



RESEARCH ARTICLE

STUDY OF THE IMPACT OF HEAT EXCHANGE ON THE DYNAMIC RESPONSE OF A FIBER-PLASTER COMPOSITE UNDER TRANSIENT CONDITIONS

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Abstract

The Cartesian coordinate study of a material forming a simple wall made of tow-plaster. The hemp-plaster thermal insulation is characterized in transient dynamic regimes by its heat transfer coefficients. The wall is 5 cm thick and composed of the hemp-plaster thermal insulation material. This article is based on determining the different profiles of temperature and heat flux density as a function of depth and time, along with the heat transfer coefficient at the front face of the material.

Nomenclature:

λ : Thermal conductivity ($\text{W.m}^{-1}.\text{°C}^{-1}$)

α : Thermal diffusivity coefficient ($\text{m}^2.\text{s}^{-1}$)

ϕ : Heat flux density (W.m^{-2})

L : Length of the material (m)

h_1 : Heat transfer coefficient at the front face ($\text{W.m}^{-2}.\text{°C}^{-1}$)

h_2 : Heat transfer coefficient at the back face ($\text{W.m}^{-2}.\text{°C}^{-1}$)

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Introduction:-

Energy efficiency in buildings and the rational use of materials are major current challenges. Some studies propose optimizing the thermal insulation materials used by determining the effective thermal insulation layer [1] or the optimal insulation thickness [2]. This study falls within the framework of improving the use of local, natural products for thermal insulation [3,4].

The wall is 5 cm long and made of a hemp-plaster thermal insulation material. Hemp, a biodegradable natural product, is used as thermal insulation in combination with plaster as a binder [5]. The thermal conductivity $\lambda = 0.15 \text{ W.m}^{-1}.\text{°C}^{-1}$ and the thermal diffusivity coefficient $\alpha = 2.07 \text{ m}^2.\text{s}^{-1}$ [6,7]. Hemp-plaster is of great interest for thermal comfort and contributes to the moderate regulation of the indoor climate of dwellings due to its thermal inertia [8].

Mathematical Model:-

We will present a profile view of a gypsum-fiber brick subjected to an excitatory temperature in a transient dynamic regime.

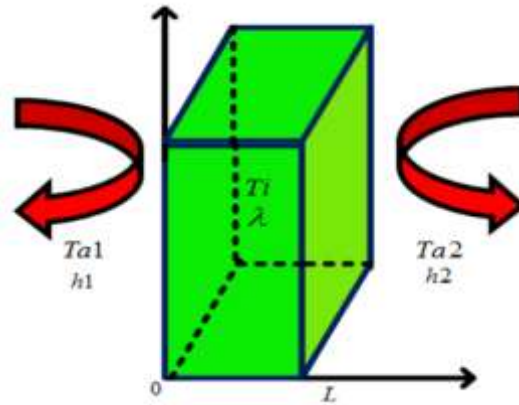


Figure 1: Sample to be studied consisting of the thermal insulation tow-plaster
 $L = 0.05\text{m}$, $T_a = 30^\circ\text{C}$ and $T_i = 10^\circ\text{C}$.

h_1 and h_2 are the heat transfer coefficients at the front and rear faces, respectively.

T_a : the ambient temperature of the material.

T_i : the initial temperature of the material.

Equation (1), which governs heat transfer in the unidirectional hemp-plaster thermal insulation without internal energy input, is given by the expression below.

$$\frac{\partial^2 T(x, h_1, h_2, t)}{\partial x^2} - \frac{1}{\alpha} \frac{\partial T(x, h_1, h_2, t)}{\partial t} = 0 \quad (1)$$

$T = T(x, h_1, h_2, t)$: is the temperature ($^\circ\text{C}$) of the material at a depth x (m) and a time t (s) with heat transfer coefficients at the front and back faces. Equation (2) gives the relationship between diffusivity α , thermal conductivity λ , density ρ , and specific heat c for the gypsum fiber.

$$\alpha = \frac{\lambda}{\rho \cdot c} \quad (2)$$

α is the thermal diffusivity coefficient ($\text{m}^2 \cdot \text{s}^{-1}$)

λ is the thermal conductivity ($\text{W} \cdot \text{m}^{-2} \cdot ^\circ\text{C}^{-1}$)

ρ is the density of the material ($\text{kg} \cdot \text{m}^{-3}$)

C is the specific heat capacity

Boundary conditions:

Equations (3) and (4) are the boundary conditions that express the conservation of heat flux at the interfaces of the tow-plaster material, and the initial condition is given by equation (5).

$$\left\{ \begin{array}{l} \lambda \frac{\partial T(x, h_1, h_2, t)}{\partial x} \Big|_{x=0} = h_1 [T(0, h_1, h_2, t) - T_a] \quad (3) \\ \lambda \frac{\partial T(x, h_1, h_2, t)}{\partial x} \Big|_{x=L} = -h_2 [T(L, h_1, h_2, t) - T_a] \quad (4) \\ T(x, h_1, h_2, t = 0) = T_i \quad (5) \end{array} \right.$$

To solve equation (1) we will perform the dimensionless decomposition of the heat equation by setting:

$$\theta(u, \tau) = \frac{T(x, h_1, h_2, t) - T_a}{T(x, h_1, h_2, t=0) - T_a} = \frac{T(x, h_1, h_2, t) - T_a}{T_i - T_a} \quad (6)$$

With $\theta(u, \tau)$: reduced temperature corresponding to the transient regime;

$u = \frac{x}{L}$; u is a reduced spatial variable

and $\tau = \frac{\alpha t}{L^2}$; $\tau = Fo$: is the reduced time variable (Fourier number).

The heat equation (1) becomes:

$$\frac{\partial^2 \theta(u, \tau)}{\partial u^2} = \frac{\partial \theta(u, \tau)}{\partial \tau} \quad (7)$$

Boundary conditions (3) and (4) become (8) and (9):

$$\left[\begin{aligned} \frac{\partial \theta(u, \tau)}{\partial \tau} \Big|_{u=0} &= \frac{h_1 L}{\lambda} \theta(0, \tau) & (8) \\ \frac{\partial \theta(u, \tau)}{\partial \tau} \Big|_{u=1} &= -\frac{h_2 L}{\lambda} \theta(1, \tau) & (9) \end{aligned} \right.$$

The boundary conditions allow us to find the transcendental equation (10):

$$\tan(\beta_n) = \frac{\frac{h_2 L}{\lambda} \beta_n + \beta_n \frac{h_1 L}{\lambda}}{\beta_n^2 - \frac{h_1 h_2 L^2}{\lambda^2}} \quad (10)$$

The transcendental equation is split into two functions:

-The trigonometric function denoted $ft(\beta_n)$

$$ft(\beta_n) = \tan(\beta_n) \quad (11)$$

-The homogeneous function denoted $fh(\beta_n)$

$$fh(\beta_n) = \frac{\frac{h_2 L}{\lambda} \beta_n + \beta_n \frac{h_1 L}{\lambda}}{\beta_n^2 - \frac{h_1 h_2 L^2}{\lambda^2}} \quad (12)$$

We plot the transcendental curve for different values of the exchange coefficient at the front and rear faces.

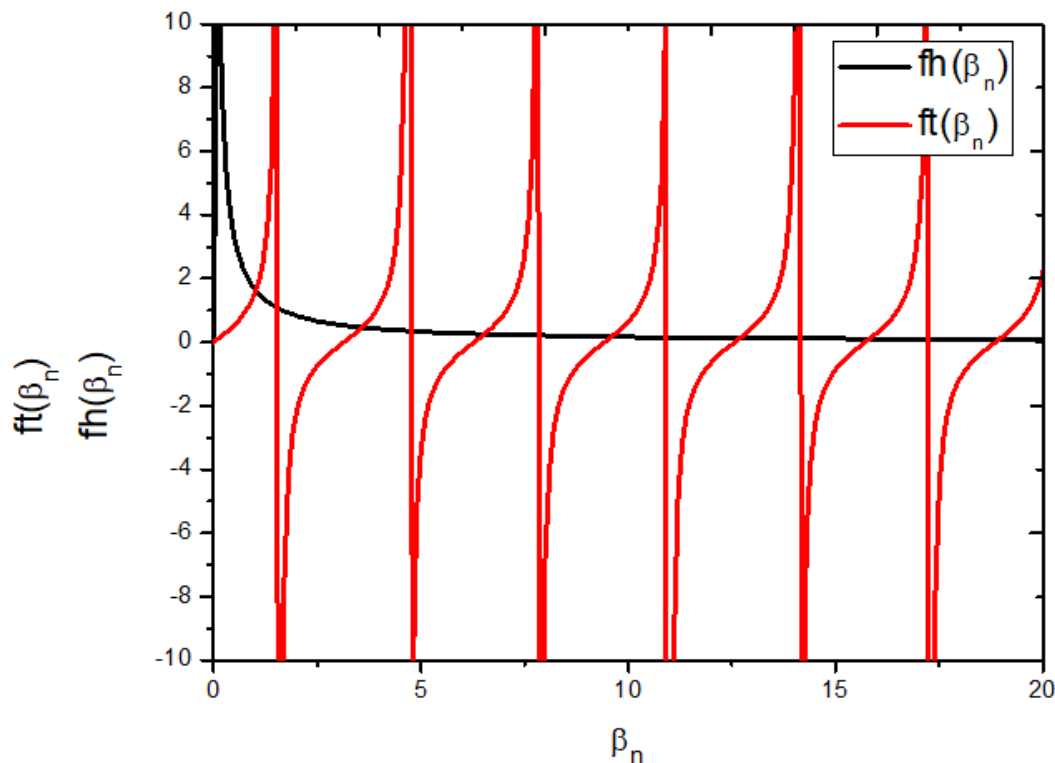


Figure 2: Graphical determination of the eigenvalues (β_n) of the transcendental equation.

In this figure, we present the evolution of $ft(\beta_n)$ and $fh(\beta_n)$ as a function of the eigenvalues of the transcendental equation for different values of the coefficients.

Table 1 summarizes the eigenvalues found for β_n .

Table 1: Eigenvalues of β_n of the equation

n	1	2	3	4	5
β_n	1.2	3.5	6.5	9.6	12.7

En utilisant les conditions aux limites et les intégrales d'Euler-Fourier. Nous avons déterminés $H_{1n}(0)$ et $H_{2n}(0)$

$$H_{1n}(0) = \frac{\sqrt{\gamma}}{(\beta_n^2 + \gamma)(\sqrt{\gamma} \cosh(\sqrt{\gamma}) + \frac{h_2 L}{\lambda} \sinh(\sqrt{\gamma}))} \left\{ (\sqrt{\gamma} \sinh(\sqrt{\gamma}) + \beta_n \sin(\beta_n)) + \frac{h_2 L}{\lambda} [\cosh(\sqrt{\gamma}) - \cos(\beta_n)] \right\} \quad (13)$$

$$H_{2n}(0) = \frac{h_1 L}{\lambda(\beta_n^2 + \gamma)(\sqrt{\gamma} \cosh(\sqrt{\gamma}) + \frac{h_2 L}{\lambda} \sinh(\sqrt{\gamma}))} \left\{ \sqrt{\gamma} [\cosh(\sqrt{\gamma}) - \sin(\beta_n)] + \frac{h_2 L}{\lambda} \left[\sinh(\sqrt{\gamma}) - \frac{\sqrt{\gamma}}{\beta_n} \cos(\beta_n) \right] \right\} \quad (14)$$

$$\theta(u, \tau) = [a_n \cos(\beta_n u) + b_n \sin(\beta_n u)] \cdot T(0) \cdot e^{-\frac{\tau}{\tau_d}}$$

We obtain the expression for the reduced temperature of the heat diffusion equation and it is given by expression (15):

$$\theta\left(\frac{x}{L}, \frac{\alpha \cdot t}{L^2}\right) = \delta \theta_0 a_n^2 \left[\cos\left(\frac{x}{L} \cdot \beta_n\right) + \frac{h_1 L}{\lambda \cdot \beta_n} \cdot \sin\left(\frac{x}{L} \cdot \beta_n\right) \right] [H_{1n}(0) + H_{2n}(0)] e^{-\frac{\alpha \cdot t}{L^2} [\beta_n^2 + \gamma]} \quad (15)$$

By combining equations (6) and (31) we obtain the expression for the final temperature:

$$T(x, h_1, h_2, t) = T_a + (T_i - T_a) \cdot \delta \theta_0 a_n^2 \cdot \left[\cos\left(\frac{x}{L} \cdot \beta_n\right) + \frac{h_1 L}{\lambda \cdot \beta_n} \cdot \sin\left(\frac{x}{L} \cdot \beta_n\right) \right] [H_{1n}(0) + H_{2n}(0)] \cdot e^{-\frac{\alpha \cdot t}{L^2} [\beta_n^2 + \gamma]} \quad (16)$$

Thermal heat flux density:-

The heat flux density [8], through the tow-gypsum material is obtained from the relation (17):

$$\Phi(x, h_1, h_2, t) = -\lambda \frac{\partial T(x, h_1, h_2, t)}{\partial x} \quad (17)$$

By differentiating the temperature with respect to the next component x, we obtain expression (18):

$$\Phi(x, h_1, h_2, t) = \lambda(T_i - T_a) \cdot \delta \theta_0 \cdot a_n^2 \cdot \left[\frac{\beta_n}{L} \cdot \sin\left(\frac{x}{L} \cdot \beta_n\right) - \frac{h_1}{\lambda} \cdot \cos\left(\frac{x}{L} \cdot \beta_n\right) \right] [H_{1n}(0) + H_{2n}(0)] e^{-\frac{\alpha_n t}{L^2} [\beta_n^2 + \gamma]} \quad (18)$$

Evolution of Temperature and Thermal Heat Flux Density as a Function of Different Parameters

Results:-

Figures 1 and 2 show the temperature and heat flux density for different values of the heat transfer coefficient at the front face, setting a very low value for h₂.

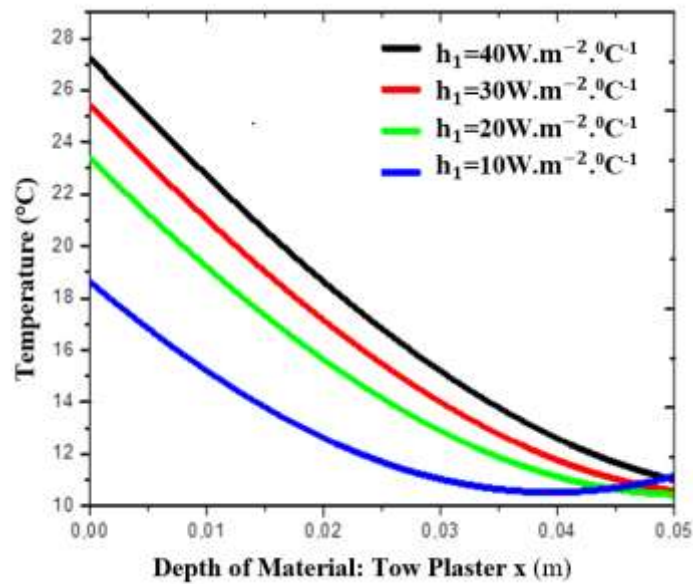


Figure 3: Temperature evolution as a function of material depth. $t = 100$ s, $h_2 = 0.005$ W.m⁻².°C⁻¹

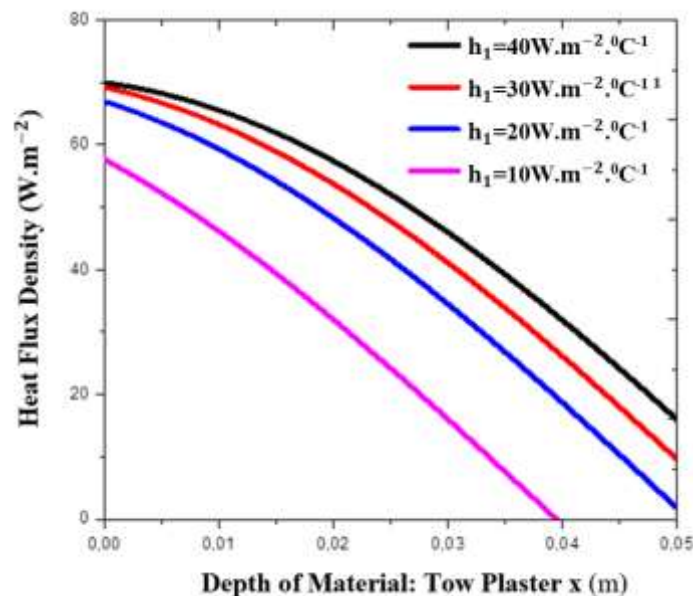


Figure 4: Evolution of heat flux density as a function of material depth. $t=100$ s, $h_2 = 0.005$ W.m⁻².°C⁻¹

In Figure 3, the temperature is maximum for $x = 0$ and begins to decrease when we move away from the front face of the material then we will reach the initial temperature T_i of the tow-plaster material when the depth becomes significant. We notice for $x = 0$ the quantity of heat exchanged at the front face is all the more important as the exchange coefficient at the front face is low. If the exchange coefficient on the front face decreases, we note a slight decrease in temperature.

The temperature curves show that the tow-gypsum insulating material tends to release heat to the interior environment. In Figure 2 we have a decrease in the heat flow density as a function of depth under the influence of the exchange coefficient on the front face. We note a decrease which is less brutal on the curve where $h_1 = 25 \text{ W.m}^{-2}.\text{°C}^{-1}$ than on the others. Low depth values correspond to high heat flux density values. On these curves we see a slight decrease in the heat flux density for low depth values.

This results in a positive gradient in the heat flux density towards the interior face.

Table 2: The different values of the temperature and heat flux density of the tow-plaster material for $t = 100\text{s}$ and $h_2 = 0.005 \text{ W.m}^{-2}.\text{°C}^{-1}$

$h_1 (\text{W.m}^{-2}.\text{s}^{-1})$	25	15	10	5
$X(\text{m})$	0.01	0.02	0.03	0.05
$T (\text{°C})$	22,7	17,1	12,9	10,6
$\Phi (\text{W.m}^{-2})$	65,6	53,7	34,4	0

For different values of the depth, we obtain figures 5 and 6. The curves of temperature and heat flux density as a function of the coefficient a at the front face h_1 .

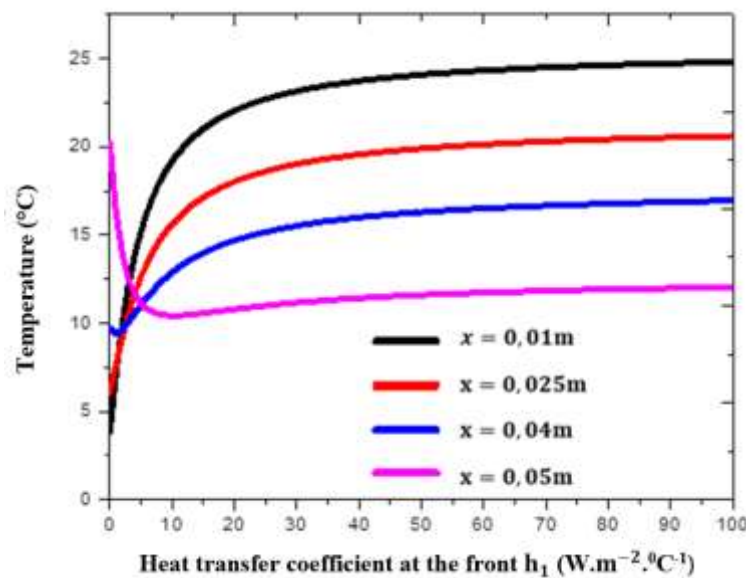


Figure 5: Temperature evolution as a function of the heat transfer coefficient a at the front face h_1 of the material. $t = 100\text{s}$, $h_2 = 0.005 \text{ W.m}^{-2}.\text{°C}^{-1}$

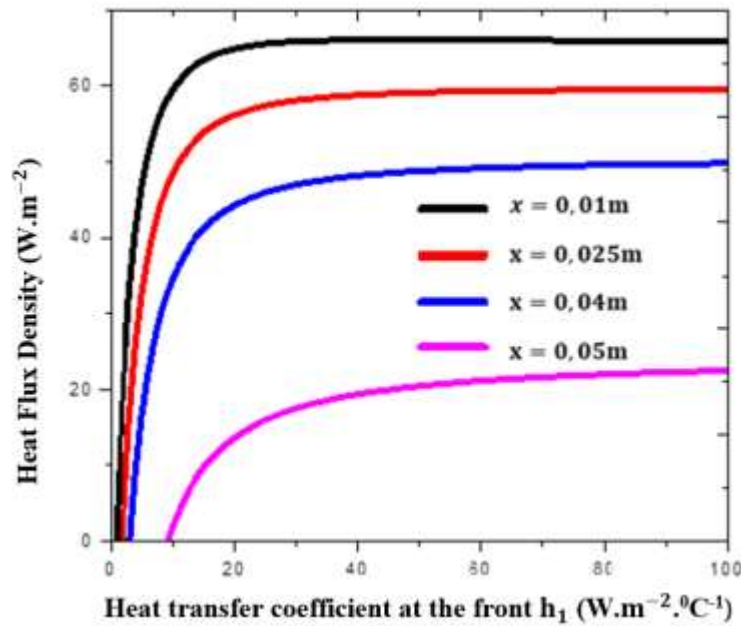


Figure 6: Evolution of the heat flux density as a function of the heat transfer coefficient a at the front face of the material h_1 . $t=100s$, $h_2 = 0.005W.m^{-2}.^{\circ}C^{-1}$

The temperature and heat flux density increase with the heat transfer coefficient at the front face, h_1 , and then become constant at high values of h_1 . Therefore, the gypsum-fiber material stores thermal energy. The temperature and heat flux density are higher when the heat transfer coefficient at the front face is greater. As the heat transfer coefficient at the front face increases, the temperature and heat flux density also increase. The temperature and heat flux density become constant beyond a certain depth.

Table 3: The different values of the temperature and heat flux density of the gypsum-fiber material for $t = 100 s$ and $h_2 = 0.005W.m^{-2}.^{\circ}C^{-1}$

$X(m)$	0.01	0.02	0.03	0.05
$h_1(W.m^{-2}.s^{-1})$	25	15	10	5
$T(^{\circ}C)$	22,7	17,1	12,9	11
$\Phi (W.m^{-2})$	65,5	53,8	34,6	0

In these figures we will plot the profiles of temperature and heat flux density as a function of time for different values of the front face heat transfer coefficient.

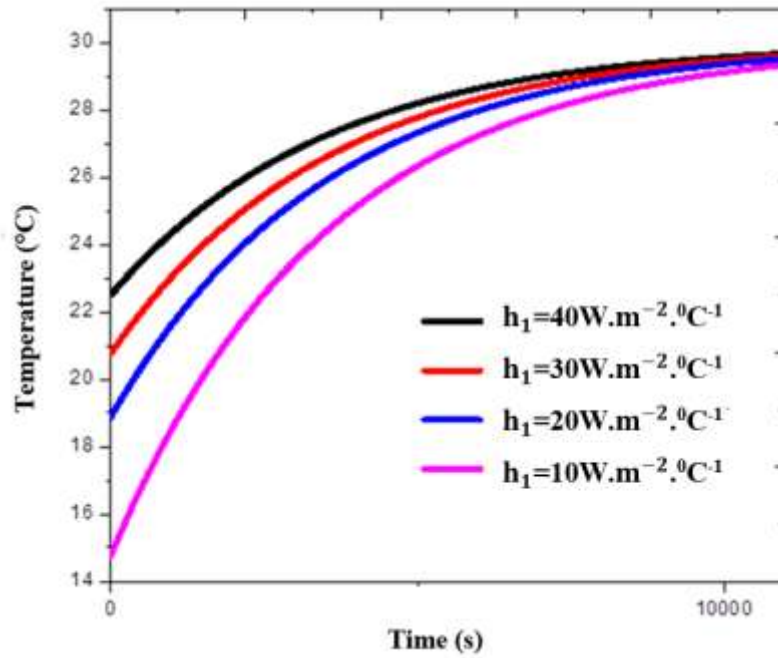


Figure 7: Temperature evolution as a function of time for the material. $t=100s$, $h_2 = 0.005 \text{ W.m}^{-2}.\text{°C}^{-1}$

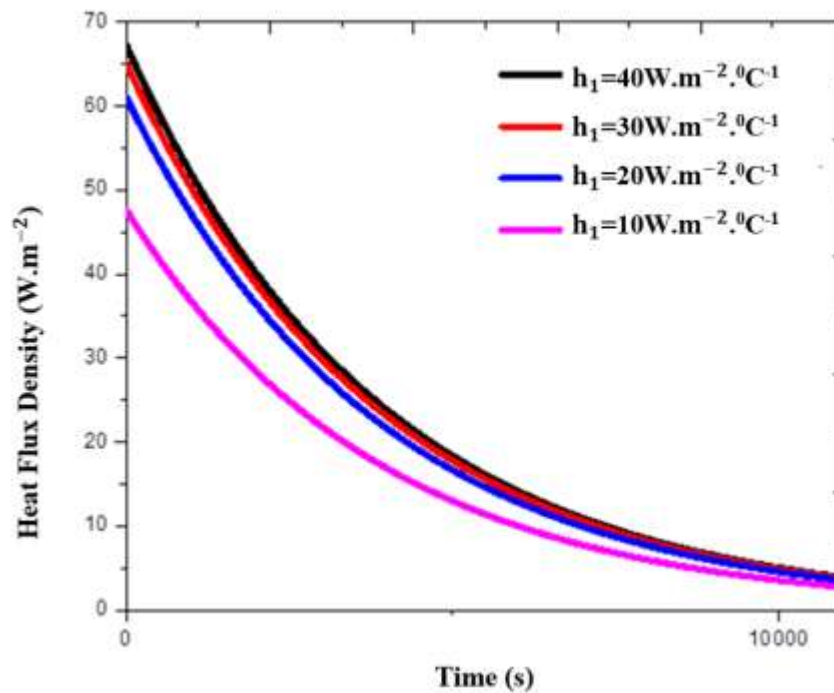


Figure 8: Evolution of heat flux density as a function of time. $t=100s$, $h_2 = 0.005 \text{ W.m}^{-2}.\text{°C}^{-1}$

In these two figures, we have plotted the temperature and heat flux density as a function of time under the influence of the heat transfer coefficient at the front surface. We observe an increase and a decrease, respectively, in the temperature and heat flux density over time. For high temperature values (Figure 7), we observe that each point in the gypsum-fiber material reaches a plateau. This increase occurs within the material. This is due to heat storage and the phenomenon of heat retention within the gypsum-fiber material.

In Figure 8, the heat flux density decreases over time for different values of the heat transfer coefficient at the front surface. This is due to the absorption and transfer of heat within the material, which leads to an increase in the heat transfer coefficient at the front surface. We observe a slight decrease in density. For each point in the material, the heat flux density is higher when the heat transfer coefficient is higher. When the heat transfer coefficient at the front face increases, the heat flux density decreases at a point deep within the material. This decrease in heat flux density is due to a gradual loss of heat.

Conclusion:-

The study shows that the thermal behavior or response of the material is highly dependent on the timing of external climatic stresses. The study of heat transfer in a simple wall highlighted the influence of the heat transfer coefficient at the front face of the gypsum-fiber material on temperature and heat flux density under transient dynamic conditions. Gypsum-fiber plays an important role in thermal insulation and in maintaining air quality and thermal comfort.

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