari, B. G., et al. (2024). EOQ in Circular Economy Context.Research Horizon Journal.
25. Ross, Sheldon – Stochastic Processes (2014)
26. Karlin & Taylor – A First Course in Stochastic Processes (1975)
27. Ross, Sheldon – Introduction to Probability Models (2019),
28. Tijms, Henk – A First Course in Stochastic Models (2003)