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RESEARCH ARTICLE

NUMERICAL SIMULATION OF THE INFLUENCE OF THE EFFECT OF TOOTH NUMBER ON THE MECHANICAL BEHAVIOR OF NATURAL FIBER COMPOSITE GEARS: THE CASE OF HDPE 40B (HDPE 40B: HIGH DENSITY POLYETHYLENE WITH 40% BIRCH FIBER)

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Abstract

The purpose of this study is to analyze stresses and deformations using a numerical simulation of the effect of the tooth number on gears made from a new composite material, HDPE40B. After presenting the methodology used to obtain the gears 3D model, up to the application of loads to determine stresses and deformations, the influence of tooth geometric characteristics and operating conditions on the distribution of stresses and deformations in these gears induced by the loads, was studied as a function of gear tooth wear. The main observation in this study is that increasing the number of teeth leads to a reduction in stress due to the decrease in the normal loads carried by the gear teeth.

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Introduction:-

The field of study of the materials is very wide and deals as well with the metals and alloys, the ceramics as on the new materials like the polymers and composites. The Laboratory of Mechanics and Eco-Materials (LMEM) of the University of Quebec à Trois-Rivières (UQTR) has developed plastic composite eco-materials reinforced by wood fibers for various applications. These composite materials are based on polyethylene and polypropylene of plant origin, reinforced by wood fibers such as aspen and birch. Studies [1] have shown that the most efficient of these materials for the manufacture of gears is the HDPE40B which is the subject of the present study. Although having some common models for their characterization, each composite material, even consisting of the same matrix and the same reinforcement, unlike their constitution rate and additive elements, has its own characteristics, so that the standards composites do not present generalized models as for metals. In addition, for certain specific applications, the existing standards are not complete as they do not cover all the aspects concerned, as is the case with gears made of plastic materials and polymer composites [2]. There is therefore a need for studies and experiments on these materials to develop models of their own for the characterization of their different properties. Moreover, the operating conditions of these gears can be at the origin of certain damages such as the wear of the active surfaces. Statistics show that, in gear-driven power transmission systems, the tooth concentrates more than 60% of the defects

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recorded [3]. It is in this context that the study of the influence of the geometrical characteristics of the teeth and the operating conditions on the distribution of the stresses and deformations induced by the loads in the gears as a function of the wear are proposed. More particularly in this study the influence of the number of teeth on stresses and deformations while varying wear will be studied in this paper.

Methodology:-

Modeling of teeth:

Before moving on to the modeling of normal and worn teeth, some characteristics of the gear will be shown (Figure 1). This study is limited to straight cylindrical gears with involute profile of a circle connected by the trochoid.

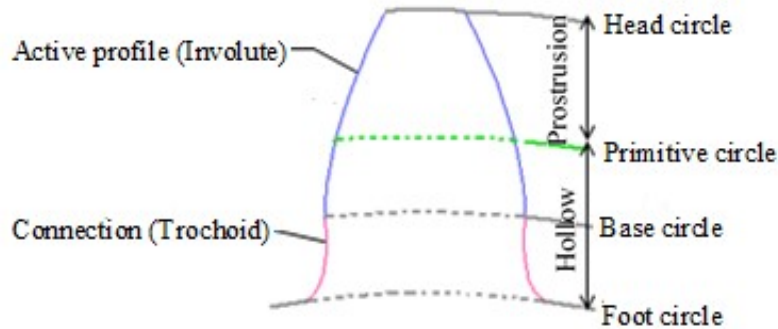


Figure 1. Characteristic of a gear tooth

Head radius: $R_a = R + m$

Primitive radius: $R = \frac{mZ}{2}$

Base radius: $R_b = R \cos \alpha$

Foot radius: $R_p = R - 1,25m$

Where m = Module;

Z = Number of teeth;

α = Pressure angle

The start and end points of contact between the teeth define the mesh of the wheel and pinion. The tooth of the driving wheel comes into contact at A with the top of the driven wheel (start of meshing) and leaves it at B (end of meshing). (Figure 2)

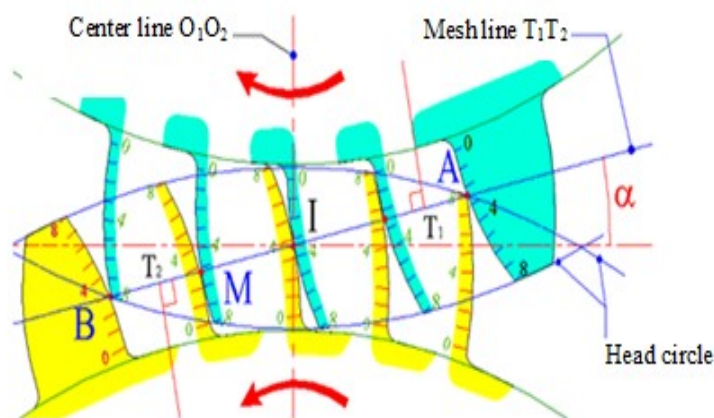


Figure 1. Mesh line (AB)

As shown in figure 3, the point of contact, at a given instant, is identified by its algebraic position S on the line of action (T_1, T_2). The point I corresponding to the primitive point on the profile of the tooth is at S equal to zero. Between points I (the primitive point) and A (the starting point of contact) defining the approach phase, S is negative and between points I and B (the end of contact point) defining the retreat phase, S is positive.

$\frac{S}{p_n}$ is the normalized position, conventionally used in gear studies.

p_n is the basic normal pitch: $p_n = \pi m \cos \alpha$

α : Pressure angle

m : Module

So according to Figure 3, the equations allowing us to obtain the points on the segment have been developed [AB] (line of action). Point I correspond to S equal to zero; between segment [IA] S is negative and between segment [IB] S is positive.

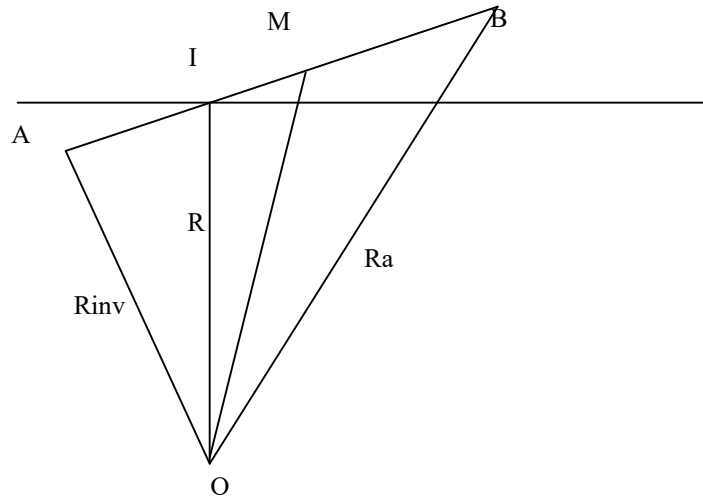


Figure 2 : Calculation of the abscissa of the meshing points

Considering the OBI triangle:

$$\frac{\sin \hat{I}}{Ra} = \frac{\sin \hat{O}}{IB} = \frac{\sin \hat{B}}{R}$$

$$\arcsin \left(\frac{\sin \hat{I}}{Ra} R \right) = \hat{B}$$

with Ra the head radius, R the pitch radius and the angle at I which is equal to $\hat{I} = \frac{\pi}{2} + \alpha$, with α the pressure angle.

$\hat{O} = \pi - (\hat{I} + \hat{B})$ which allows us to obtain the segment [IB],

$$IB = \frac{\sin \hat{O}}{\sin \hat{I}} Ra, [IB] \text{ is the retreat distance.}$$

Thus, for radius going from the head to the clearance radius, for a point M of radius R_i , its abscissa S on the action line which corresponds to the segment [IM] is found, I being the origin.

With R_i the radius included between the head and clearance radius.

Modelling:

For the modeling of straight cylindrical gears before and after wear in SOLIDWORKS, the method of guiding curves has been adopted. This method makes it possible to plot the curves from precise coordinates obtained in a text file provided by the VBA computer programming software. Indeed, the SOLIDWORKS software considers the macros from VBA, can read a file of points that it can represent with much precision in a form of curve. After reading all the curves of the tooth by the same method, the representation of the latter consisting of the two main

profiles which are the involute of circle and the trochoid, is obtain precisely. To pass from this two-dimensional curve to the representation of the tooth in three dimensions, it is sufficient to make an extrusion of the guide curve. Figure 4, shows the tooth profiles obtained after a uniform wear of 2%, 4%, 6% and 8%. Here the wear is simulated and assumed to be uniform to simplify calculations. To obtain the worn profile, the coordinates of the normal profile are reduced according to the desired wear rate. This is done using a program in Visual Basic. And the profile of the tooth is drawn. [4] The SOLIDWORKS software has a library of materials ranging from metals to plastics and ceramics. However, the material that is the subject of this study is a new material not listed in the SOLIDWORKS material library. As a reminder, it is the composite with a thermoplastic matrix (high density polyethylene) reinforced with 40% of birch wood fibers called HDPE 40B. Therefore, this new material will be integrated into the library of materials. The Table 1 below shows the full characteristics of the HDPE 40B material implemented in the software.

Table 1. Characteristics of HDPE 40B in the SOLIDWORKS Materials Library

Property	Value	Unity
elasticity module	3450	N/mm ²
Poisson coefficient	0.33	
shear modulus	1.85	N/mm ²
Volumic mass	1185.6	kg/m ³
tensile limit	38.5	N/mm ²
coefficient of thermal expansion	0.00028	/°K
thermal conductivity	0.787	W/(m.°K)
specific heat	1325.1	J/(kg.°K)
material damping ratio	0.06	

The calculation of the forces in thermoplastic-based composite gearing differs from that of conventional metal gears since the meshing of these types of gears has certain particularities. Indeed, there is an extension of the contact between the teeth more at least outside the line of action, before and after the theoretical start and end of the meshing [5] [6]. Thus, the significant deformation of the tooth during the meshing tends to relieve each pair of teeth in contact since the transmitted load is then distributed better [7]. This particular behavior of thermoplastic gears has already been studied by authors who studied the magnitudes which characterize it in his previous studies [8,9]. In a practical approach, a simplified model for the calculation of the load distribution factor have been proposed [8]:

For plastic / plastic mesh:

$$\frac{W_i}{W_n} \left(\frac{S}{P_n} \right) = \frac{W_i}{W_n} \Big|_0 \cos \left(\frac{\pi S/P_n}{2 S_2''/P_n} \right) \quad (1)$$

with:

$$\frac{W_i}{W_n} \Big|_0 = 0.48 E_2^{-0.28} (W_0 P \cos \alpha)^{-0.22} Z_2^{-0.4} \left(\frac{Z_2}{Z_1} \right)^{0.1} \quad (2)$$

Where $\frac{W_i}{W_n} \Big|_0$ is the value of the load distribution factor at the primitive point ($s=0$).

$$\frac{s}{p_n} = \frac{\sin \left(\frac{\pi}{2} - \alpha - \sin^{-1} \left(\cos \alpha \frac{R}{R_i} \right) \right) \sqrt{x^2 + y^2}}{\pi m \cos \alpha \cos \alpha} \quad (3)$$

where:

α : Pression angle

R : Primitive radius

Ra : Head radius

Ri : Radius of any point taken on the profile with coordinate (x,y)

x, y : Coordinates of any point on the tooth profile

m : Module of the tooth

P : Circular step

$\frac{s}{p_n}$: Normalized position

p_n : Basic normal pitch: $p_n = \pi m \cos \alpha$

Z_1, Z_2 : Number of teeth of wheel 1 and wheel 2

S_2''/P_n and S_1''/P_n : normalized positions on the action line of the start and end of real contact.

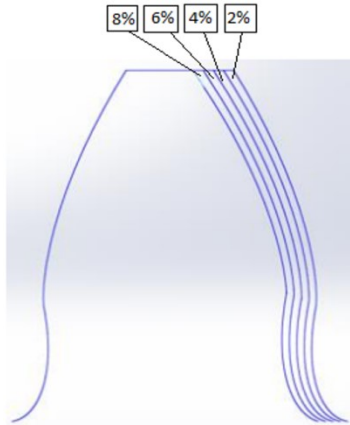


Figure 3. Teeth profile wear of 2%, 4%, 6%, et 8%

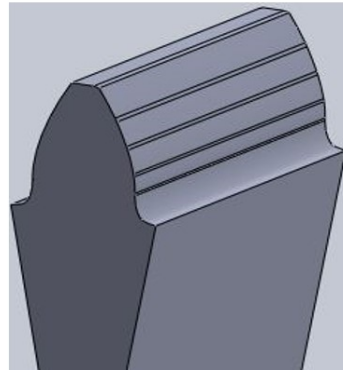


Figure 5: Areas of application of forces

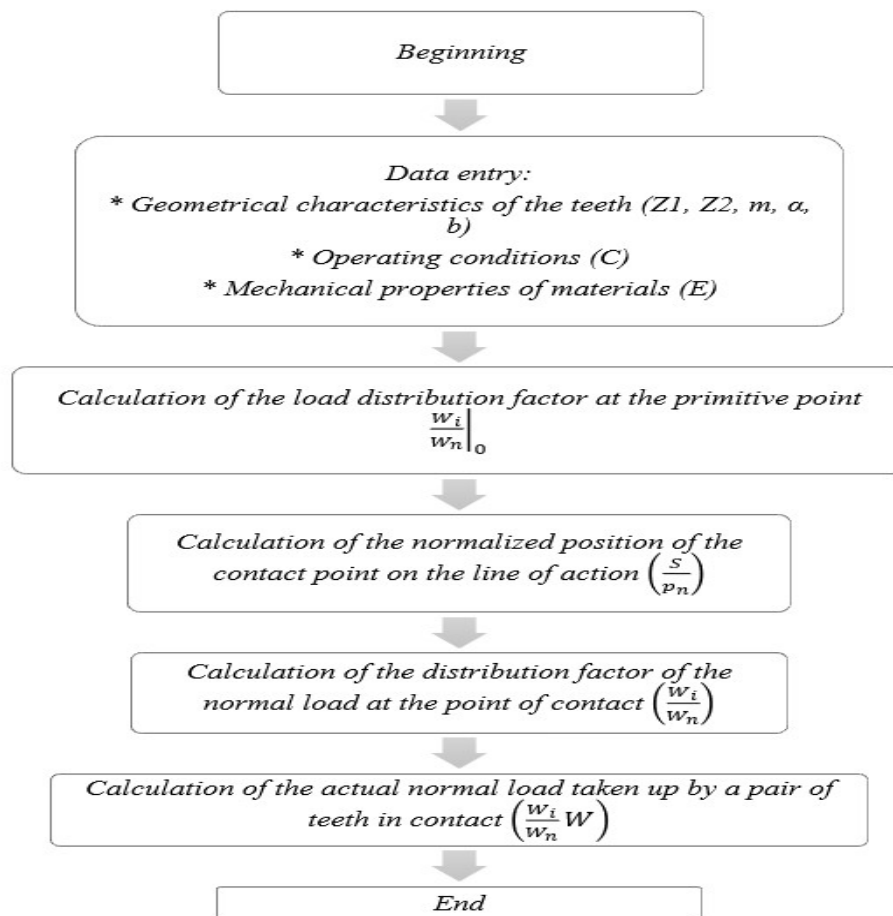


Figure 6: Force calculation chart

The calculation of the forces on the profile of a gear tooth passes through the various steps listed in the above flowchart (figure.6): The different cases of contact forces that exist are point, linear or surface force. The version of SOLIDWORKS used in our study only allows the application of surface force. So small areas of a few tenths of a square millimeter were created to apply these forces. They were created small so as not to deviate too much from the real conditions (figure 5). Five contact zones on the tooth profile where the forces will be applied have been selected. With regard to the imposed displacements, this involves embedding on the lower surface of the tooth as shown in figure 7. Meshing is the process by which the elements and nodes of a field of study are defined. This

mesh under SOLIDWORKS is automatic once the meshing parameters are introduced. For our case study, Figure 8 shows an example of a mesh that the software proposes to us. Once the mesh is executed the calculation of stresses and deformations is launched (Figure 9). To study the influence of the module, all other parameters are fixed and the module varied by 3, 5 and 8 mm; and this for different values of wear rate (0%, 4%, 8%). Experience has shown that when the wear rate reaches 12%, the tooth deteriorates, then the wear rates of 0%, 4% and 8% are chosen to remain within allowable limits. For this study, we fixed the module, the pressure angle, and the torque, and varied the number of gear teeth.

The fixed parameters are:

- Module: 5 mm (Intermediate between the modules used: 3, 5, 8)
- Pressure angle: 20° (The most commonly used angle in the ISO system)
- Torque: 10 Nm (Intermediate between the torques used: 5, 10, 15)

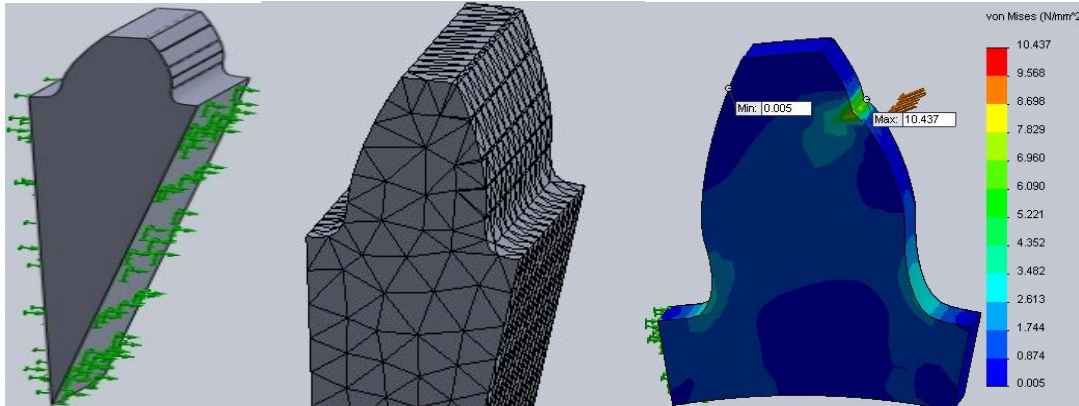


Figure 7. Tooth boundary

Figure 8. Meshing

Figure 9. Von Mises stress

Results:-

Figure 10 shows the effect of the number of teeth on contact stresses, and Figure 11 the effect on tooth root bending stresses.

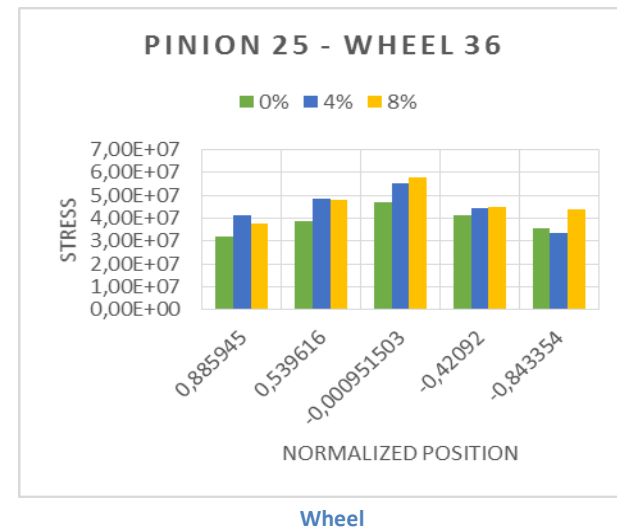
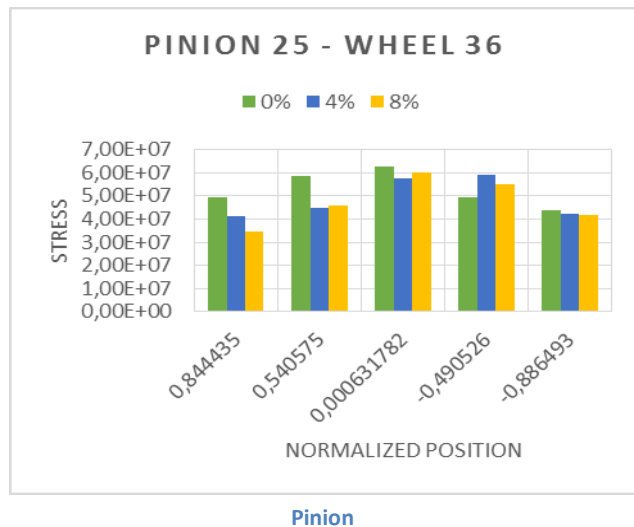
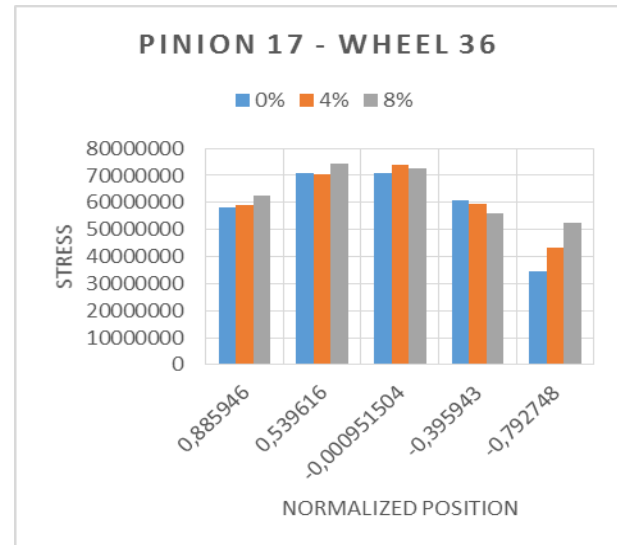
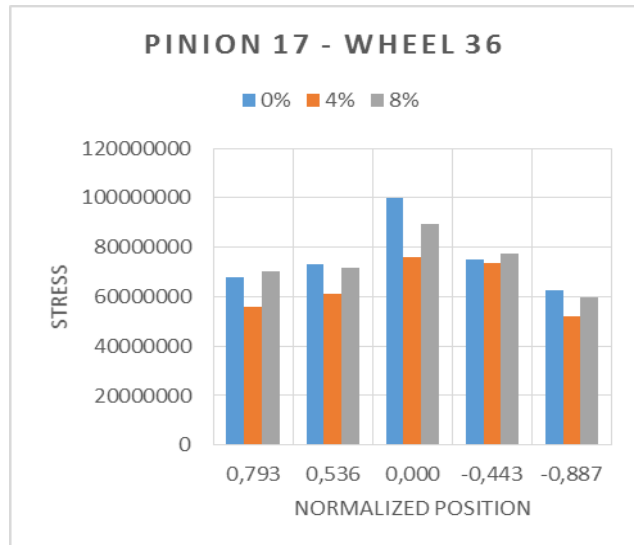
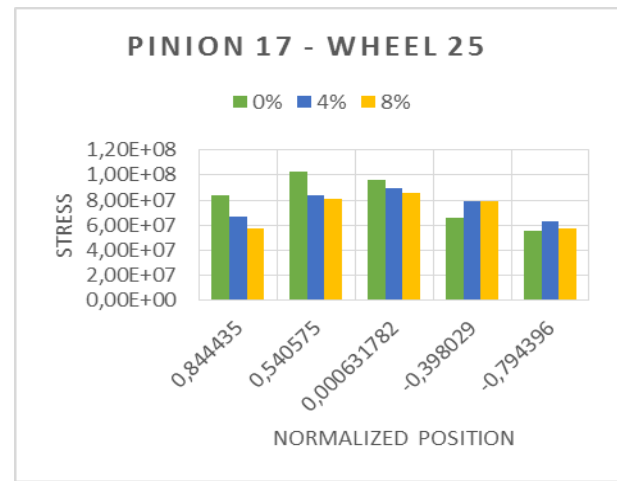
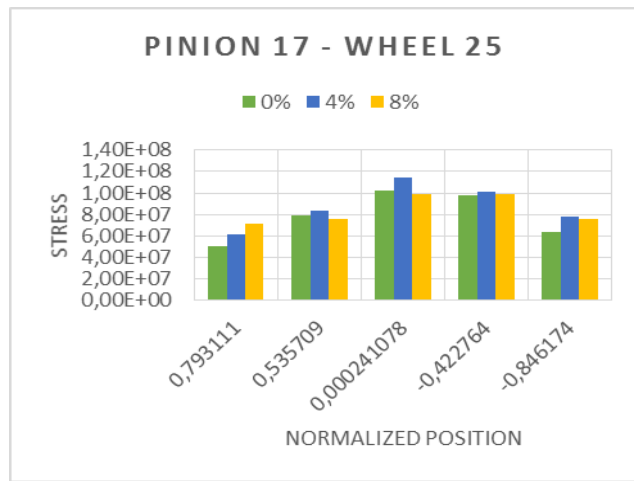


Figure 10: Contact stresses



Figure 11: Bending stresses

The increase in the number of teeth (Figure 10, where the wheel goes from 25 to 36 teeth) leads to a reduction in contact stress, owing to the decrease in the normal loads carried by the gear teeth, as shown by Equation (2) for the calculation of the normal load distribution factor at the pitch point, where Z_2 is raised to the power of -0.4. The increase in the contact ratio (as illustrated by Equation (4) for the calculation of the normalized deviation, where Z_2 is raised to the power of 0.7) also contributes to this stress reduction. Indeed, increasing the number of teeth on the wheels increases the gear ratio and consequently reduces the actual loads carried by each pair of teeth in contact, hence the decrease in contact stresses. The same observation applies (Figure 10), where the pinion goes from 17 to 25 teeth. The bending stresses at the tooth root, presented in Figure 11, increase as the wear rate increases. Indeed, as a result of the material loss caused by wear, the tooth becomes thinner, which in turn leads to higher stresses. Moreover, it is worth noting that bending stresses generally decrease as the number of teeth increases, as shown in Figure 11, where the number of teeth of the wheel goes from 25 to 36, for the same reasons as those given above. The same is observed in, where the number of teeth of the pinion goes from 17 to 25.

$$\frac{\delta S}{p_n} = 0.131 E_2^{-0.34} \left(Z_2 \sqrt{W_0 P \cos \theta} \right)^{0.7} \left(\frac{Z_2}{Z_1} \right)^{-0.55} \quad (4)$$

Conclusion:-

The aim of this study was to see how variation in number of teeth and wear rate acts on stress and deformation when gears are in operation in order to predict the effect of number of teeth and allowable wear. The study of the influence of number of teeth on the stresses and deformations for the pinion as well as for the wheel, generally indicate that as the number of teeth increases, the contact stresses and the bending stresses at the root of the tooth decrease, because increasing the number of teeth leads to a reduction in stress due to the decrease in the normal loads carried by the gear teeth. The stresses have the same shape for the pinion as well as for the wheel. The deformations have the same paces as the stresses which induced them in all cases, given that the deformations are proportional to the stresses. This study is only a part of the study of the influence of the geometrical characteristics of the gears and of the operating conditions on the distribution of stresses and deformations induced by the loads in the gears as a function of wear. It is envisaged the mathematical characterization of stresses as a function of torque, modules, pressure angle, number of teeth, tooth width and wear rate to encompass all the parameters influencing the stresses in a single formula.

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